ME 476
Solar Energy

UNIT FIVE
THERMAL ANALYSIS OF PARABOLIC TROUGH COLLECTORS

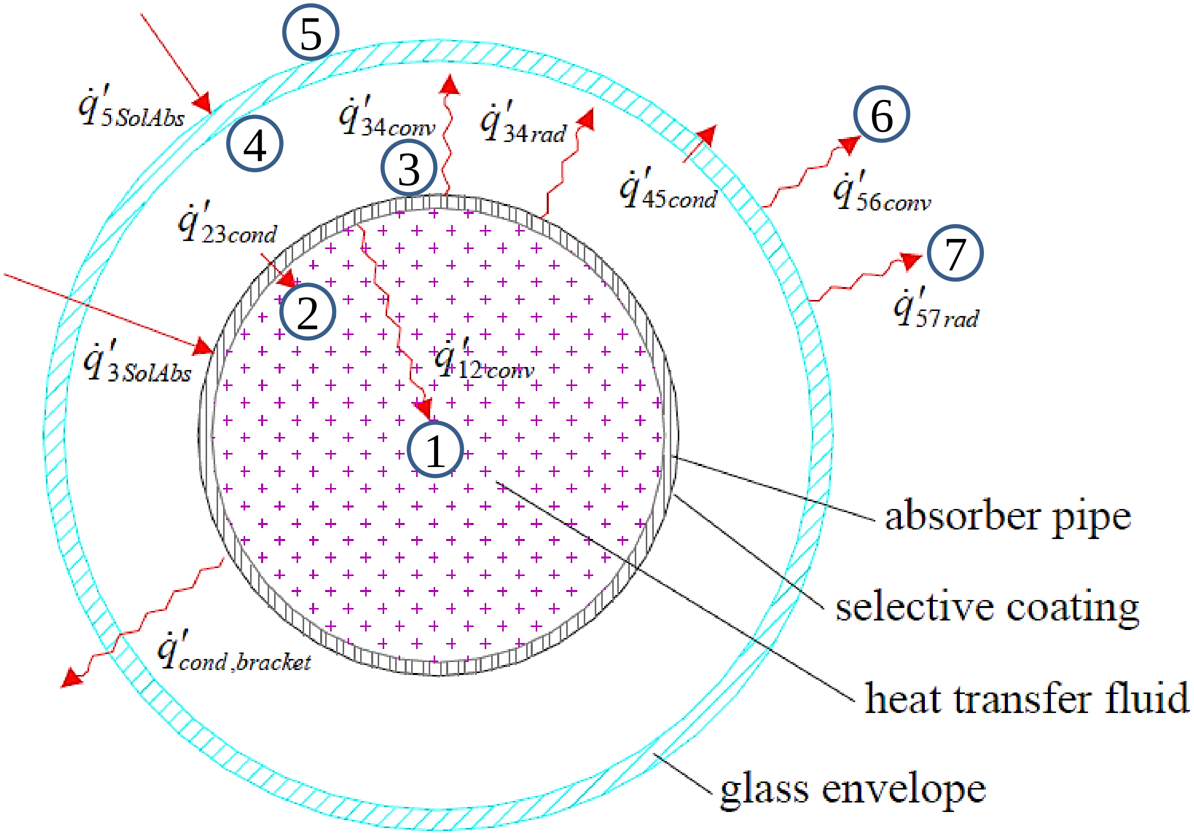
Thermal Analysis of PTCs

- The objective of this unit is to formulate a simplified analysis of the useful energy gain of a parabolic trough collector.
- This analysis will be only one-dimensional.
- It will look at the heat loss and energy gain in the radial direction only.
    - **The temperature of the fluid will be assumed to be constant along the absorber tube.**
- This analysis is suitable for a short section of a parabolic trough receiver.

Thermal Analysis of PTCs

ENERGY GAIN AND LOSS MECHANISMS

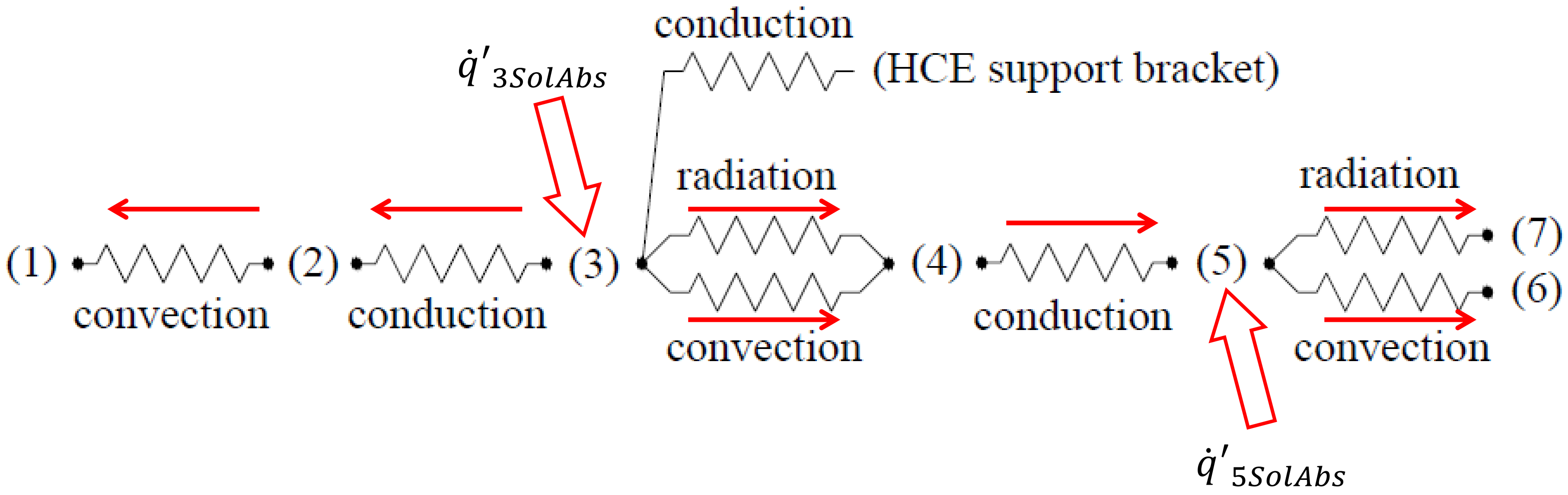
(1) heat transfer fluid
(2) absorber inner surface
(3) absorber outer surface
(4) glass envelope inner surface
(5) glass envelope outer surface
(6) surrounding air
(7) sky



