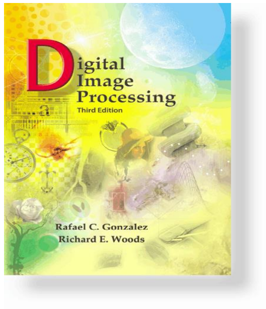
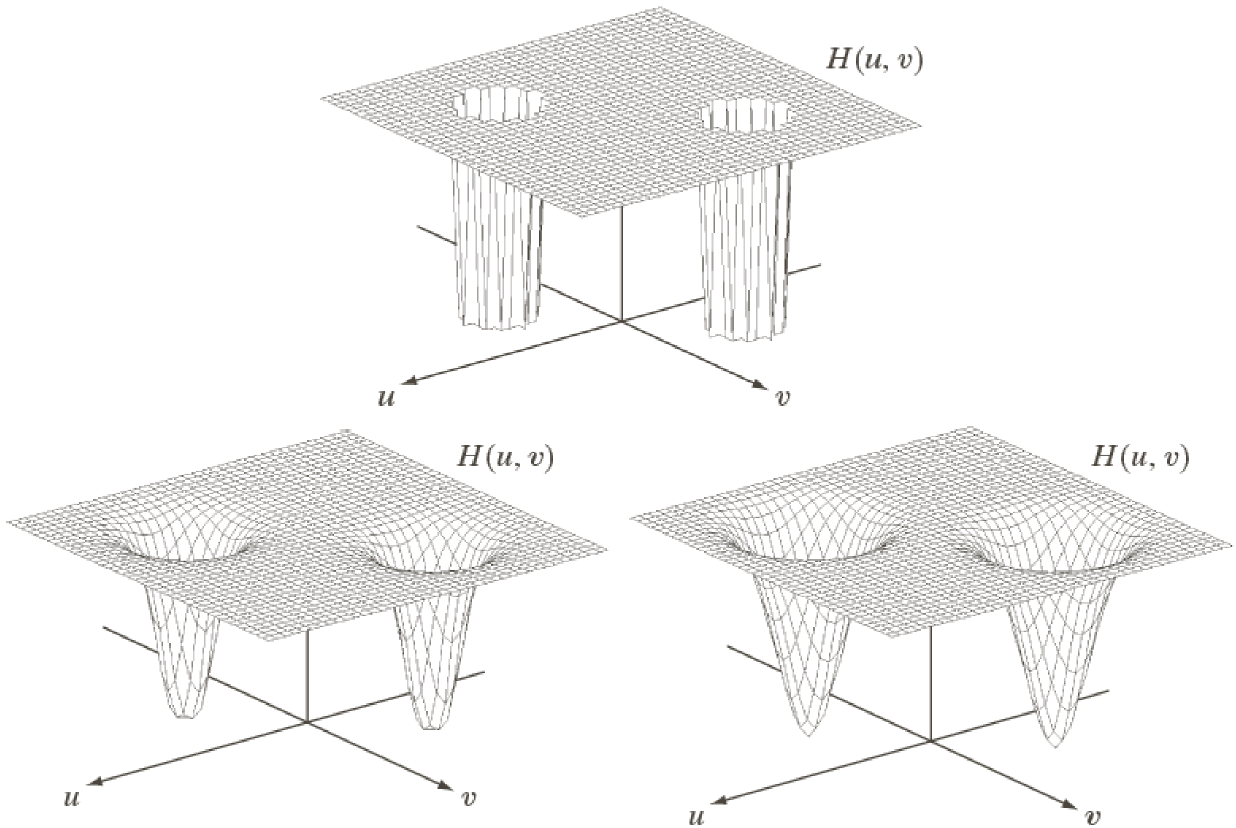


Notch Filtering

a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



Optimum Notch Filtering - I

Methods discussed before: filter too much image information.

Optimum: minimizes local variances of the restored image, $\hat{f}(x,y)$

First Step:

Extract principal freq. components of the interference pattern.

→ Place a notch pass filter at the location of each spike.

$$N(u,v) = H_{NP}(u,v)G(u,v)$$

Second Step:

Find corresponding pattern in spatial domain.

$$n(x,y) = \mathfrak{I}^{-1}\{H_{NP}(u,v)G(u,v)\} \qquad \text{Eq.1}$$

Optimum Notch Filtering - II

Third Step:

Subtract weighted noise estimation from noisy image.

$$\hat{f}(x,y) = g(x,y) - w(x,y)n(x,y) \qquad \text{Eq.2}$$

How to get w(x,y)?

Select w(x,y) so that the variance of $\hat{f}(x,y)$ is minimized over a neighborhood.

