Lect 2

* Transfer function:

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

$$G(s) = \frac{Y(s)}{U(s)}$$

- to find the transfer function of a system, we consider the input $u(t)$ to be impulse function, then we observe the output

$\Rightarrow Y(s) = G(s)$

- If we have the differential equation of the system, we can obtain the transfer function.

$$\frac{d^n}{dt^n} y(t) + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} y(t) + a_{n-2} \frac{d^{n-2}}{dt^{n-2}} y(t) + \cdots + a_1 \frac{d}{dt} y(t) + a_0 y(t) = b_m \frac{d^m}{dt^m} u(t) + \cdots + b_0 u(t)$$

we know: $\frac{d^n}{dt^n} y(t) \overset{\mathcal{L}}{\longleftrightarrow} s^n Y(s)$

$$\Rightarrow s^n Y(s) + a_{n-1} s^{n-1} Y(s) + a_{n-2} s^{n-2} Y(s) + \cdots + a_1 s Y(s) + a_0 Y(s) = b_m s^m U(s) + \cdots + b_0 U(s)$$

$$G(s) = \frac{Y(s)}{U(s)}$$

$$\Rightarrow [s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 s + a_0] Y(s) = [b_m s^m + \cdots + b_0] U(s)$$

$$G(s) = \frac{b_m s^m + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 s + a_0}$$

Example Find the T.F. of a system with this D.E.

$$\frac{d^2}{dt^2} y(t) + 2 \frac{d}{dt} y(t) - 3 y(t) = 3 \frac{d}{dt} u(t) + 2 u(t)$$

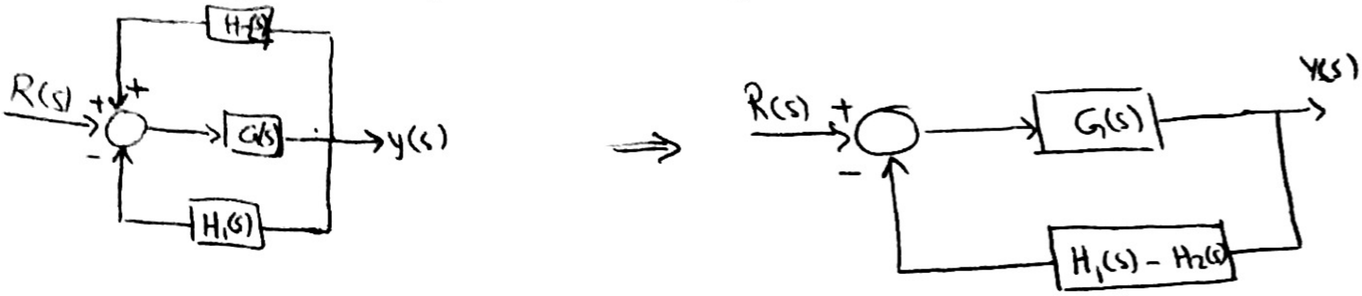
Solution: find the Laplace transform of this D.E.

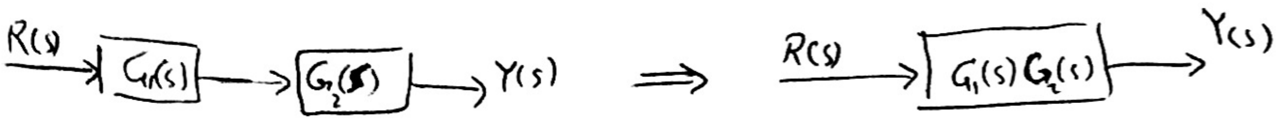
$$s^2 Y(s) + 2s Y(s) - 3 Y(s) = 3s U(s) + 2 U(s)$$