- **The prime symbol (’) denotes “per unit length”**

Thermal Analysis of PTCs

THERMAL RESISTANCE MODEL



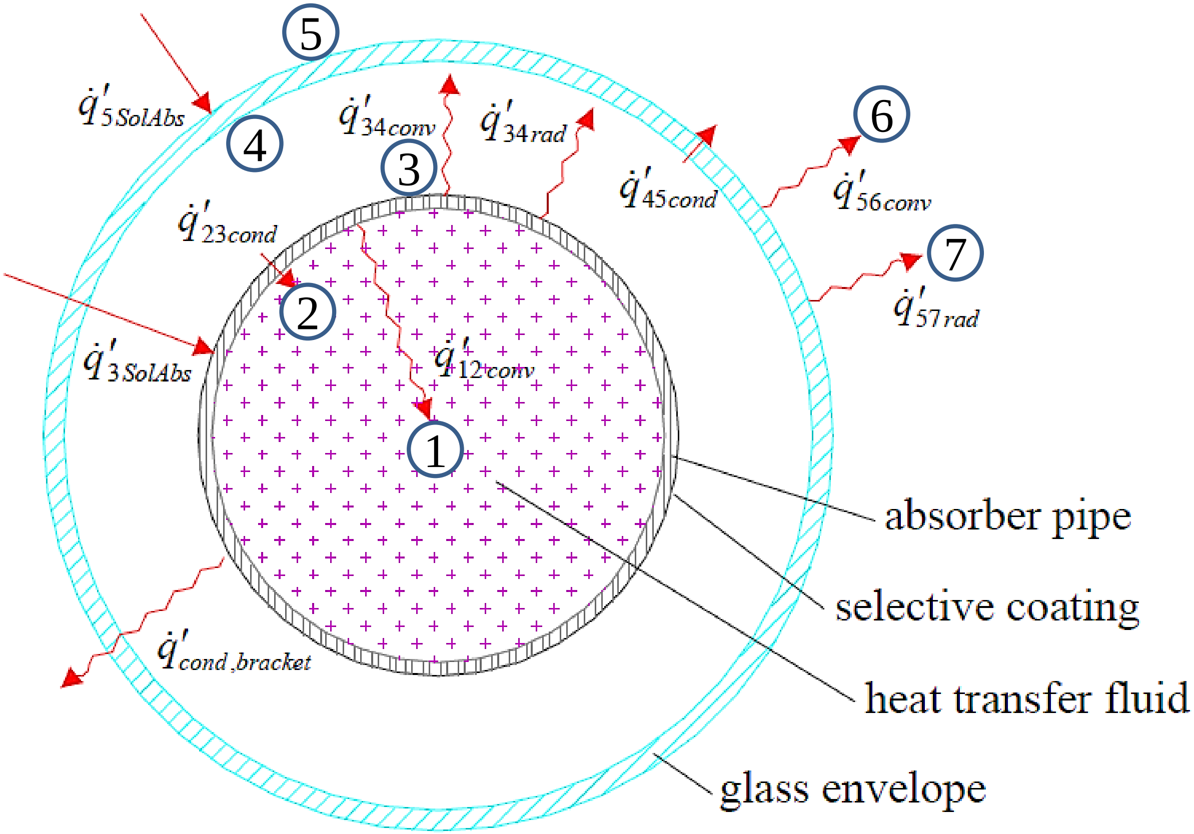
(1) heat transfer fluid
(2) absorber inner surface
(3) absorber outer surface
(4) glass envelope inner surface

(5) glass envelope outer surface
(6) surrounding air
(7) sky

Thermal Analysis of PTCs

ENERGY BALANCE ON SURFACE (2)