Consider: neighborhood size: (2a+1) by (2b+1)

Average: $$\bar{\hat{f}}(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s, y+t)$$

Local variance: $$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x,y) \right]^2$$

Optimum Notch Filtering - III

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left\{ \begin{array}{l} [g(x+s,y+t) - w(x,y)n(x+s,y+t)] \\ -[\bar{g}(x,y) - w(x,y)\bar{n}(x,y)] \end{array} \right\}^2$$

To minimize, $\dfrac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$

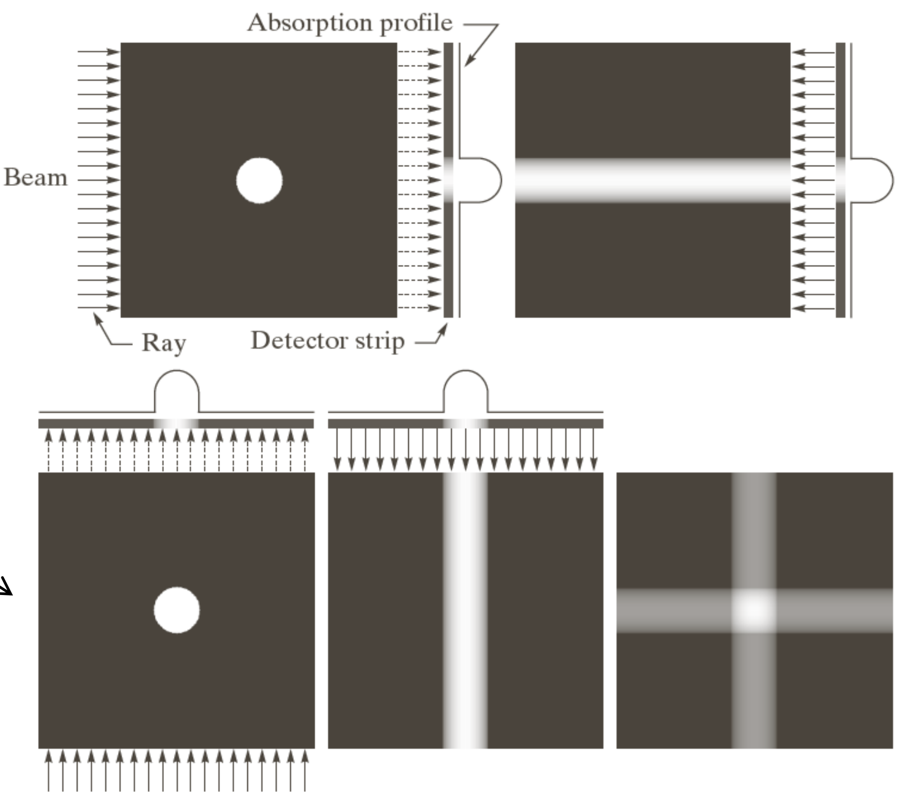
We find, $w(x,y) = \dfrac{\overline{g(x,y)n(x,y)} - \bar{g}(x,y)\bar{n}(x,y)}{\overline{n^2}(x,y) - \bar{n}^2(x,y)}$ Eq.3

Get noise reduced image from Eq. 1, 2, 3.

Image Reconstruction : projection

Computed Tomography (CT)

X-Rays from different angles.