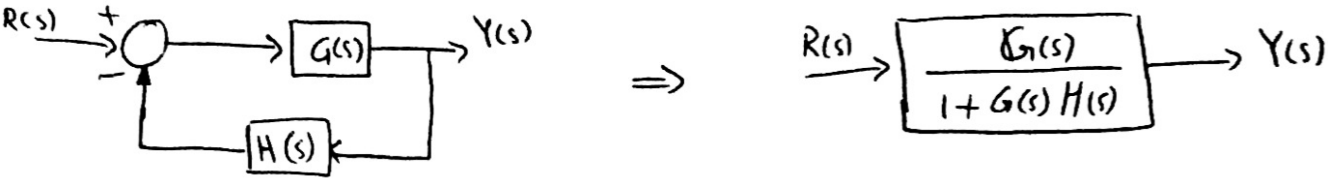
$$\Rightarrow [s^2 + 2s - 3] Y(s) = [3s + 2] U(s)$$

$$\Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{3s+2}{s^2+2s-3}$$

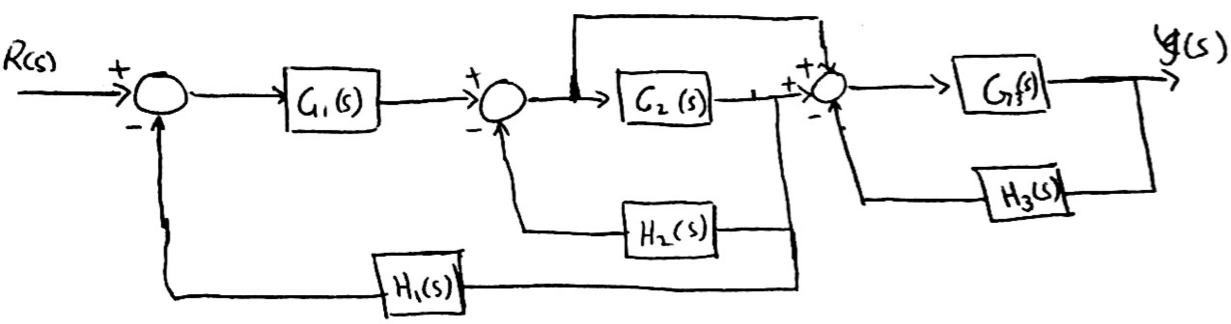
★ Block diagram of control systems.