$$\dot{q}'_{12conv} = \dot{q}'_{23cond}$$

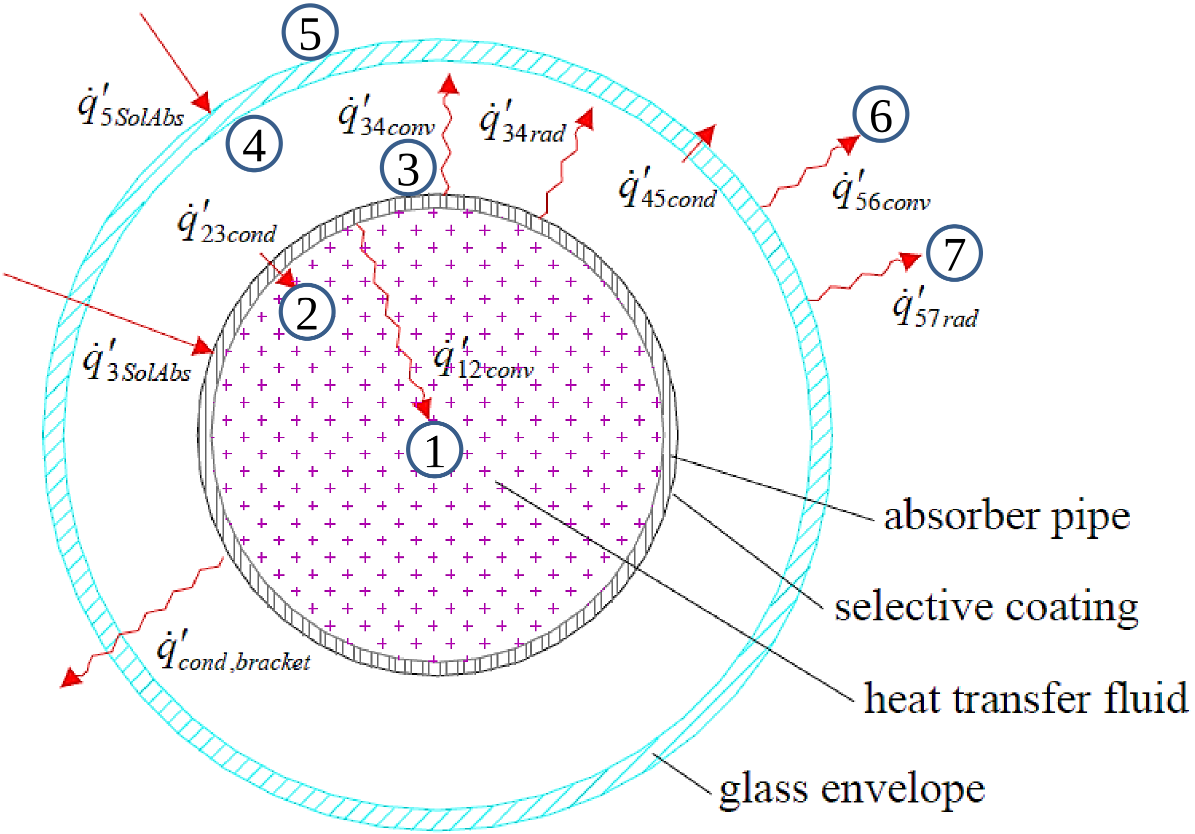


- Either $\dot{q}'_{12conv}$ or $\dot{q}'_{23cond}$ are the ***useful energy gain***.

Thermal Analysis of PTCs

ENERGY BALANCE ON SURFACE (3)

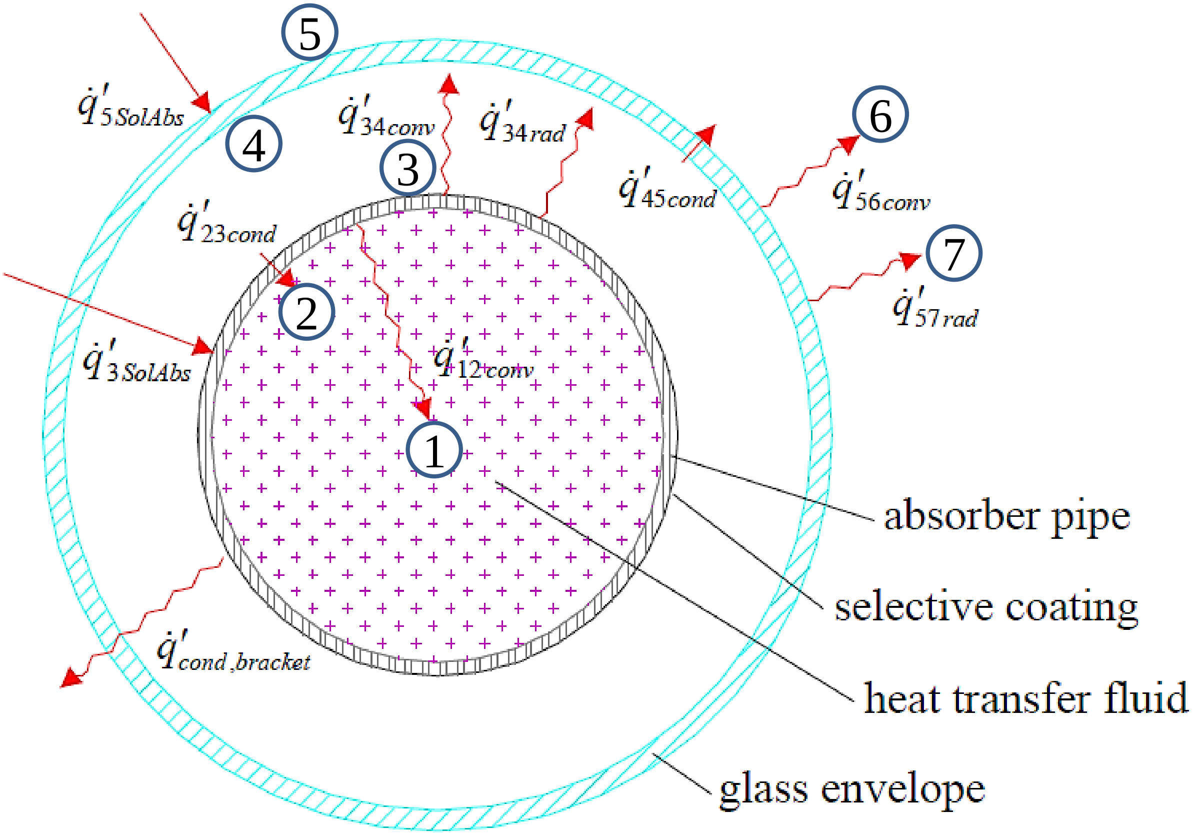
$$\dot{q}'_{3SolAbs} = \dot{q}'_{34conv} + \dot{q}'_{34rad} + \dot{q}'_{23cond} + \dot{q}'_{cond,bracket}$$



Thermal Analysis of PTCs

ENERGY BALANCE ON SURFACE (4)