Back projection

Angle varied: 90°

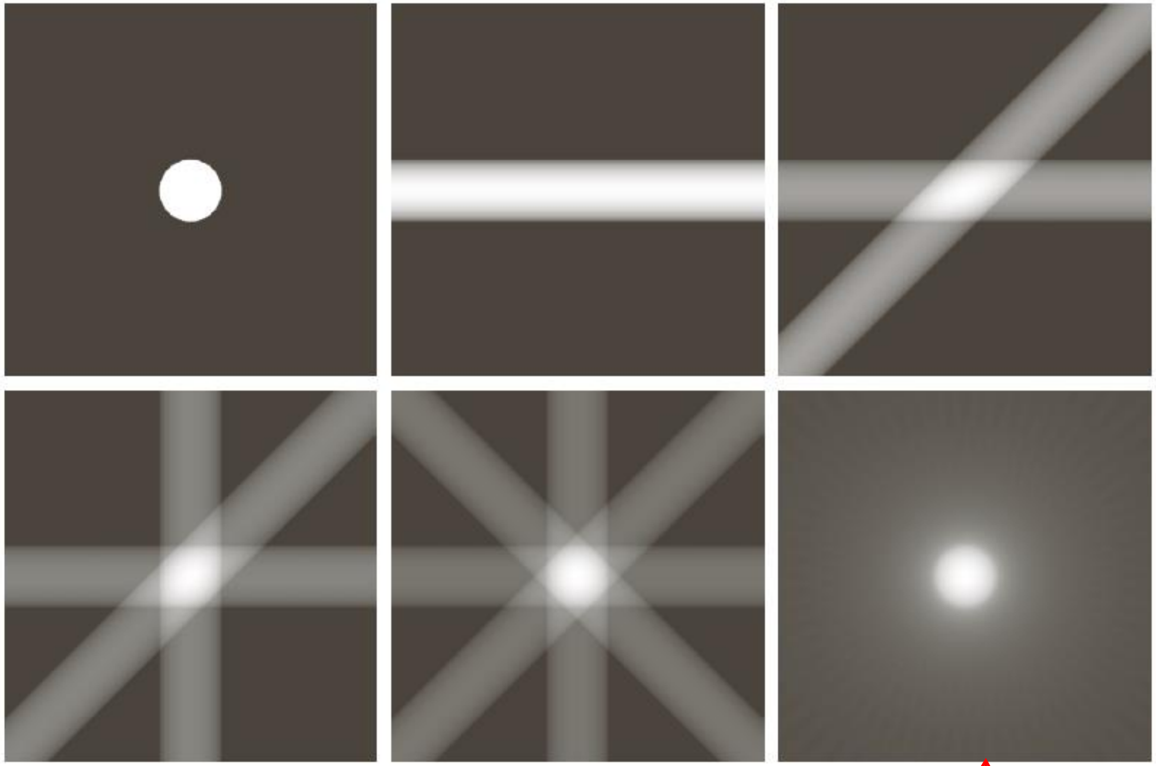
Sum of two back projections

Projection

Projection using many angles:
more true construction of the original image.

a	b	c
d	e	f

FIGURE 5.33
(a) Same as Fig. 5.32(a).
(b)–(e) Reconstruction using 1, 2, 3, and 4 backprojections 45° apart.
(f) Reconstruction with 32 backprojections 5.625° apart (note the blurring).



Sum of 32 back projections.

Principles of CT

G1: Pencil X-Ray beam; one detector; angle [0° ~ 180°].

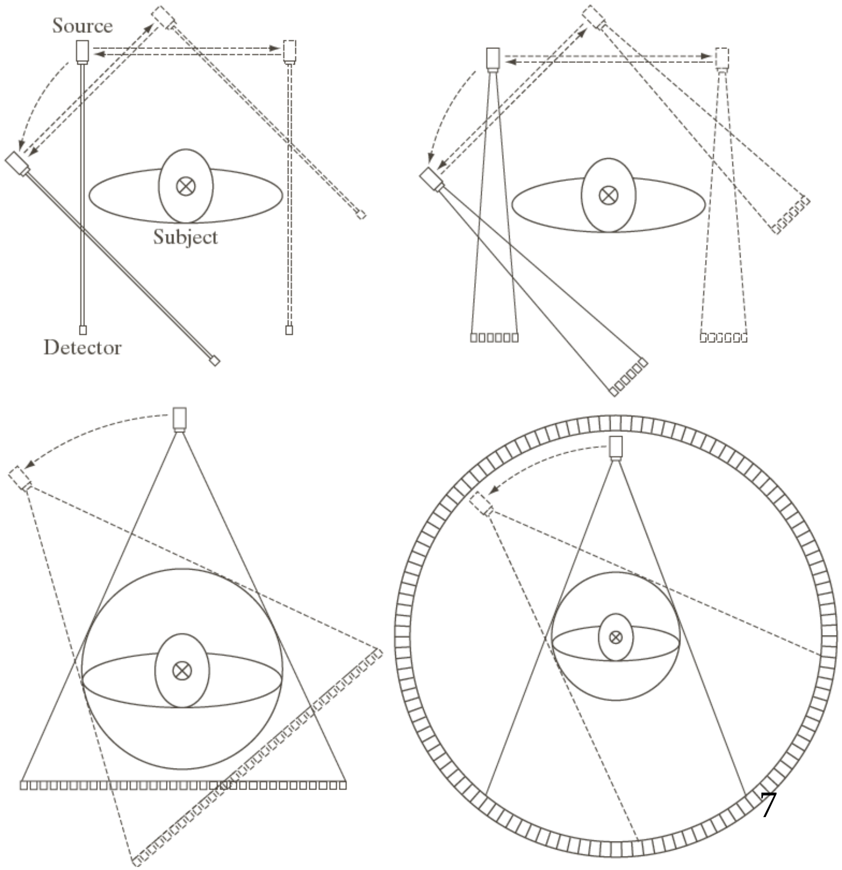
G2: Fan X-Ray beam; multiple detectors.

G3: Wider X-Ray beam; a bank of detectors (1000).

G4: Circular positioned detectors (5000).

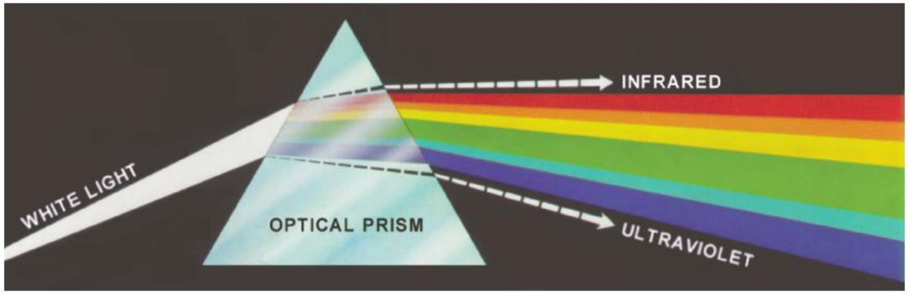
G5: No mechanical motion, uses electron beams controlled electromagnetically.