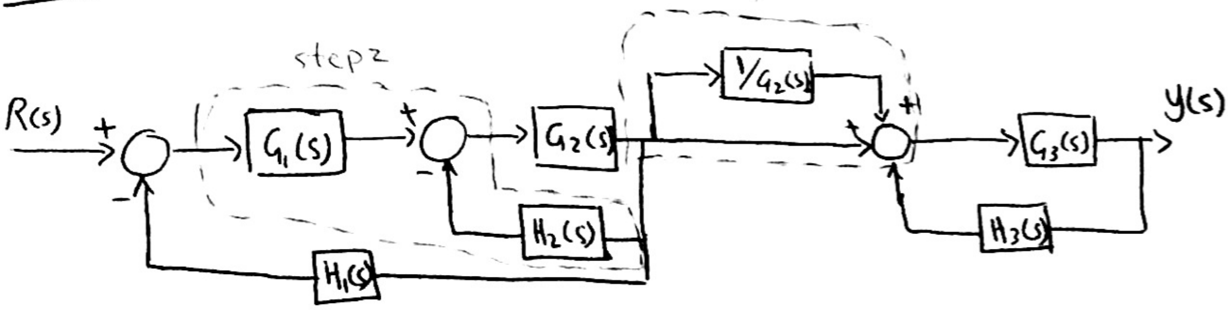




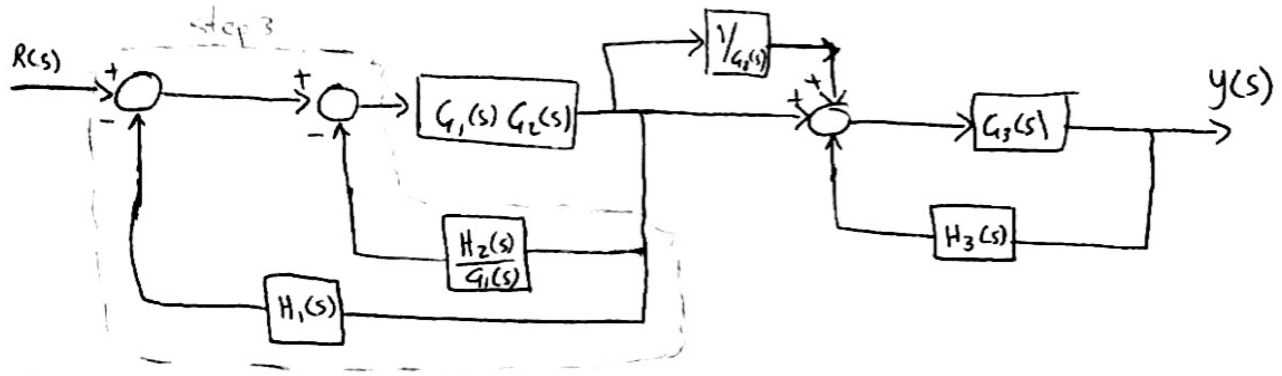
<u>Example:</u> Reduce this system to a single transfer function:



