Chapter 2

6.3 The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution with A = 7 and B = 10. Find the probability that on a given day the amount of coffee dispensed by this machine will be:
(a) at most 8.8 liters;
(b) more than 7.4 liters but less than 9.5 liters;
(c) at least 8.5 liters.

$$f(x) = \frac{1}{b-a} \quad , a \le x \le b \; ; \; E(X) = \frac{a+b}{2} \; and \; \sigma^2 = \frac{(b-a)^2}{12}$$

$X \sim \text{Uniform}(7{,}10)$, $f(x) = \frac{1}{3} \quad , 7 \le x \le 10$

a) $p(x \le 8.8) = \int_7^{8.8} \frac{1}{3} dx = \frac{1}{3} \, [x]_7^{8.8} = \frac{8.8-7}{3} = 0.6$

b) $p(7.4 \le x \le 9.5) = \int_{7.4}^{9.5} \frac{1}{3} dx = \frac{1}{3} \, [x]_{7.4}^{9.5} = \frac{9.5-7.4}{3} = 0.7$

c) $p(x \ge 8.5) = \int_{8.5}^{10} \frac{1}{3} dx = \frac{1}{3} \, [x]_{8.5}^{10} = \frac{10-8.5}{3} = 0.5$

6.4 A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.
(a) What is the probability that the individual waits more than 7 minutes?
(b) What is the probability that the individual waits between 2 and 7 minutes?

$X \sim \text{Uniform}(0{,}10)$, $f(x) = \frac{1}{10} \quad , 0 \le x \le 10$

a) $p(x > 7) = \int_7^{10} \frac{1}{10} dx = \frac{1}{10} \, [x]_7^{10} = \frac{10-7}{10} = 0.3$

b) $p(2 < x < 7) = \int_2^{7} \frac{1}{10} dx = \frac{1}{10} \, [x]_2^{7} = \frac{7-2}{10} = 0.5$

6.45 The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes?

$$f(x) = \lambda e^{-\lambda x} \;, x > 0 \quad ; \; E(X) = \frac{1}{\lambda} \; and \; \sigma^2 = \frac{1}{\lambda^2}$$

$$X \sim EXP\left(\frac{1}{\lambda} = \frac{1}{4}\right); \;\; f(x) = \frac{1}{4}\, e^{-\frac{x}{4}}, X > 0$$

$$p(x < 3) = \int_0^3 \frac{1}{4} e^{-\frac{x}{4}} dx = \left[-e^{-\frac{x}{4}}\right]_0^3 = -e^{-\frac{3}{4}} + e^{-\frac{0}{4}} = 1 - e^{-\frac{3}{4}} = 0.5276$$

What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

First , calculate the probability of served less than 3 min .

Consider "served less then 3" is success thus, p=0.5276.

Now, let Y be the number of success in 6 days.

Y~Bin(n=6,p=0.5276)

$$f(x) = \binom{6}{x}(0.5276)^x(0.4724)^{6-x} \quad ; x = 0,1,2,\dots,6$$

$$p(Y \geq 4) = f(4) + f(5) + f(6) = \sum_{Y=4}^{6} \binom{6}{x}(0.5276)^x(0.4724)^{6-x} = 0.3986$$

H.W

Q2. Suppose that the random variable X has the following uniform distribution: $f(x) = \begin{cases} 3 & , \frac{2}{3} < x < 1 \\ 0 & , other\ wise \end{cases}$

1) p(0.33< x <0.5)=

2) p(x >1.25)=

(3) The variance of X is

Q2. Suppose that the failure time (in hours) of a certain electrical device is distributed with a probability density function given by:

f(x)= (1/70) $e^{-x/70}$, x > 0

1) the probability that a randomly selected device will fail within the first 50 hours is:

2) the probability that a randomly selected device will last more than 150 hours is:

3) the average failure time of the electrical device is:

4) the variance of the failure time of the electrical device is:

Chapter 3

If we have $X \sim N(\mu, \sigma^2)$ then $\bar{X}_1 \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