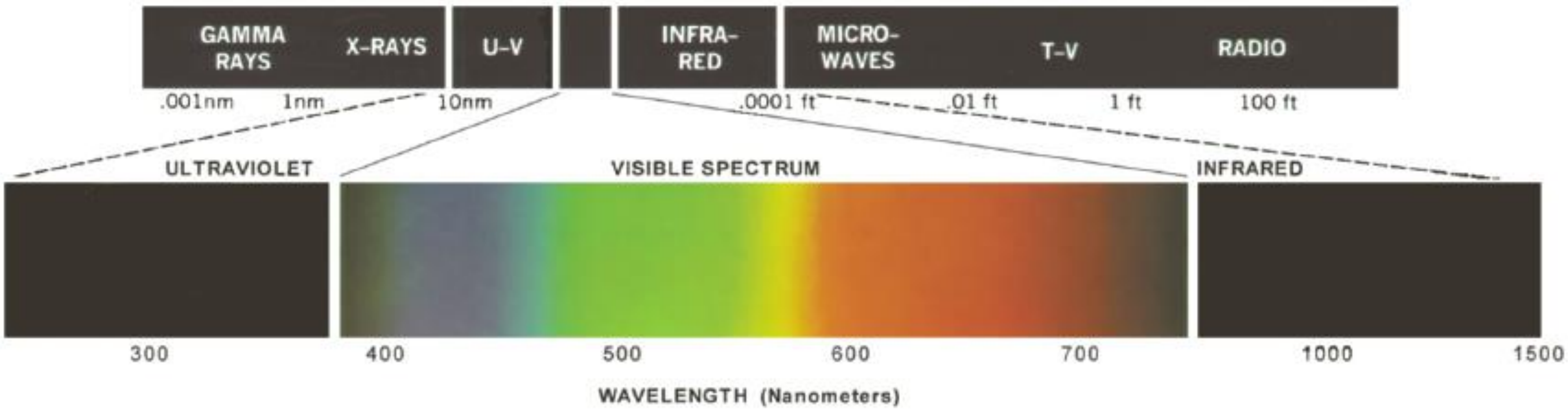
G6, G7, …..



Color Fundamentals

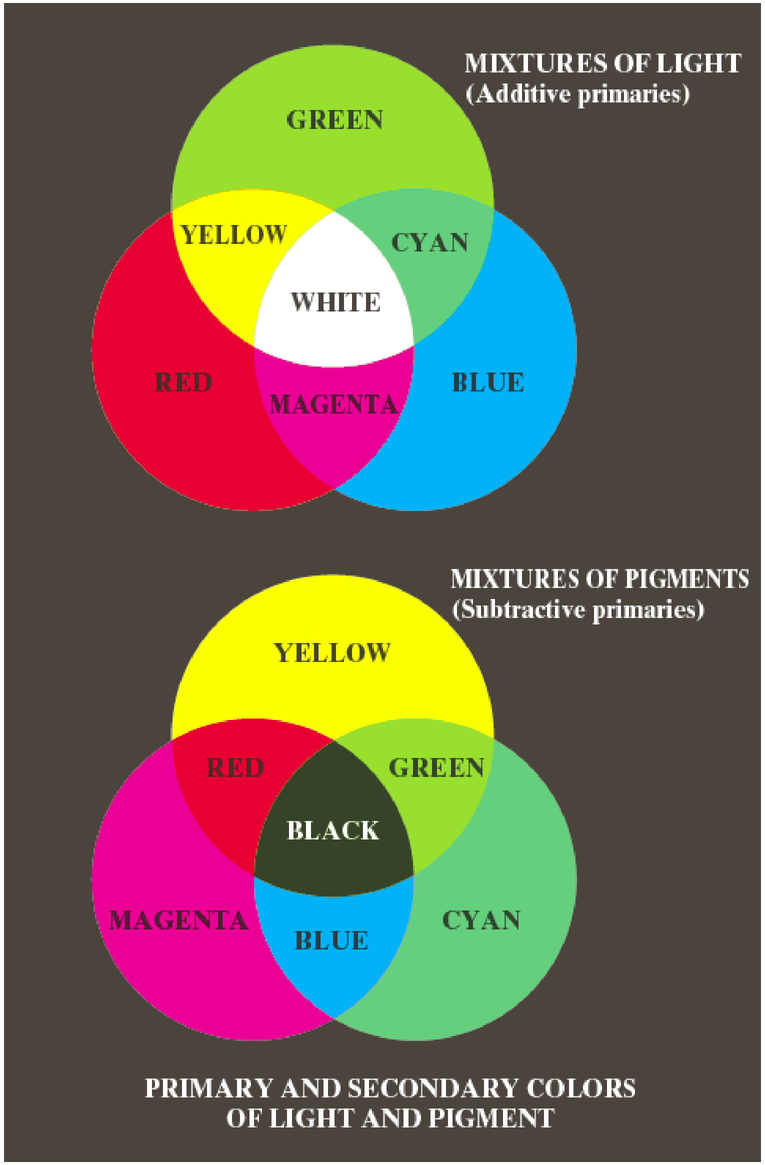
White color: composed of 6 visible colors.