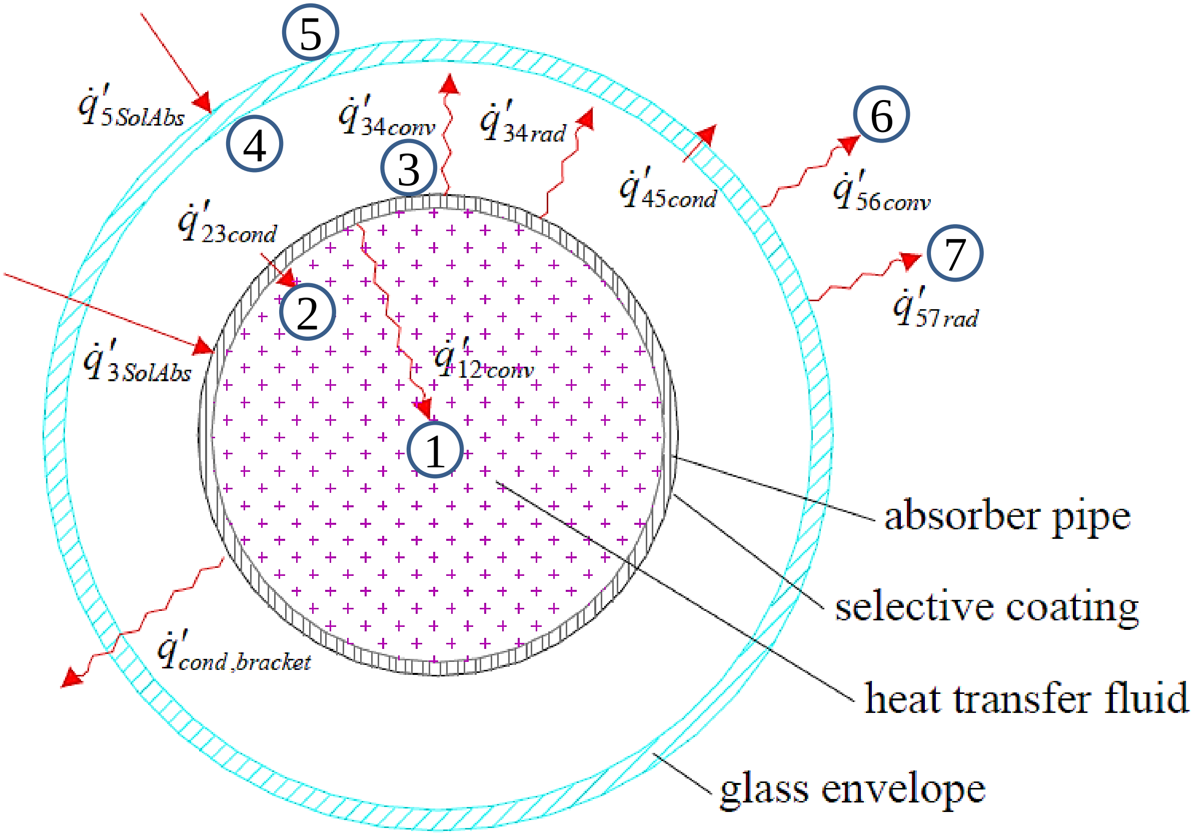
$$\dot{q}'_{34conv} + \dot{q}'_{34rad} = \dot{q}'_{45cond}$$



Thermal Analysis of PTCs

ENERGY BALANCE ON SURFACE (5)

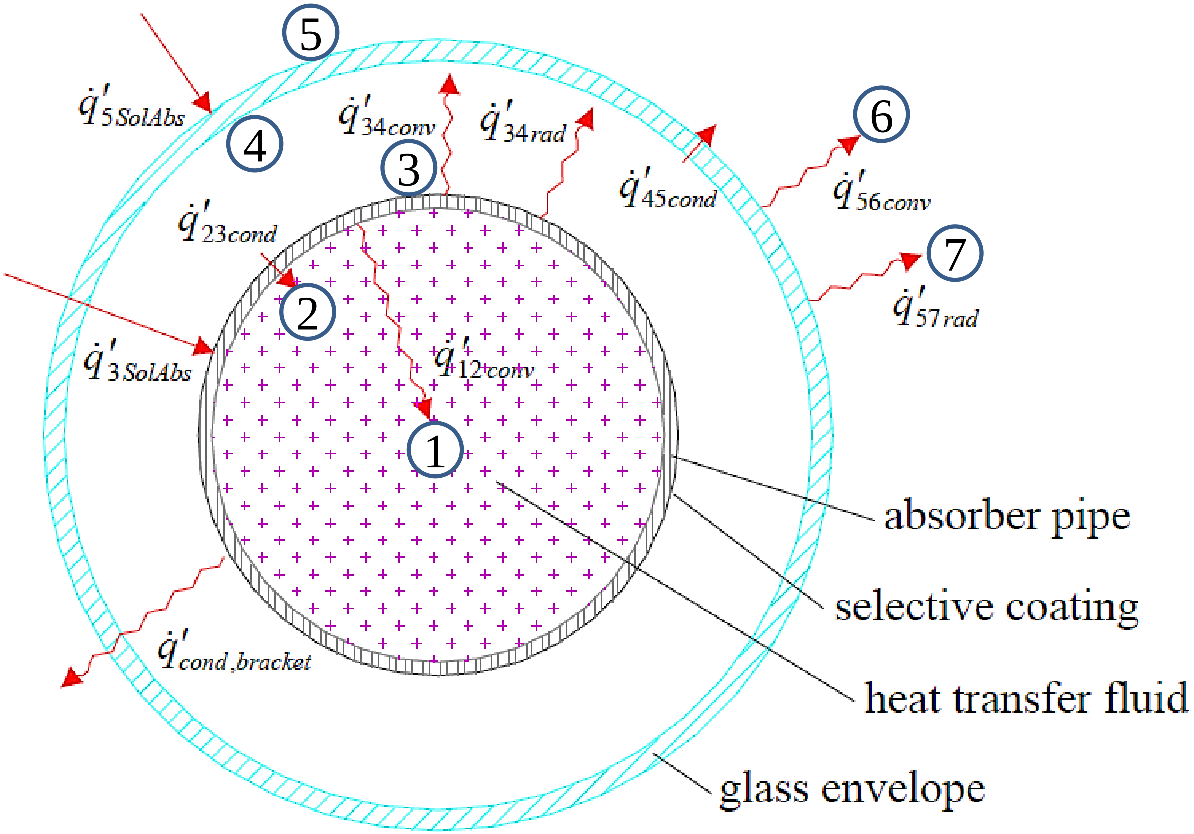
$$\dot{q}'_{45cond} + \dot{q}'_{5SolAbs} = \dot{q}'_{56conv} + \dot{q}'_{57rad}$$



Thermal Analysis of PTCs

HEAT LOSS

$$\dot{q}'_{HeatLoss} = \dot{q}'_{56conv} + \dot{q}'_{57rad} + \dot{q}'_{cond,bracket}$$



SUMMARY OF EQUATIONS

$$\dot{q}'_{12conv} = \dot{q}'_{23cond}$$

$$\dot{q}'_{3SolAbs} = \dot{q}'_{34conv} + \dot{q}'_{34rad} + \dot{q}'_{23cond} + \dot{q}'_{cond,bracket}$$

$$\dot{q}'_{34conv} + \dot{q}'_{34rad} = \dot{q}'_{45cond}$$

$$\dot{q}'_{45cond} + \dot{q}'_{5SolAbs} = \dot{q}'_{56conv} + \dot{q}'_{57rad}$$

$$\dot{q}'_{HeatLoss} = \dot{q}'_{56conv} + \dot{q}'_{57rad} + \dot{q}'_{cond,bracket}$$

- Each term has to be determined from the principles of heat transfer.

