Problem:

Given the following FIR filter:

$$y(n) = 0.1x(n) + 0.25x(n-1) + 0.2x(n-2),$$

a. Determine the transfer function, filter length, nonzero coefficients, and impulse response.

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Solution:

$$Y(z) = 0.1X(z) + 0.25X(z)z^{-1} + 0.2X(z)z^{-2}.$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.1 + 0.25z^{-1} + 0.2z^{-2}.$$

The filter length is $K + 1 = 3$

$$b_0 = 0.1 \;\; b_1 = 0.25 \text{ and } b_2 = 0.2.$$

$$h(n) = 0.1\delta(n) + 0.25\delta(n-1) + 0.2\delta(n-2).$$

Problem:

Design a bandpass FIR filter with the following specifications:

Lower stopband = 0–500 Hz
Passband = 1,600–2,300 Hz
Upper stopband = 3,500–4,000 Hz
Stopband attenuation = 50 dB
Passband ripple = 0.05 dB
Sampling rate = 8,000 Hz

Window Type	Window Function $w(n)$, $-M \le n \le M$	Window Length, N	Passband Ripple (dB)	Stopband Attenuation (dB)
Rectangular	1	$N = 0.9/\Delta f$	0.7416	21
Hanning	$0.5 + 0.5\cos\left(\frac{\pi n}{M}\right)$	$N = 3.1/\Delta f$	0.0546	44
Hamming	$0.54 + 0.46\cos\left(\frac{\pi n}{M}\right)$	$N = 3.3/\Delta f$	0.0194	53
Blackman	$0.42 + 0.5\cos\left(\frac{n\pi}{M}\right) + 0.08\cos\left(\frac{2n\pi}{M}\right)$	$N = 5.5/\Delta f$	0.0017	74

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Solution:

$$\Delta f_1 = |1600 - 500|/8000 = 0.1375 \text{ and } \Delta f_2 = |3500 - 2300|/8000 = 0.15$$

$$N_1 = 3.3/0.1375 = 24 \text{ and } N_2 = 3.3/0.15 = 22$$

Choosing $N = 25$ filter coefficients using the Hamming window method:

$$f_1 = (1600 + 500)/2 = 1050\,\text{Hz and } f_2 = (3500 + 2300)/2 = 2900\,\text{Hz}.$$

The normalized lower and upper cutoff frequencies are calculated as:

$$\Omega_L = \frac{1050 \times 2\pi}{8000} = 0.2625\pi \text{ radians and}$$

$$\Omega_H = \frac{2900 \times 2\pi}{8000} = 0.725\pi \text{ radians,}$$

Problem:

Design a bandstop FIR filter with the following specifications:

Lower cutoff frequency = 1,250 Hz
Lower transition width = 1,500 Hz
Upper cutoff frequency = 2,850 Hz
Upper transition width = 1,300 Hz
Stopband attenuation = 60 dB
Passband ripple = 0.02 dB
Sampling rate = 8,000 Hz

Window Type	Window Function $w(n)$, $-M \le n \le M$	Window Length, N	Passband Ripple (dB)	Stopband Attenuation (dB)
Rectangular	1	$N = 0.9/\Delta f$	0.7416	21
Hanning	$0.5 + 0.5\cos\left(\frac{\pi n}{M}\right)$	$N = 3.1/\Delta f$	0.0546	44
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Lower transition width = 1,500 Hz
Upper cutoff frequency = 2,850 Hz
Upper transition width = 1,300 Hz
Stopband attenuation = 60 dB
Passband ripple = 0.02 dB
Sampling rate = 8,000 Hz

Solution:

$$\Delta f_1 = 1500/8000 = 0.1875, \quad \text{and } \Delta f_2 = 1300/8000 = 0.1625.$$

The filter lengths are determined using the Blackman window as:

$$N_1 = 5.5/0.1875 = 29.33, \quad \text{and } N_2 = 5.5/0.1625 = 33.8.$$

$$N = 35.$$

$$\Omega_L = \frac{2\pi \times 1250}{8000} = 0.3125\pi \text{ radian and}$$

$$\Omega_H = \frac{2\pi \times 2850}{8000} = 0.7125\pi \text{ radians,}$$

Problem:

Given the following IIR filter:

$$y(n) = 0.2x(n) + 0.4x(n-1) + 0.5y(n-1),$$

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a. Determine the transfer function, nonzero coefficients, and impulse response.

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.2 + 0.4z^{-1}}{1 - 0.5z^{-1}}.$$

$$b_0 = 0.2,\ b_1 = 0.4,\ \text{and}\ a_1 = -0.5.$$

$$H(z) = \frac{0.2}{1 - 0.5z^{-1}} + \frac{0.4z^{-1}}{1 - 0.5z^{-1}}.$$

$$h(n) = 0.2(0.5)^n u(n) + 0.4(0.5)^{n-1} u(n-1).$$

$$h(0) = 0.2,\ h(1) = 0.7,\ h(2) = 0.25, \ldots.$$

Given an analog filter whose transfer function is

$$H(s) = \frac{10}{s + 10},$$

Convert it to the digital filter transfer function

$$T = 0.01$$

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Applying the BLT, we have

$$H(z) = H(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \left.\frac{10}{s + 10}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}}.$$

$$H(z) = \frac{10}{\frac{200(z-1)}{z+1} + 10} = \frac{0.05}{\frac{z-1}{z+1} + 0.05} = \frac{0.05(z+1)}{z - 1 + 0.05(z+1)} = \frac{0.05z + 0.05}{1.05z - 0.95}.$$

$$H(z) = \frac{(0.05z + 0.05)/(1.05z)}{(1.05z - 0.95)/(1.05z)} = \frac{0.0476 + 0.0476z^{-1}}{1 - 0.9048z^{-1}}.$$