Primary & Secondary Colors

- Green
- Red
- Blue



- Yellow
- Magenta
- Cyan

Characteristics of Colors

1. **Brightness**: how bright (intensity) the color is. [Perception.]

2. **Hue**: dominant wavelength in a mixture of light waves.

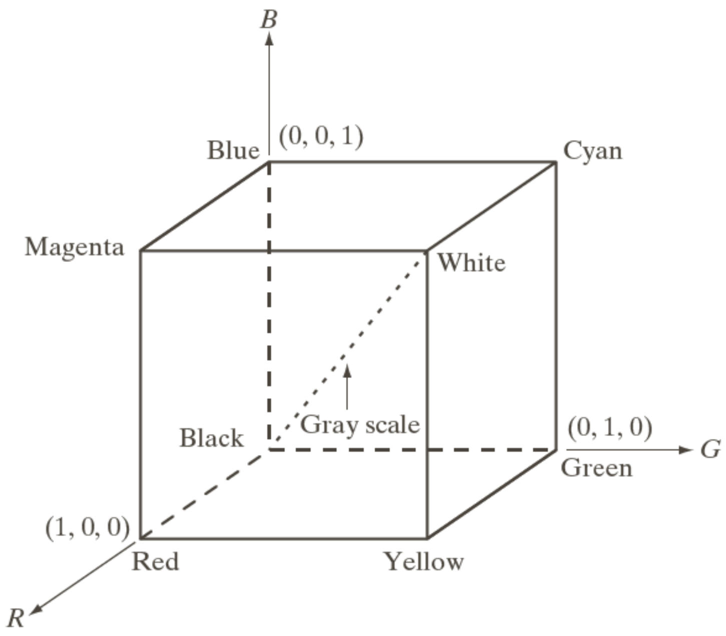
3. **Saturation**: the amount of white light mixed with a hue.

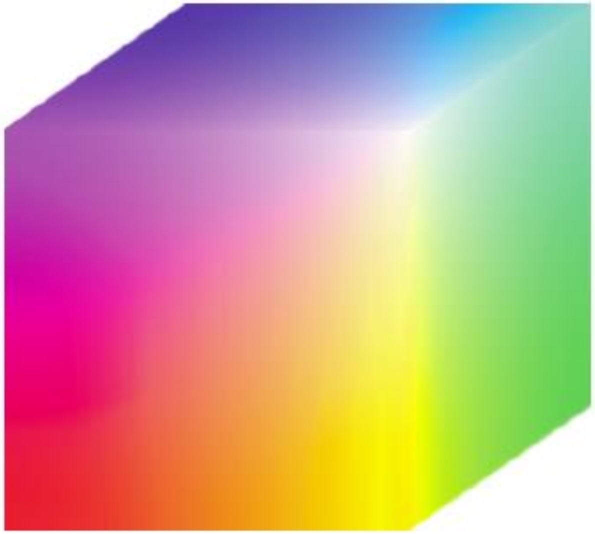
↓

Inversely proportional

Hue + Saturation = Chromaticity

RGB Color Model





24-bit color cube.

Black (0,0,0)
White (1,1,1)

All R, G, B values are normalized to [0, 1] range.

Safe Color

Many systems in use today are limited to 256 colors.

Forty (40) of these are processed differently by various operating systems.

The rest 216 are common: de facto standard for safe colors.

Number System	Color Equivalents					
Hex	00	33	66	99	CC	FF
Decimal	0	51	102	153	204	255

TABLE 6.1 Valid values of each RGB component in a safe color.

$(6)^3 = 216$

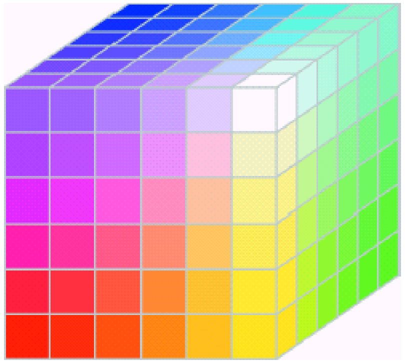


FIGURE 6.11 The RGB safe-color cube.