Thermal Analysis of PTCs

Convection Heat Transfer Between the Fluid and Absorber

$$\dot{q}'_{12\mathrm{conv}} = h_1 D_2 \pi \left(T_2 - T_1 \right)$$

$$h_1 = Nu_{D2} \frac{k_1}{D_2}$$

h_1 = HTF convection heat transfer coefficient at T_1 (W/m^2-K)
D_2 = inside diameter of the absorber pipe (m)
T_1 = mean (bulk) temperature of the HTF ($^{\circ}$C)
T_2 = inside surface temperature of absorber pipe ($^{\circ}$C)
Nu_{D2} = Nusselt number based on D_2
k_1 = thermal conductance of the HTF at T_1 (W/m-K)

Thermal Analysis of PTCs

Convection Heat Transfer Between the Fluid and Absorber

TURBULENT FLOW

$$Nu_{D2} = \frac{f_2/8(\mathrm{Re}_{D2} - 1000)\mathrm{Pr}_1}{1 + 12.7\sqrt{f_2/8}\left(\mathrm{Pr}_1^{2/3} - 1\right)}\left(\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\right)^{0.11} \qquad f_2 = \left(1.82\log_{10}(\mathrm{Re}_{D2}) - 1.64\right)^{-2}$$

f_2 = friction factor for the inner surface of the absorber pipe
Pr_1 = Prandtl number evaluated at the HTF temperature, T_1
Pr_2 = Prandtl number evaluated at the absorber inner surface temperature, T_2

- This correlation was developed by Gnielinski.
- It is valid for $0.5 < \mathrm{Pr}_1 < 2000$ and $2300 < \mathrm{Re}_{D2} < 5\times10^6$
- For $Re > 10{,}000$, a simpler correlation is: $Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$

LAMINAR FLOW

- Nusselt number is always equal to 4.36

Thermal Analysis of PTCs

Conduction Heat Transfer through the Absorber Wall

$$\dot{q}'_{23cond} = 2\pi k_{23}\left(T_2 - T_3\right) / \ln\left(D_3 / D_2\right)$$

k_{23} = absorber thermal conductance at the average absorber temperature $(T_2+T_3)/2$ (W/m-K)

T_2 = absorber inside surface temperature (K)

T_3 = absorber outside surface temperature (K)

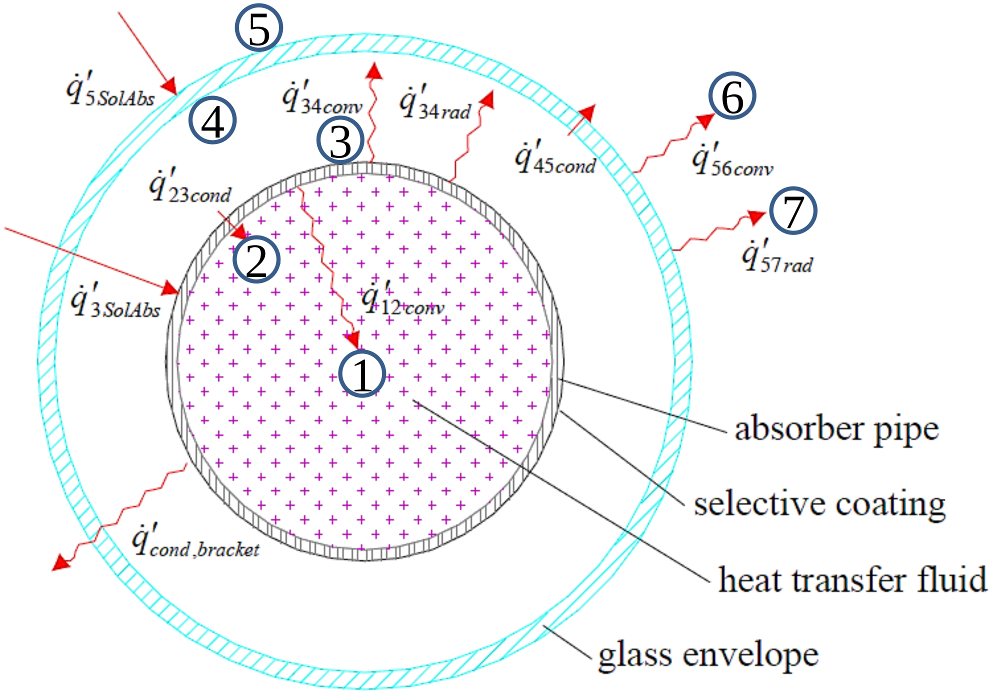
D_2 = absorber inside diameter (m)

D_3 = absorber outside diameter (m)

Thermal Analysis of PTCs

Heat Transfer from the Absorber to the Glass Envelope

- There are two possible mechanisms for heat transfer:
    - **Convection**
    - **Radiation**
- If the receiver is under vacuum, convection losses are negligible.
- If the receiver is filled with ai convection losses must be calculated.



Thermal Analysis of PTCs

Heat Transfer from the Absorber to the Glass Envelope

CONVECTION FROM AIR-FILLED ANNULUS

$$q'_{34conv} = \frac{2.425 k_{34} (T_3 - T_4)\left(\Pr Ra_{D3} / (0.861 + \Pr_{34})\right)^{1/4}}{\left(1 + (D_3 / D_4)^{3/5}\right)^{5/4}}$$

$$Ra_{D3} = \frac{g\beta (T_3 - T_4) D_3{}^3}{\alpha \nu} \qquad \beta = \frac{1}{T_{avg}}$$

k_{34} = thermal conductance of annulus gas at T_{34} (W/m-K)
T_3 = outer absorber surface temperature (°C)
T_4 = inner glass envelope surface temperature (°C)
D_3 = outer absorber diameter (m)
D_4 = inner glass envelope diameter (m)
$\Pr_{34}$ = Prandtl number
Ra_{D3} = Rayleigh number evaluated at D_3
β = volumetric thermal expansion coefficient (1/K)
T_{34} = average temperature, $(T_3 + T_4)/2$ (°C)