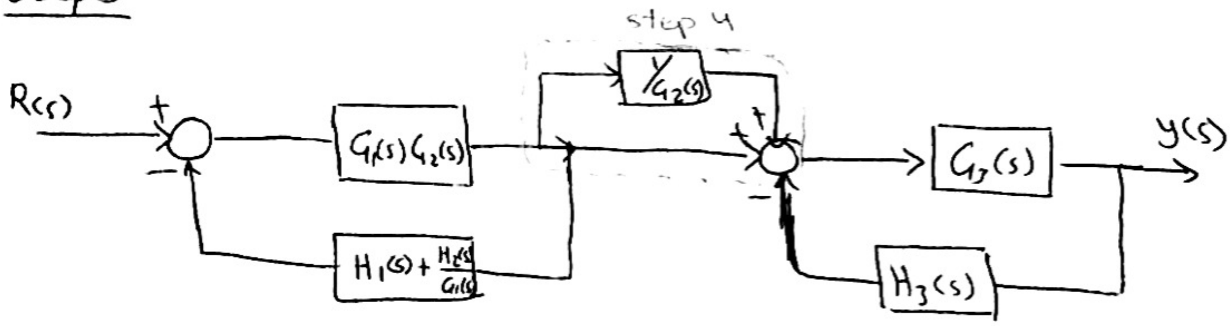
<u>step 1</u>



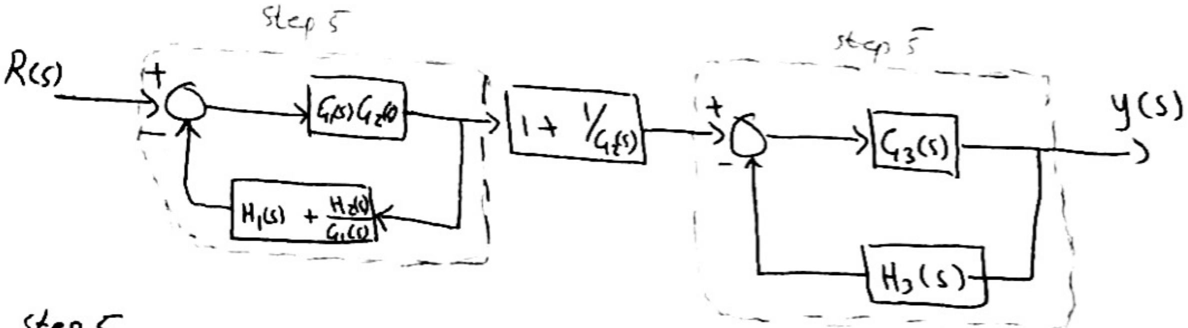
step 2



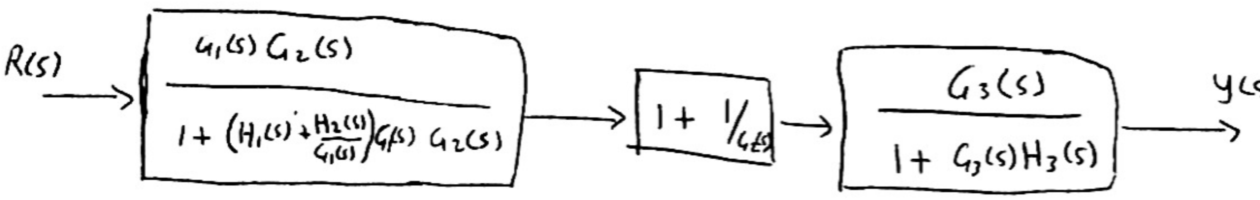
step 3



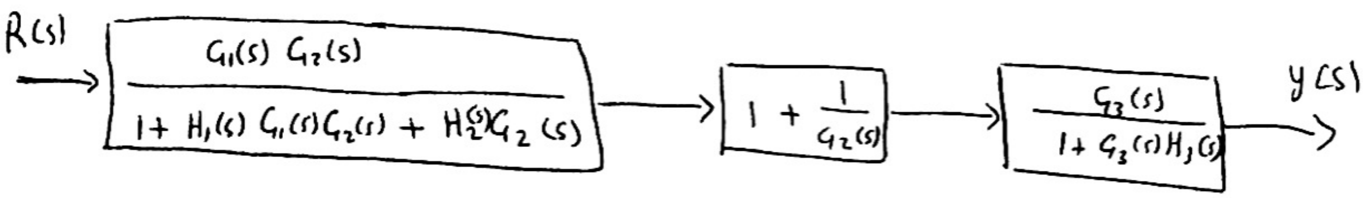
step 4

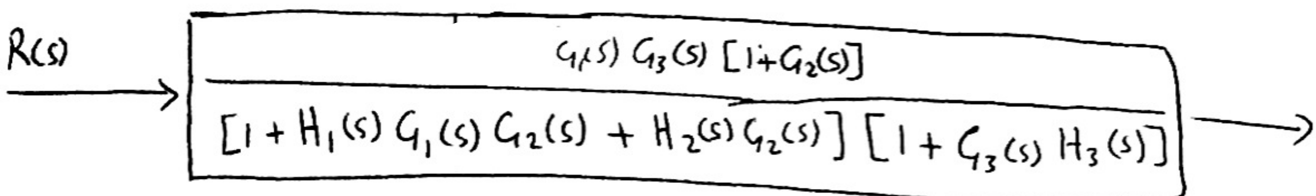


step 5



Simplify, then multiply





Gain Formula: (Mason's Gain Formula)

$$T = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \, \Delta_k}{\Delta}$$

N: number of forewarded paths between y_{in}, y_{out}

M_k: gain of the k^{th} foreward path between y_{in}, y_{out}

Δ: determinant of the graph

$\Delta = 1 -$ (sum of all different loop gains)

+ (sum of ~~the~~ gain products of all combinations of 2 possible non touching loops)

− (sum of the gain products of all combinations of 3 possible non touching loops)

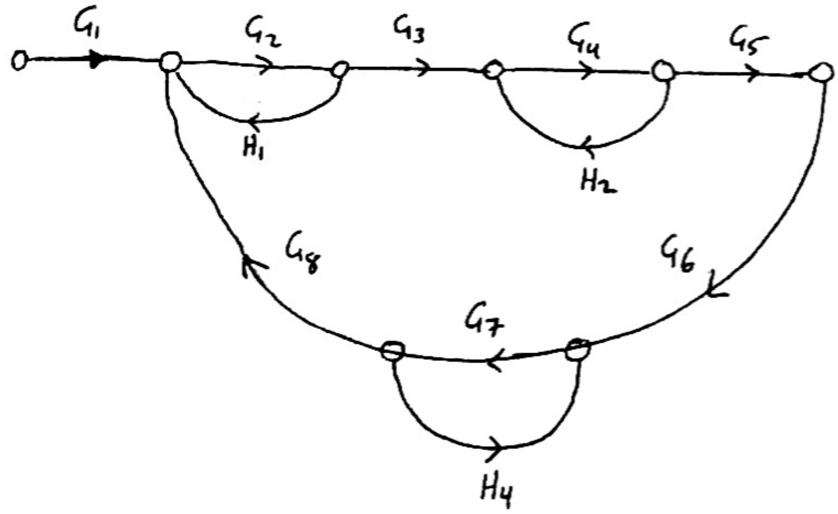
+ (- - - - - - - -)

