work sheet - Chapter 1 :

(1) Let A, B, C and D be matrices where

$$(A+BA)^t + C^tA^t = D$$

if $size(D) = 10 \times 5$, find $size(A)$, $size(B)$ and $size(C)$?

Solution

we have $\quad (A+BA)^t + C^tA^t = \underbrace{D}_{10\times 5}$.

So, $size(A+BA)^t = 10 \times 5$

$\Rightarrow size(A+BA) = 5 \times 10$

$size(A) = 5 \times 10$ $\qquad$ $size(\underbrace{BA}_{5\times 10}) = 5 \times 10$

$\Rightarrow size(B) = 5 \times 5$

$size(\underbrace{C^tA^t}_{10\times 5}) = 10 \times 5$

$\Rightarrow size\ C^t = 10 \times 10$

$\Rightarrow \underline{size(C) = 10 \times 10}$ ∎

(2) Let A, B and I be matrices of $M_3(\mathbb{R})$. Find

$$tr(AB - (B^tA^t + I)^t)$$

Solution

$$\begin{aligned} AB - (B^tA^t + I)^t &= AB - (B^tA^t)^t + I_3^t \\ &= AB - AB + I_3 \\ &= I_3 \end{aligned}$$

$$\therefore tr(AB - (B^tA^t + I)^t) = tr(I_3) = 1+1+1 = 3$$ ∎

(3) If A is a square matrix such that $A^2 = A$ then $(I+A)^2 - 3A = I$, Prove that ?

Solution $L.H.S = (I+A)^2 - 3A$
$= I^2 + IA + AI + A^2 - 3A$
$= I^2 + A + A + A^2 - 3A$
$= I = R.H.S$

~~(4) If AB = BA = I~~

(4) Complete the sentence :
The main diagnal elements of a skew-symmetric matrix are all zeros

why ?
Let A be a skew-symmetric. Then
$\boxed{a_{ij} = -a_{ji}}$.

So, $a_{ii} = -a_{ii}$ (Elements of the main diagnal)
$\Rightarrow 2a_{ii} = 0$
$\Rightarrow a_{ii} = 0$

(5) Let A be matrix where A is both symmetric and skew-symmetric. Find A ?

Solution : $A = A^t = -A^t$
$\Rightarrow a_{ij} = \underbrace{a_{ji} = -a_{ji}}$.
$\Rightarrow$ $a_{ji} = 0$
$\Rightarrow A$ is a zero-matrix

(6) Simplify :

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{\cos^2\theta} & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \boxed{\sin^2\theta} & -\sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \quad \left(\text{Notice that } \cos^2\theta + \sin^2\theta = 1\right)$$

(7) Let $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + x^4 + x^8 + x^{16}$

find $f(A)$?

Solution $f(A) = I + A + A^2 + A^4 + A^8 + A^{16}$

$$A^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$A^4 = A^2 A^2 = 0$

$A^8 = 0$

$A^{16} = 0$

So, $f(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

(8) Find a matrix B such that

$$2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} + B = 0$$

Solution : (Try to solve it !!)