All physical properties are evaluated at the average temperature (T3 + T4)/2

Thermal Analysis of PTCs

Heat Transfer from the Absorber to the Glass Envelope

RADIATION

$$\dot{q}'_{34rad} = \frac{\sigma \pi D_3 \left(T_3{}^4 - T_4{}^4 \right)}{\left(1/\varepsilon_3 + \left(1 - \varepsilon_4 \right) D_3 / \left(\varepsilon_4 D_4 \right) \right)}$$

σ = Stefan-Boltzmann constant (W/m^2-K^4)

D_3 = outer absorber diameter (m)

D_4 = inner glass envelope diameter (m)

T_3 = outer absorber surface temperature (K)

T_4 = inner glass envelope surface temperature (K)

ε_3 = Absorber selective coating emissivity

ε_4 = glass envelope emissivity

Thermal Analysis of PTCs

Conduction Heat Transfer through the Glass Envelope

$$\dot{q}'_{45cond} = 2\pi k_{45}(T_4 - T_5)/\ln(D_5/D_4)$$

k_{45} = glass envelope thermal conductance at the average glass temperature ($T_4 + T_5$)/2 (W/m-K)

T_4 = glass inside surface temperature (K)

T_5 = glass outside surface temperature (K)

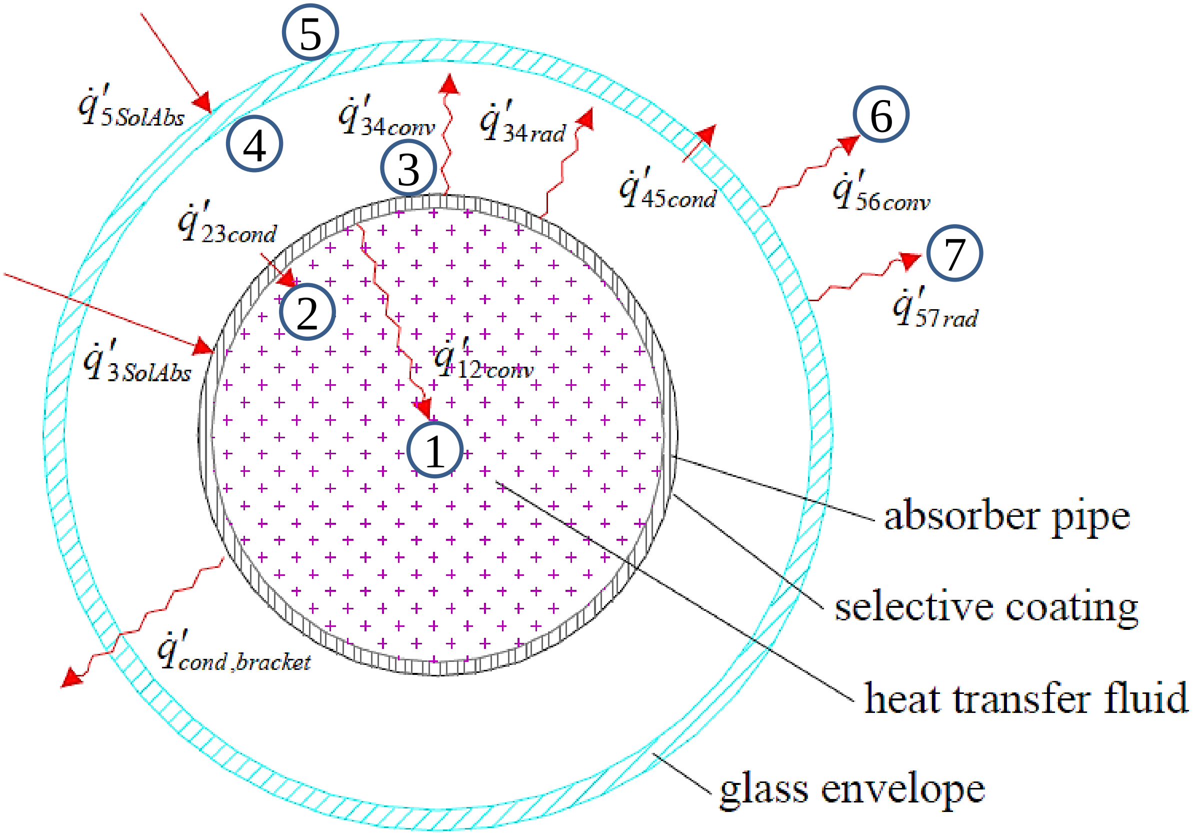
D_4 = glass inside diameter (m)

D_5 = glass outside diameter (m)

Thermal Analysis of PTCs

Heat Transfer from the Glass Envelope to the Atmosphere

- There are two possible mechanisms for heat transfer:
  - **Convection**
  - **Radiation**



Thermal Analysis of PTCs

Heat Transfer from the Glass Envelope to the Atmosphere

CONVECTION

$$\dot{q}'_{56conv} = h_{56} \pi D_5 \left(T_5 - T_6\right)$$

$$h_{56} = \frac{k_{56}}{D_5} Nu_{D5}$$

T_5 = glass envelope outer surface temperature (°C)

T_6 = ambient temperature (°C)

h_{56} = convection heat transfer coefficient for air at $(T_5 - T_6)/2$ $(W/m^2\text{-}K)$

k_{56} = thermal conductance of air at $(T_5 - T_6)/2$ (W/m-K)

D_5 = glass envelope outer diameter (m)

Nu_{D5} = average Nusselt number based on the glass envelope outer diameter

Thermal Analysis of PTCs

Heat Transfer from the Glass Envelope to the Atmosphere

CONVECTION

$$\overline{Nu}_{D5} = C\,\mathrm{Re}_{D5}{}^{m}\,\mathrm{Pr}_6{}^{n}\left(\frac{\mathrm{Pr}_6}{\mathrm{Pr}_5}\right)^{1/4}$$

$n = 0.37$, for $Pr <= 10$
$n = 0.36$, for $Pr > 10$

Re_D	C	m
1-40	0.75	0.4
40-1000	0.51	0.5
1000-200000	0.26	0.6
200000-1000000	0.076	0.7

- Valid for $0.7 < Pr_6 < 500$.
- All fluid properties are evaluated at atmospheric temperature, T_6, except Pr_5.