CMY and CMYK Color Models

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

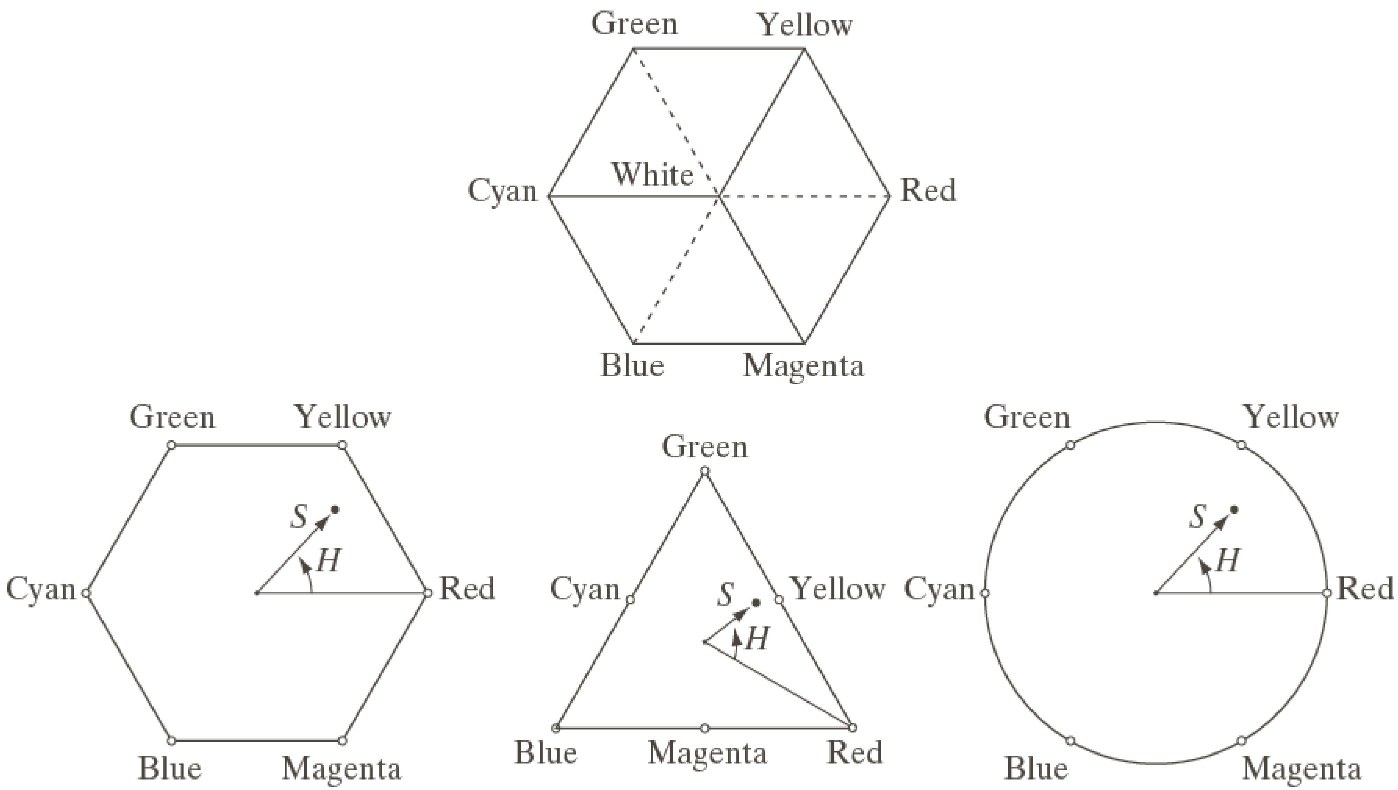
- Used as primary colors in printers.

- Equal amounts of C, M, Y should produce Black. But in practice it produces muddy-looking black.

- Four-color printing: add a fourth color: Black to CMY producing CMYK model.

HSI Color Model - I

Hue, Saturation, Intensity.

