Chapter 4

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of **40** hours. If a sample of **30** bulbs has an average life of **780** hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

Population normal and $\sigma = 40$ *"known"*, $n = 30$, $\bar{X} = 780$

96% $C.I\ for\ \mu$ **is:** $\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$1)\alpha = 1 - 0.96 = 0.04$, $2)\ 1 - \frac{\alpha}{2} = 1 - \frac{0.04}{2} = 0.98$, $3) Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 2.05$

$4)\ 780 \pm 2.05 \frac{40}{\sqrt{30}}$ >> 780 ± 14.971

$(780 - 14.971,\ 780 + 14.971) = (765.03, 794.971)$

9.6 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within **10** hours of the true mean?

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}}.\sigma}{e}\right)^2 = \left(\frac{2.05\ (40)}{10}\right)^2 = 67.24 \approx 68$$

"we always rounded the number up"

9.4 The heights of a random sample of **50** college students showed a mean of **174.5** centimeters and a standard deviation of **6.9** centimeters.
(a) Construct a 98% confidence interval for the mean height of all college students.

(b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be **174.5** centimeters?

$n = 50$, $\bar{X} = 174.5$, $S = 6.9$ (σ **unknown**)

a) **98%** $C.I\ for\ \mu$ **is:** $\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$1)\alpha = 1 - 0.98 = 0.02$, $2)\ 1 - \frac{\alpha}{2} = 1 - \frac{0.02}{2} = 0.99$, $3) Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.33$

$4)\ 174.5 \pm 2.33 \frac{6.9}{\sqrt{50}}$ >> 174.5 ± 2.2736

98% $C.I\ for\ \mu \in (172.23, 176.77)$

b) The error will not exceed $Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 2.2736$

H.W 9.5 A random sample of **100** automobile owners in the state of Virginia shows that an automobile is driven on average **23,500** kilometers per year with a standard deviation of **3900** kilometers. Assume the distribution of measurements to be approximately normal.

(a) Construct a 99% confidence interval for the average number of kilometers an automobile is driven annually in Virginia.

(b) What can we assert with **99%** confidence about the possible size of our error if we estimate the average number of kilometers driven by car owners in Virginia to be **23,500** kilometers per year?

$$n = 100\,, \quad \bar{X} = 23500, \quad S = 3900\ (\boldsymbol{\sigma\ unknown})$$

a) $\mathbf{99\%\ C.I\ for\ \mu}$ **is:** $\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$$\mathbf{1)\alpha = 1 - 0.99 = 0.01 \quad ,2)\ 1 - \frac{\alpha}{2} = 1 - \frac{0.01}{2} = 0.995 \ , \quad 3) Z_{1-\frac{\alpha}{2}} = Z_{0.995} = 2.575}$$

$$\mathbf{4)\quad 23500 \pm 2.575 \frac{3900}{\sqrt{100}} \quad >> \ 23500 \pm 1004.25}$$

$$\mathbf{99\%\ C.I\ for\ \mu \in (22495.75, 24504.25)}$$

b) The error will not exceed $\mathbf{Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 1004.25}$

Q. A group of **10** college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained: 7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2. Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend.

(a) Find the sample mean and the sample variance.

$\bar{X} = 7.575\,,\ S^2 = (1.724)^2\ (\sigma^2\ unknown)$

(b) Find a point estimate for μ

$\bar{X} = 7.575$

(c) Construct a 80% confidence interval for μ.

$$\bar{X} = 7.575\,, \quad S = 1.724\ (\sigma\ unknown)\,, \quad df = n - 1 = 9$$

80% $C.I$ for is: $\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

1) $\alpha = 1 - 0.80 = 0.20$ 2) $\frac{\alpha}{2} = \frac{0.20}{2} = 0.1$ 3) $t_{\frac{\alpha}{2}} = t_{0.1} = 1.383$

4) $7.575 \pm 1.383\,\frac{1.724}{\sqrt{10}}$ >> 7.575 ± 0.754 (error=e=0.754)