Thermal Analysis of PTCs

Heat Transfer from the Glass Envelope to the Atmosphere

RADIATION

$$\dot{q}'_{57rad} = \sigma D_5 \pi \varepsilon_5 \left(T_5{}^4 - T_7{}^4 \right)$$

σ = Stefan-Boltzmann constant (5.670E-8) (W/m^2-K^4)
D_5 = glass envelope outer diameter (m)
ε_5 = emissivity of the glass envelope outer surface
T_5 = glass envelope outer surface temperature (K)
T_7 = effective sky temperature (K)

- Sky temperature depends on many variables.
- A simplified model is to assume that it is less than ambient temperature by 8°C.

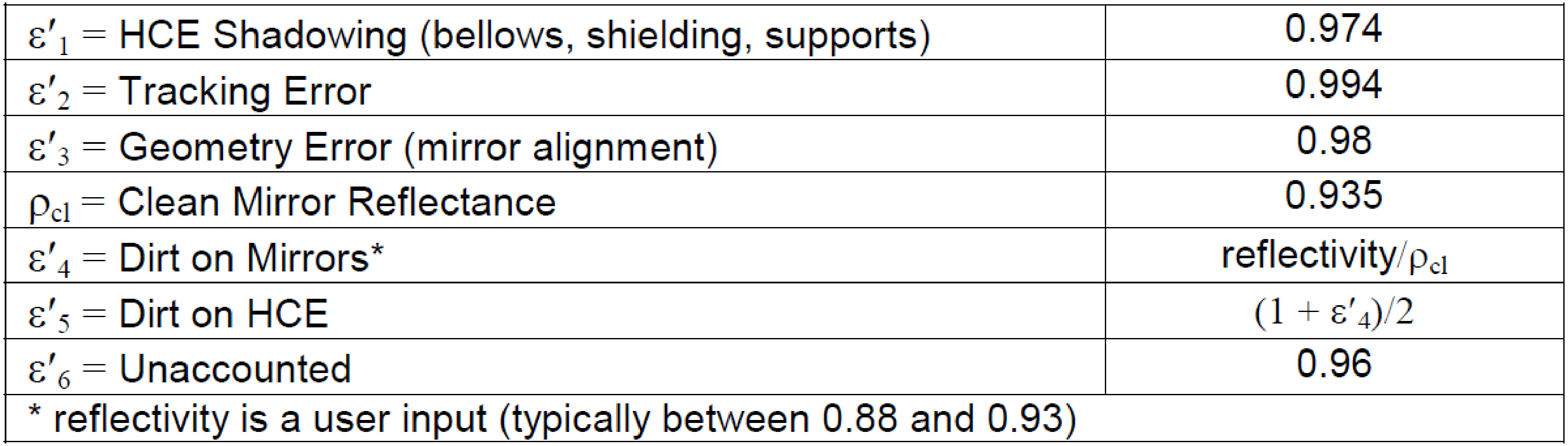
Thermal Analysis of PTCs

OPTICAL EFFICIENCY OF THE PTC

- Optical efficiency is defined as the ratio of irradiation incident on the outside of the receiver to the irradiation incident on the collector (mirrors).
- There are many sources of “error” that cause the optical efficiency to be less than 1.

Thermal Analysis of PTCs

SOURCES OF LOSSES IN OPTICAL EFFICIENCY



ε'_1 = HCE Shadowing (bellows, shielding, supports)	0.974
ε'_2 = Tracking Error	0.994
ε'_3 = Geometry Error (mirror alignment)	0.98
ρ_{cl} = Clean Mirror Reflectance	0.935
ε'_4 = Dirt on Mirrors*	reflectivity/ρ_{cl}
ε'_5 = Dirt on HCE	$(1 + \varepsilon'_4)/2$
ε'_6 = Unaccounted	0.96
* reflectivity is a user input (typically between 0.88 and 0.93)	

- Values in the table are typical values.
- They can vary from one PTC model to another and from one time to another.
- $[\varepsilon'_1\ \varepsilon'_2\ \varepsilon'_3\ \varepsilon'_4\ \varepsilon'_5\ \varepsilon'_6\ \rho_{cl}]$ is the optical efficiency.
- The optical efficiency is denoted by η_{opt}

Thermal Analysis of PTCs

Solar Irradiation Absorption in the Absorber

$$\dot{q}'_{3SolAbs} = G_{ND} W_a [\varepsilon_1' \; \varepsilon_2' \; \varepsilon_3' \; \varepsilon_4' \; \varepsilon_5' \; \varepsilon_6' \; \rho_{cl}] \; K \; \tau_{env} \; \alpha_{abs}$$

OR

$$\dot{q}'_{3SolAbs} = G_{ND} W_a \, \eta_{opt} \; K \; \tau_{env} \; \alpha_{abs}$$

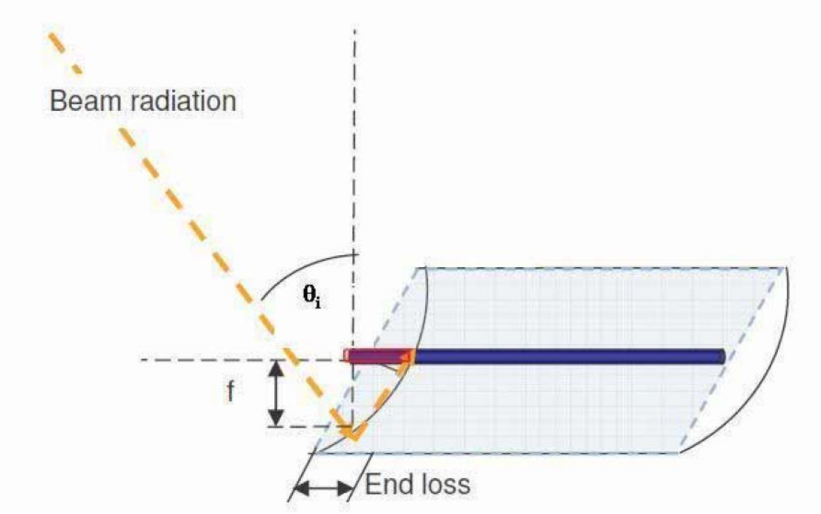
Where,

- W_a is the aperture width
- K is the incidence angle modifier (explained in next slide)
- τ_{env} is the glass tube's transmittance.
- α_{abs} is the absorber tube's absorptance.