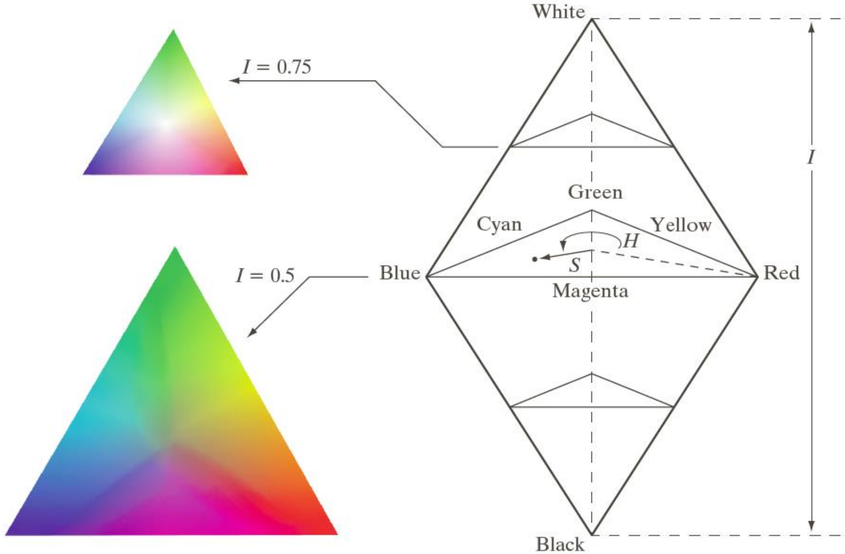


a
b c d

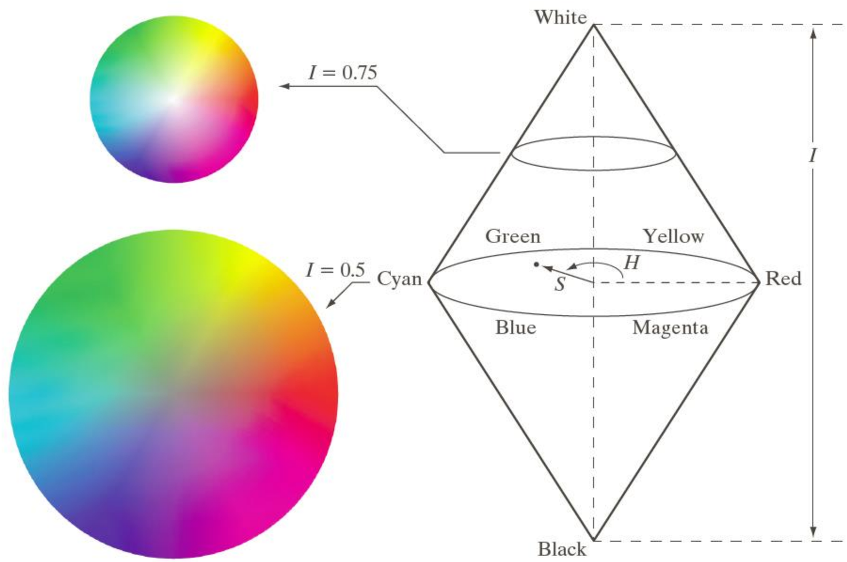
FIGURE 6.13 Hue and saturation in the HSI color model. The dot is an arbitrary color point. The angle from the red axis gives the hue, and the length of the vector is the saturation. The intensity of all colors in any of these planes is given by the position of the plane on the vertical intensity axis.

HSI Color Model - II

Triangular Plane



Circular Plane



Fundamentals of Image Compression

Relative data redundancy, $R = 1 - \frac{1}{C}$

Compression ratio: $C = \frac{b}{b'}$

- b ← Bits required in the method
- b' ← Bits required in the *base* method

Average number of bits required to represent each pixel:

$$L_{Avg} = \sum_{k=0}^{L-1} l(r_k) P_r(r_k)$$

$$P_r(r_k) = \frac{n_k}{MN}, k = 0, 1, 2, ..., L-1$$

Variable Length Coding

Table 1.

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

For code 1: $l_1(r_k)$ = 8 bits for al r_k. Average = 8.

For code 2: $L_{Avg} = 0.25(2) + 0.47(1) + 0.25(3) + 0.03(3) = 1.81$ bits

$$C = \frac{256 \times 256 \times 8}{256 \times 256 \times 1.81} \approx 4.42 \qquad R = 1 - \frac{1}{4.42} = 0.774$$

77.4% of the data in the original 8-bit intensity array is redundant.