$(7.575 - 0.754,\quad 7.575 + 0.754) = (6.821, 8.329)$

9.35 A random sample of size $\mathbf{n_1 = 25}$, taken from a normal population with a standard deviation $\boldsymbol{\sigma_1 = 5}$, has a mean $\bar{X}_1 = \mathbf{80}$. A second random sample of size $\mathbf{n_2 = 36}$, taken from a different normal population with a standard deviation $\boldsymbol{\sigma_2 = 3}$, has a mean $\bar{X}_2 = \mathbf{75}$. Find a 94% confidence interval for $\mu_1 - \mu_2$.

$$94\%\ C.I\ for\ \mu_1 - \mu_2\ is: (\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$1)\ \alpha = 1 - 0.94 = 0.04 \quad 2)\ 1 - \frac{\alpha}{2} = 1 - \frac{0.04}{2} = 0.97 \quad 3)\ Z_{1-\frac{\alpha}{2}} = Z_{0.97} = 1.88$$

4) $(80 - 75) \pm 1.88\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} \gg 5 \pm 2.1019$ **(error=e=2.1019)**

$\mu_1 - \mu_2 \in$ **(2.8981, 7.1019)**

9.38 Two catalysts(محفز) in a batch chemical process, are being compared for their effect on the output of the process reaction. A sample of **12** batches was prepared using catalyst **1**, and a sample of **10** batches was prepared using catalyst **2**. The **12** batches for which catalyst **1** was used in the reaction gave an average yield of **85** with a sample standard deviation of **4**, and the **10** batches for which catalyst **2** was used gave an average yield of **81** and a sample standard deviation of **5**.
Find a 90% confidence interval for the difference between the population means, assuming that the populations are approximately normally distributed with equal variances.

$n_1 = 12, \bar{X}_1 = 85, s_1 = 4$

$n_2 = 10, \bar{X}_2 = 81, s_2 = 5$

σ_1^2 and σ_2^2 unknown but equal

$$90\%\ C.I\ for\ \mu_1 - \mu_2\ is: (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$1)\ \alpha = 1 - 0.90 = 0.1 \quad 2)\ \frac{\alpha}{2} = \frac{0.1}{2} = 0.05 \quad 3)\ t_{\frac{\alpha}{2}} = t_{0.05} = 1.725$$

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{11\,(16) + 9(25)}{20}} = 4.4777$$

4) $(85 - 81) \pm (1.725)(4.4777)\sqrt{\frac{1}{12} + \frac{1}{10}} \gg 4 \pm 3.3072$

$(error = e = 3.3072)$

$\mu_1 - \mu_2 \in$ **(0.693, 7.307)**

H.W 9.41 The following data represent the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication 1	**Medication 2**
$n_1 = 14$	$n_2 = 16$
$\bar{x}_1 = 17$	$\bar{x}_2 = 19$
$s_1^2 = 1.5$	$s_2^2 = 1.8$

Find a 99% confidence interval for the difference $\boldsymbol{\mu_2 - \mu_1}$

σ_1^2 *and* σ_2^2 *unknown* **but equal**

$$90\%\ C.I\ for\ \mu_2 - \mu_1\ is: (\bar{X}_2 - \bar{X}_1) \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\ s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$1)\ \alpha = 1 - 0.99 = 0.01 \qquad 2)\ \frac{\alpha}{2} = \frac{0.01}{2} = 0.005 \qquad 3)\ t_{\frac{\alpha}{2}} = t_{0.005} = 2.763$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{13\,(1.5^2) + 15(1.8^2)}{28}} = 1.3336$$

$$4)\ \ (19-17) \pm (2.763)(1.3336)\sqrt{\frac{1}{14} + \frac{1}{16}} \gg 2 \pm 1.348$$

$(error = e = \ 1.348\ )$

$\mu_2 - \mu_1 \in$ **(0.65, 3.35)**

9.44 A taxi company is trying to decide whether to purchase brand "A" or brand "B" tires for its fleet of taxis (اسطول من سيارات التكسي). The experiment is conducted using 12 of each brand and the tires are run until they wear out.

I) Compute a 99% confidence interval for $\boldsymbol{\mu_1 - \mu_2}$, assuming the populations to be approximately normally.

II) Find a 99% confidence interval for $\boldsymbol{\mu_1 - \mu_2}$ if tires of the two brands are assigned at random to the left and right rear wheels of **8** taxis and the following distances, in kilometers, are recorded:

Taxi	**Brand A**	**Brand B**
1	34,400	36,700
2	45,500	46,800
3	36,700	37,700
4	32,000	31,100
5	48,400	47,800
6	32,800	36,400
7	38,100	38,900
8	30,100	31,500

Assume that the differences of the distances are approximately normally distributed.