Thermal Analysis of PTCs

Incidence Angle Modifier (*K*)

- *K* accounts for cases when the incidence angle is not 0°.
- It also accounts for the "end loss".
- *K* depends on the exact geometry of the PTC.
- The following is an example of the formula for calculating *K* :

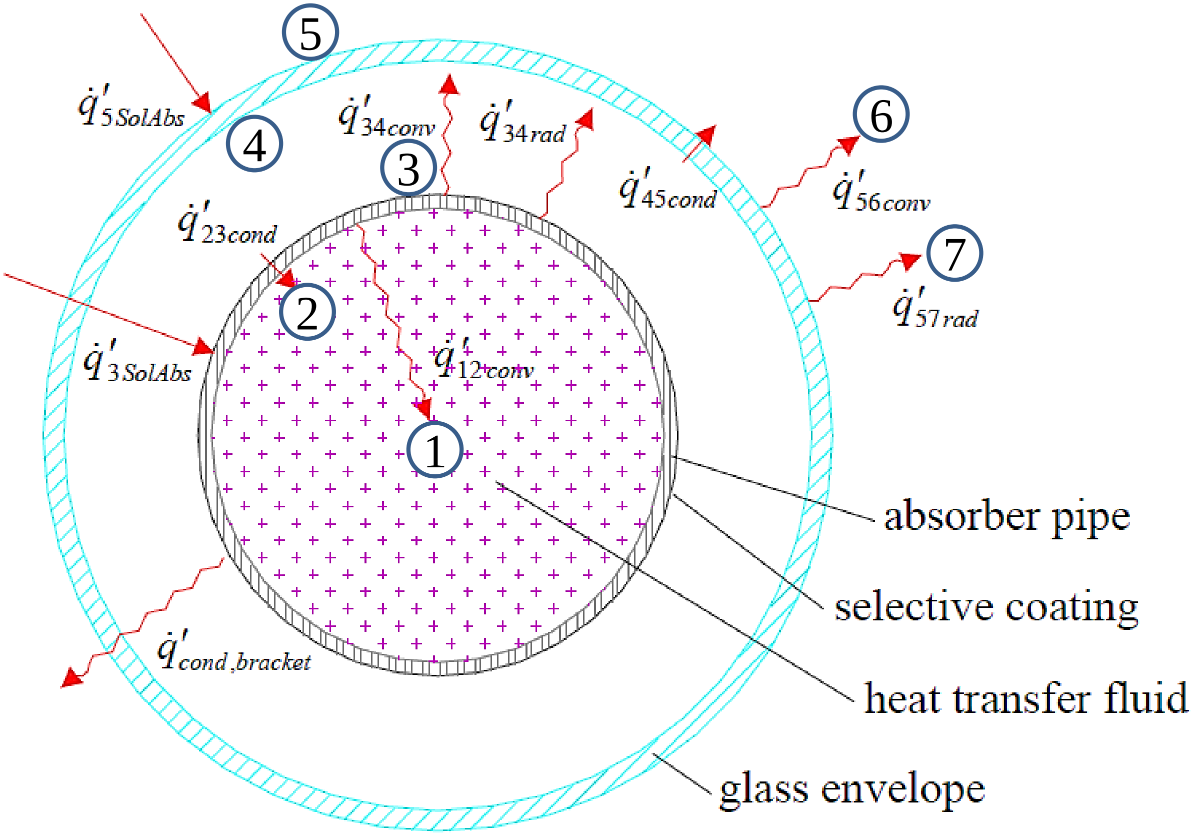


$$K = \cos(\theta) + 0.000884\theta - 0.00005369\theta^2$$

Thermal Analysis of PTCs

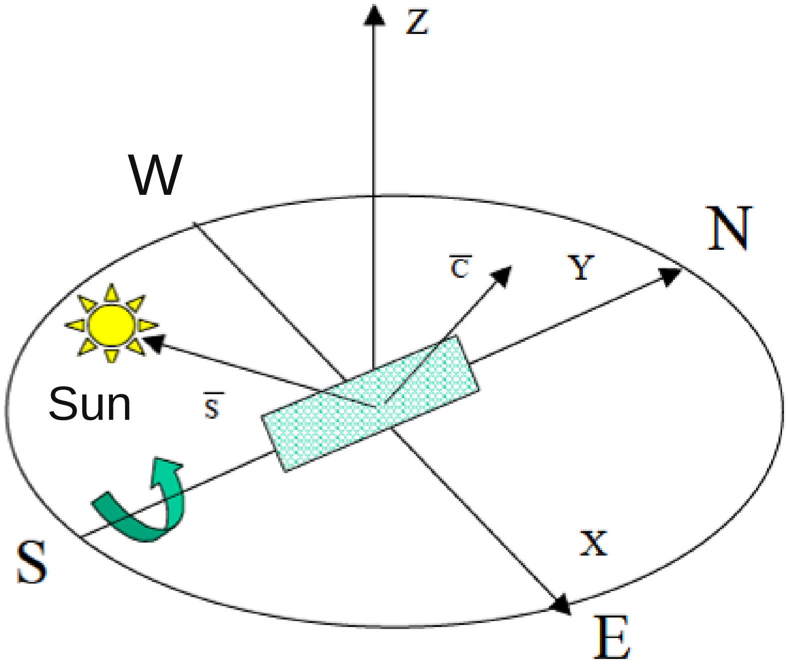
Solar Irradiation Absorption in the Glass Envelope

$$\dot{q}'_{5SolAbs} = G_{ND} W_a [\varepsilon'_1\ \varepsilon'_2\ \varepsilon'_3\ \varepsilon'_4\ \varepsilon'_5\ \varepsilon'_6\ \rho_{cl}]\ K\ \alpha_{env}$$



Thermal Analysis of PTCs

- The tilt angle (α_{PTC}) varies with the time of the day since PTCs track the sun continuously.
- For PTC systems with a north-south tracking axis, the tilt angle can be given by:



$$\sin\alpha_{PTC} = \cos\beta\ \cos|\phi - 90|\ \ if\ h \le 0$$

$$\sin\alpha_{PTC} = \cos\beta\ \cos|\phi - 270|\ \ if\ h > 0$$