8.17 If all possible samples of size 16 are drawn from a normal population with mean equal to 50 and standard deviation equal to 5, what is the probability that a sample mean $\bar{x}$ will fall in the interval from $\mu_{\bar{X}} - 1.9\sigma_{\bar{X}}$ To $\mu_{\bar{X}} - 0.4\sigma_{\bar{X}}$?
Assume that the sample means can be measured to any degree of accuracy.

$$\mu = 50, \qquad \sigma = 5, n = 16$$

$$P(\mu_{\bar{X}} - 1.9\sigma_{\bar{X}} < \bar{X} < \mu_{\bar{X}} - 0.4\sigma_{\bar{X}}) = P\left(\frac{\mu_{\bar{X}} - 1.9\sigma_{\bar{X}} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < Z < \frac{\mu_{\bar{X}} - 0.4\sigma_{\bar{X}} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right)$$

$$= P(-1.9 < Z < -0.4) = P(Z < -0.4) - P(Z < -1.9) = 0.3446 - 0.0287 = 0.3159$$

8.20 Given the discrete uniform population

$$f(x) = \begin{cases} \frac{1}{3}, & x = 2,4,6 \\ 0, & elswhere \end{cases}$$

Find the probability that a random sample of size **54**, selected with replacement, will yield a **sample mean** greater than 4.1 but less than 4.4. Assume the means are measured to the nearest tenth.

Because $n \geq 30$, then $X \sim N(\mu, \sigma^2)$

$$n = 54, \quad \mu = E(X) = \frac{2+4+6}{3} = 4, \qquad E(X^2) = \frac{4+16+36}{3} = \frac{56}{3}$$

$$\sigma^2 = \frac{56}{3} - 4^2 = 2.667, \qquad \sigma = \sqrt{2.667} = 1.63$$

$$\mu_{\bar{X}} = \mu = 4, \quad \sigma_{\bar{X}} = \frac{1.63}{\sqrt{54}} = 0.22$$

$$p(4.1 < \bar{X} < 4.4) = p\left(\frac{4.1-4}{0.22} < Z < \frac{4.4-4}{0.22}\right) = p(0.45 < z < 1.82)$$
$$= 0.9656 - 0.6736 = 0.292$$

8.23 The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

(a) Find the mean μ and the variance σ^2 of X.
(b) Find the mean $\mu_{\bar{X}}$ and the variance $\sigma^2_{\bar{X}}$ of the mean $\bar{X}$ for random samples of **36** cherry puffs.
(c) Find the probability that the average number of cherries in **36** cherry puffs will be less than **5.5**.

a) $\mu = E(X) = 4(0.2) + 5(0.4) + 6(0.3) + 7(0.1) = 5.3$
$E(X^2) = 16(0.2) + 25(0.4) + 36(0.3) + 49(0.1) = 28.9$

$$\sigma^2 = 28.9 - (5.3)^2 = 0.81\,,\quad \sigma = \sqrt{0.81} = 0.9$$

b) $\mu_{\bar{X}} = \mu = 5.3$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.9}{\sqrt{36}} = 0.15$$

c) $p(\bar{X} < 5.5) = p\left(Z < \frac{5.5-5.3}{0.15}\right) = p(z < 1.33) = 0.9082$

8.26 The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\boldsymbol{\mu}$ **= 3.2** minutes and a standard deviation $\boldsymbol{\sigma}$ **= 1.6** minutes. If a random sample of **64** customers is observed, find the probability that their **mean time** at the teller's window is

(a) at most 2.7 minutes;
(b) more than 3.5 minutes;
(c) at least 3.2 minutes but less than 3.4 minutes.

$\boldsymbol{\mu = 3.2}$ **,** $\boldsymbol{\sigma = 1.6}$ **, n=64 ,** $\mu_{\bar{X}} = \mu = 3.2\,, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = 0.2$

a) $p(\bar{X} \le 2.7) = p\left(z \le \frac{2.7-3.2}{0.2}\right) = p(Z \le -2.5) = 0.0062$

b) $p(\bar{X} > 3.5) = p\left(z > \frac{3.5-3.2}{0.2}\right) = p(Z > 1.5) = 1 - 0.9332 = 0.0668$

c) $p(3.2 \leq \bar{X} \leq 3.4) = p\left(\frac{3.2-3.2}{0.2} \leq z \leq \frac{3.4-3.2}{0.2}\right) = p(0 \leq Z \leq 1) =$
$p(z < 1) - p(z < 0) = 0.8413 - 0.5 = 0.3413$

8.28 A random sample of size **25** is taken from a normal population having a mean of **80** and a standard deviation of **5**. A second random sample of size **36** is taken from a different normal population having a mean of **75** and a standard deviation of **3**. Find the probability that the sample mean computed from the **25** measurements will exceed the sample mean computed from the **36** measurements by at least **3.4** but less than **5.9**. Assume the difference of the means to be measured to the nearest tenth.

