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Test Exercise – XI

Answer *all* the questions. Take your time over them and work carefully.

1. The 3rd term of an A.P. is 34 and the 17th term is −8. Find the sum of the first 20 terms.

2. For the series 1, 1·2, 1·44, ... find the 6th term and the sum of the first 10 terms.

3. Evaluate $\sum_{n=1}^{8} n(3 + 2n + n^2)$.

4. Determine whether each of the following series is convergent.

   (i) $\frac{2}{2.3} + \frac{2}{3.4} + \frac{2}{4.5} + \frac{2}{5.6} + \ldots\ldots$

   (ii) $\frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2^3}{3^2} + \frac{2^4}{4^2} + \ldots + \frac{2^n}{n^2} + \ldots$

   (iii) $u_n = \frac{1 + 2n^2}{1 + n^2}$

   (iv) $u_n = \frac{1}{n!}$

5. Find the range of values of x for which each of the following series is convergent or divergent.

   (i) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ \ldots$

   (ii) $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \ldots\ \ldots$

   (iii) $\sum_{n=1}^{\infty} \frac{(n+1)}{n^3} x^n$

Further Problems – XI

1. Find the sum of n terms of the series

$$S_n = 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2$$

2. Find the sum to n terms of

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \ldots$$

3. Sum to n terms, the series

$$1.3.5 + 2.4.6 + 3.5.7 + \ldots$$

4. Evaluate the following:

$$\text{(i)}\ \sum_1^n r(r+3) \qquad \text{(ii)}\ \sum_1^n (r+1)^3$$

5. Find the sum to infinity of the series

$$1 + \frac{4}{3!} + \frac{6}{4!} + \frac{8}{5!} + \ldots$$

6. For the series

$$5 - \frac{5}{2} + \frac{5}{4} - \frac{5}{8} + \ldots + \frac{(-1)^{n-1}5}{2^{n-1}} + \ldots$$

find an expression for S_n, the sum of the first n terms. Also, if the series converges, find the sum to infinity.

7. Find the limiting values of

$$\text{(i)}\ \frac{3x^2 + 5x - 4}{5x^2 - x + 7} \quad \text{as} \quad x \to \infty$$

$$\text{(ii)}\ \frac{x^2 + 5x - 4}{2x^2 - 3x + 1} \quad \text{as} \quad x \to \infty$$

8. Determine whether each of the following series converges or diverges.

$$\text{(i)}\ \sum_1^\infty \frac{n}{n+2} \qquad \text{(ii)}\ \sum_1^\infty \frac{n}{n^2+1}$$

$$\text{(iii)}\ \sum_1^\infty \frac{1}{n^2+1} \qquad \text{(iv)}\ \sum_0^\infty \frac{1}{(2n+1)!}$$

9. Find the range of values of x for which the series

$$\frac{x}{27}+\frac{x^2}{125}+\ldots+\frac{x^n}{(2n+1)^3}+\ldots$$

is absolutely convergent.

10. Show that the series

$$1+\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots$$

is absolutely convergent when $-1<x<+1$.

11. Determine the range of values of x for which the following series is convergent

$$\frac{x}{1.2.3}+\frac{x^2}{2.3.4}+\frac{x^3}{3.4.5}+\frac{x^4}{4.5.6}+\ldots$$

12. Find the range of values of x for convergence for the series

$$x+\frac{2^4x^2}{2!}+\frac{3^4x^3}{3!}+\frac{4^4x^4}{4!}+\ldots$$

13. Investigate the convergence of the series

$$\frac{1}{1.2}+\frac{x}{2.3}+\frac{x^2}{3.4}+\frac{x^3}{4.5}+\ldots \text{ for } x>0$$

14. Show that the following series is convergent

$$2+\frac{3}{2}\cdot\frac{1}{4}+\frac{4}{3}\cdot\frac{1}{4^2}+\frac{5}{4}\cdot\frac{1}{4^3}+\ldots$$

15. Prove that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\ldots \text{ is divergent}$$

and that

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots \text{ is convergent.}$$

16. Determine whether each of the following series is convergent or divergent.

(i) $\Sigma\dfrac{1}{2n(2n+1)}$ (ii) $\Sigma\dfrac{1+3n^2}{1+n^2}$

(iii) $\Sigma\dfrac{n}{\sqrt{(4n^2+1)}}$ (iv) $\Sigma\dfrac{3n+1}{3n^2-2}$

17. Show that the series

$$1 + \frac{2x}{5} + \frac{3x^2}{25} + \frac{4x^3}{125} + \ldots \text{ is convergent}$$

if $-5 < x < 5$ and for no other values of x.

18. Investigate the convergence of

$$\text{(i)}\ 1 + \frac{3}{2.4} + \frac{7}{4.9} + \frac{15}{8.16} + \frac{31}{16.25} + \ldots$$

$$\text{(ii)}\ \frac{1}{1.2} + \frac{1}{2.2^2} + \frac{1}{3.2^3} + \frac{1}{4.2^4} + \ldots$$

19. Find the range of values of x for which the following series is convergent.

$$\frac{(x-2)}{1} + \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3} + \ldots + \frac{(x-2)^n}{n} + \ldots$$

20. If $u_r = r(2r+1) + 2^{r+1}$, find the value of $\sum_1^n u_r$.