− (. . . . . .)

Δ_k: Δ for the part of the SFG with the loops touching the k^{th} Path removed

" ——————— "

Example Find the transfer function for this SFG



ans: we have only one foreward path => $k=1$

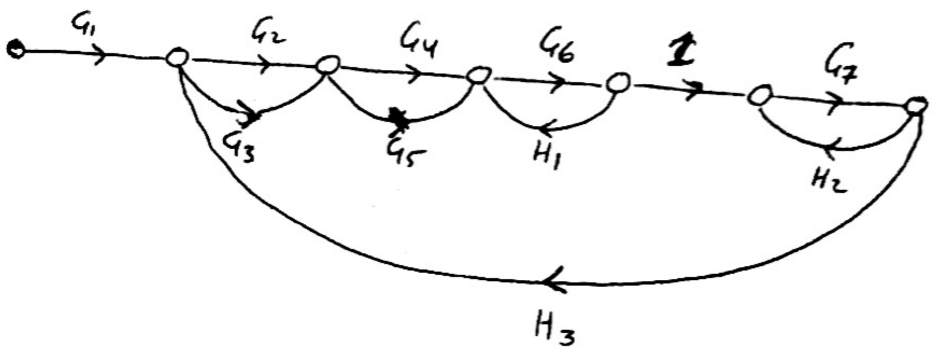
$$M_1 = G_1 G_2 G_3 G_4 G_5$$

$$\Delta_1 = 1 - H_4 G_7$$

$$\Delta = 1 - (H_1 G_2 + H_2 G_4 + G_2 G_3 G_4 G_5 G_6 G_7 G_8 + G_7 H_4)$$
$$+ (H_1 G_2 H_4 G_7 + H_2 G_4 H_4 G_7 + G_2 H_1 G_4 H_2)$$
$$- (H_1 G_2 H_2 G_4 H_4 G_7)$$

$$\Rightarrow T = \frac{(G_1 G_2 G_3 G_4 G_5)(1 - H_4 G_7)}{\Delta}$$

Example 2 find the transfer function using Mason's rule



ans: we have 4 foreward paths => $k=2$

$$M_1 = G_1 G_2 G_4 G_6 (1) G_7$$

$$\Delta_1 = 1$$

$$M_2 = G_1 G_2 G_5 G_6 (1) G_7$$

$$\Delta_2 = 1$$

$$M_3 = G_1 G_3 G_4 G_6 (1) G_7$$

$$\Delta_3 = 1$$

$$M_4 = G_1 G_3 G_5 G_6 (1) G_7$$

$$\Delta_4 = 1$$

$$\Delta = 1 - (H_1 G_6 + H_2 G_7 + H_3 G_2 G_4 G_6 G_7 + H_3 G_3 G_4 G_6 G_7 + H_3 G_3 G_5 G_6 G_7$$
$$+ H_3 G_2 G_5 G_6 G_7) + (H_1 G_6 H_2 G_7)$$

$$\Rightarrow T = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4}{\Delta}$$

$$\Rightarrow T = \frac{G_1 G_2 G_4 G_6 G_7 + G_1 G_2 G_5 G_6 G_7 + G_1 G_3 G_4 G_6 G_7 + G_1 G_3 G_5 G_6 G_7}{\Delta}$$