$n_1 = 25, \mu_1 = 80, \boldsymbol{\sigma}_1 = 5$
$n_2 = 36, \mu_2 = 75, \boldsymbol{\sigma}_2 = 3$
$p(3.4 \leq \bar{X}_1 - \bar{X}_2 \leq 5.9) = ?$

if we have $X_1 \sim N(\mu_1, {\sigma^2}_1)$ and $X_2 \sim N(\mu_2, {\sigma^2}_2)$ which are independent then

$$\bar{X}_1 \sim N\left(\mu_1, \frac{{\sigma^2}_1}{n_1}\right) \quad , \quad \bar{X}_2 \sim N\left(\mu_1, \frac{{\sigma^2}_2}{n_2}\right) \text{, and} \quad \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{{\sigma^2}_1}{n_1} + \frac{{\sigma^2}_2}{n_2}\right)$$

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 80 - 75 = 5$$

$$\boldsymbol{\sigma}^2_{\bar{X}_1 - \bar{X}_2} = \frac{{\sigma^2}_1}{n_1} + \frac{{\sigma^2}_2}{n_2} = \frac{5^2}{25} + \frac{3^2}{36} = 1.25, \quad \boldsymbol{\sigma}_{\bar{X}_1 - \bar{X}_2} = 1.118$$

$$\bar{X}_1 - \bar{X}_2 \sim N(5, 1.25)$$

$$p(3.4 \leq \bar{X}_1 - \bar{X}_2 \leq 5.9) = p\left(\frac{3.4-5}{1.118} \leq \frac{\bar{X}_1 - \bar{X}_2 - \mu_{\bar{X}_1 - \bar{X}_2}}{\boldsymbol{\sigma}_{\bar{X}_1 - \bar{X}_2}} \leq \frac{5.9-5}{1.118}\right)$$

$$= p(-1.43 < Z < 0.81) = p(Z < 0.81) - p(Z < -1.43)$$

$$= 0.791 - 0.0764 = 0.8946$$

8.29 The distribution of heights of a certain breed of terrier has a mean of **72** centimeters and a standard deviation of **10** centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of **28** centimeters with a standard deviation of **5** centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of **64** terriers exceeds the sample mean for a random sample of heights of **100** poodles by at most **44.2** centimeters.

$n_1 = 64,\ \mu_1 = 72\ ,\boldsymbol{\sigma}_1 = 10$
$n_2 = 100, \mu_2 = 28\ ,\boldsymbol{\sigma}_2 = 5$

$$\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2 = 72 - 28 = 44$$

$$\boldsymbol{\sigma^2}_{\bar{X}_1-\bar{X}_2} = \frac{\sigma^2{}_1}{n_1} + \frac{\sigma^2{}_2}{n_2} = \frac{10^2}{64} + \frac{5^2}{100} = \frac{29}{16}\ , \qquad \boldsymbol{\sigma}_{\bar{X}_1-\bar{X}_2} = 1.3463$$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(44\ , \frac{29}{16}\right)$$

$$p(\ \bar{X}_1 - \bar{X}_2 \le 44.2) = p\left(\ Z \le \frac{44.2 - 44}{1.3463}\right) = p(Z \le 0.15) = 0.5596$$

8.41 Assume the **sample variances** to be continuous measurements. Find the probability that a random sample of **25** observations, from a normal population with variance $\boldsymbol{\sigma^2} = \mathbf{6}$, will have a sample variance S^2
(a) greater than 9.1;
(b) between 3.462 and 10.745.