σ_1^2 *and* σ_2^2 *unknown*, $\sigma_1^2 \neq \sigma_2^2$.

I)

From the table, we calculate:

$$\bar{X}_1 = 37,250, s_1 = 6546.755$$

$$\bar{X}_2 = 38,362.5, s_2 = 6181.063$$

99% *C.I for* $\mu_1 - \mu_2$ *is* : $(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$1)\alpha = 1 - 0.99 = 0.01 \quad ,2)\ \frac{\alpha}{2} = \frac{0.01}{2} = 0.005 \ , \quad 3)t_{\frac{\alpha}{2}} = t_{0.005,14} = 2.977$

$$4)\quad (37250 - 38362.5) \pm (2.977)\sqrt{\frac{6546.755^2}{8} + \frac{6181.063^2}{8}}$$

$$\gg -1112.5 \pm 9476.587$$

$(error = e = \ 205469\ )$

$\mu_2 - \mu_1 \in$ **(-10589.0873 , 8364.0873)**

II)

99% *C.I for* μ_d *is* : $[\ \bar{d} \pm t_{\frac{\propto}{2}, n-1} \frac{S_d}{\sqrt{n}}\ ]$

$$\bar{d} = \frac{\sum d_i}{n} = \ , \qquad S_d = 1454.488$$

$1)\alpha = 1 - 0.99 = 0.01 \quad ,2)\ \frac{\alpha}{2} = \frac{0.05}{2} = 0.005 \ , \quad 3)t_{\frac{\alpha}{2}, n-1} = t_{0.005,7} \ = 3.499$

$4)\ -1112.5 \ \pm (3.499)\left(\frac{1454.488}{\sqrt{8}}\right) \gg \ -1112.5 \ \pm 1799.3$

$$\mu_d \in [\ -2911.8\ , 686.8\ ]$$

Values of Z	
$Z_{0.90}$	1.285
$Z_{0.95}$	1.645
$Z_{0.97}$	1.885
$Z_{0.975}$	1.96
$Z_{0.98}$	2.055
$Z_{0.99}$	2.325
$Z_{0.995}$	2.575

H.W

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation σ=2.0. Suppose that a random sample of size n=49 from this population gave a sample mean =4.5. $\overline{X}$

(1) The distribution of $\overline{X}$ is .

(2) A good point estimate of μ is = $\overline{X} = 4.5$

(3) The standard error of $\overline{X}$ is = **0.2875**

(4) A 95% confidence interval for μ is $(3.94\ , 5.06)$

(5) If the upper confidence limit of a confidence interval is **5.2**, then the lower confidence limit is

(6) The confidence level of the confidence interval (3.88, 5.12) is

$$\overline{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 5.12 \quad >> \quad 4.5 + Z_{1-\frac{\alpha}{2}} \frac{2}{\sqrt{49}} = 5.12$$

$$Z_{1-\frac{\alpha}{2}} = (5.12 - 4.5)\ \frac{\sqrt{49}}{2} = 2.17$$

$$P(Z < 2.17) = 1 - \frac{\alpha}{2} = 0.985 \quad \gg \quad \alpha = 2(0.015) = 0.03$$

$$\text{Then, the confidence level} = 1 - \alpha = 0.97 \gg \ 0.97(100) = 97\%$$

Note: we will get the same result if use $\overline{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 3.88$

(7) If we use $\overline{X}$ to estimate μ, then we are 95% confident that our estimation error will not exceed. $e = 0.56$

(8) If we want to be 95% confident that the estimation error will not exceed e=0.1 when we use $\overline{X}$ to estimate μ, then the sample size n must be equal to
$n = 1537$

Q1. A survey of 500 students from a college of science shows that 275 students own computers. In another independent survey of 400 students from a college of engineering shows that 240 students own computers.

(a) a 99% confidence interval for the true proportion of college of science's student who own computers is $(\mathbf{0.4927}, \mathbf{0.6073})$

(b) a 95% confidence interval for the difference between the proportions of students owning computers in the two colleges is

$$(\mathbf{-0.1148}, \mathbf{0.0148})$$