n=25, $\sigma^2 = 6$
we know that $x^2 = \frac{(n-1)s^2}{\sigma^2} \sim x^2_{n-1}\ , thus, x^2 = \frac{(24)s^2}{6} = 4s^2\ has\ x^2_{24}$

a) $p(s^2 > 9.1) = p\big(x^2 > 4(9.1)\big) = p(x^2 > 36.4) = 0.05$
b) $p(3.462 < s^2 < 10.745) = p\big(\ 4(3.462) < x^2 < 4(10.745)\big)$
$= \ p(\ 13.86 < x^2 < 42.98) = p(x^2 < 42.98) - p(x^2 < 13.86)$
$= 1 - p(x^2 > 42.98) - [1 - p(x^2 > 13.86)]$
$p(x^2 > 13.86) - p(x^2 > 42.98)$
$= 0.95 - 0.01 = 0.94$

8.45 Using t-distrbution
(a) Find $P(T < 2.365)$ when $v = 7$.
(b) Find $P(T > 1.318)$ when $v = 24$.
(c) Find $P(\ -1.356 < T < 2.179)$ when $v = 12$.
(d) Find $P(T > \ -2.567)$ when $v = 17$.

a) $p(T_7 < 2.365) = 1 - 0.025 = 0.975$
b) $p(T_{24} > \ 1.318) = 0.1$
c) $p(-1.356 < T_{12} < 2.179) = p(T_{12} < 2.179) - p(T_{12} < -1.356) =$
$= 1 - p(T_{12} > 2.179) - p(T_{12} > 1.356) = \ 0.975 - 0.1 = 0.875$
d) $p(T_{17} > \ -2.567) = 1 - 0.01 = 0.99$

8.51 For an *F*-distribution, find

(a) $f_{0.05}$ with $v_1 = 7$ and $v_2 = 15$;
(b) $f_{0.05}$ with $v_1 = 15$ and $v_2 = 7$:
(c) $f_{0.01}$ with $v_1 = 24$ and $v_2 = 19$;
(d) $f_{0.95}$ with $v_1 = 19$ and $v_2 = 24$;
(e) $f_{0.99}$ with $v_1 = 28$ and $v_2 = 12$.

a) $f_{0.05,7,15} = 2.71$
b) $f_{0.05,15,7} = 3.51$
c) $f_{0.01,24,19} = 2.92$
d) $f_{0.95,19,24} = \frac{1}{f_{0.05,24,19}} = \frac{1}{2.11} = 0.47$
e) $f_{0.99,28,12} = \frac{1}{f_{0.01,12,28}} = \frac{1}{2.90} = 0.34$

H.W

Q13. The average rainfall in a certain city for the month of March is 9.22 centimeters. Assuming a normal distribution with a standard deviation of 2.83 centimeters, then the probability that next March, this city will receive:

(1) less than 11.84 centimeters of rain is: Answer: 0.8238
(2) more than 5 centimeters but less than 7 centimeters of rain is: Answer: 0.1496

(3) more than 13.8 centimeters of rain is: Answer: 0.0526

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1) The sample mean X of a random sample of 5 batteries selected from this product has a mean $E(\bar{X}) = \mu_{\bar{x}}$ equal to: Answer: 5

2) The variance $var(\bar{X}) = \sigma^2{}_{\bar{x}}$ of the sample mean X of a random sample of 5 batteries selected from this product is equal to: Answer: 0.2

3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is: Answer: 0.9224

4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is: Answer: 0.9772

5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is: Answer: 0.8413

6) If $p(\bar{X} > a) = 0.1492$ where X represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is: Answer: 5.347

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let X_1 and X_2 be the averages of the first and second samples, respectively.

a) Find $E(\bar{X}_1)$ and $Var(\bar{X}_1)$. Answer: 70 and $\frac{16}{36}$
b) Find $E(\bar{X}_1 - \bar{X}_2)$ and $Var(\bar{X}_1 - \bar{X}_2)$. Answer: -15 and $\frac{421}{441}$
c) Find $P(70 < \bar{X}_1 < 71)$. Answer: 0.4332
d) Find $P(\bar{X}_1 - \bar{X}_2 > -16)$. Answer: 0.8461