Irrelevant Information

Spatial redundancy

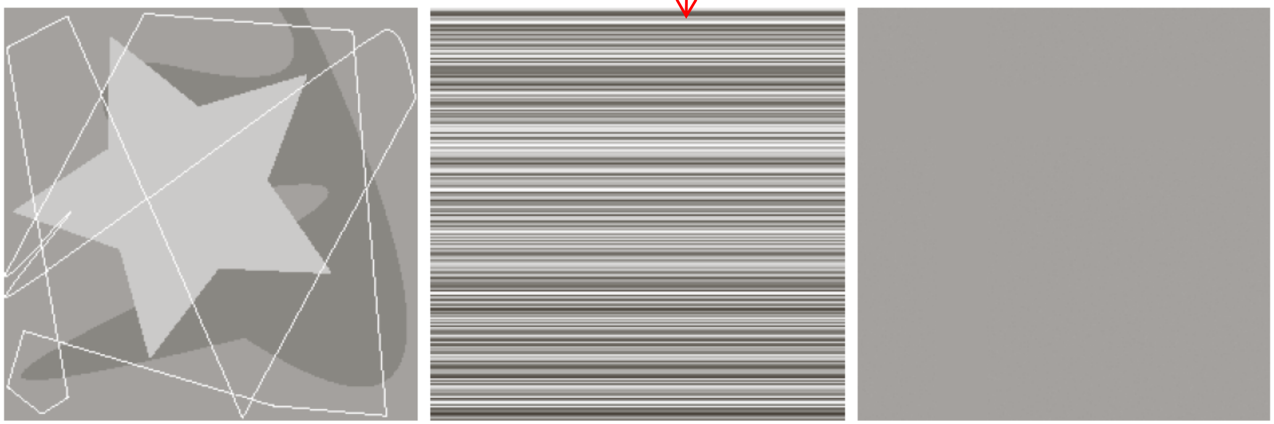
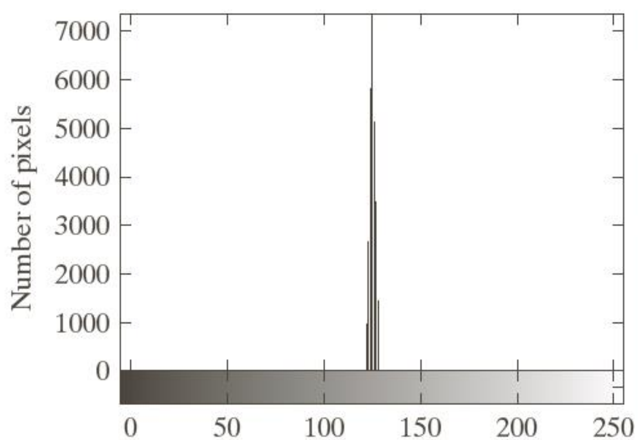
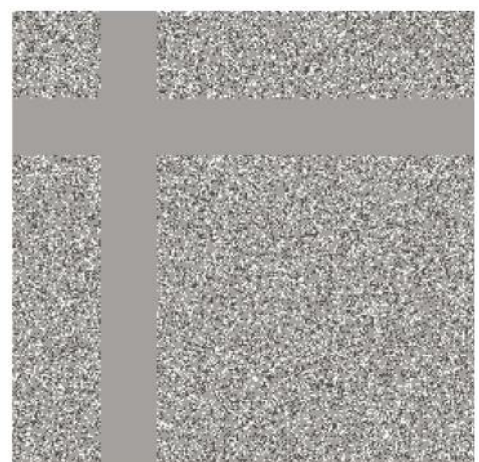


Figure 8.1

Histogram

Histogram
equalized image





Entropy

Entropy: $$H = -\sum_{k=0}^{L-1} p_r(r_k) \log_2 p_r(r_k)$$

Entropy in Table 1.: 1.6614 bits / pixel → Figure 1 (a)

Entropy of Figure 1 (b) = 8 bits / pixel → Higher entropy than Fig. 1 (a)

Entropy of Figure 1 (c) = 1.566 bits / pixel

↓ Little or no information

↘ But comparable entropy with Fig. 1(a)!

Entropy of an image is far from intuitive.

Fidelity Criteria - I

Quantifying the nature of the loss, after removal of 'noise'.

Objective: mathematical expression such as *rms error*, SNR, etc.

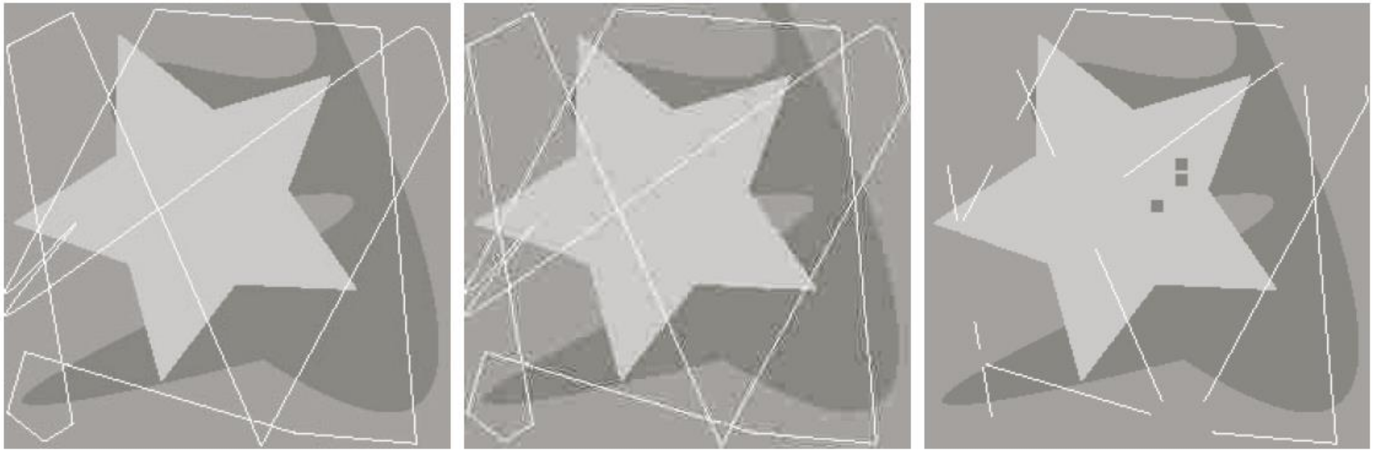
$$e_{rms} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x, y) - f(x, y) \right]^2 \right]^{1/2}$$

Subjective: Human evaluation.

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Fidelity Criteria - II

Misleading image



rms error: 5.17 15.67 14.17

Objective criteria fails.

Image Compression Models

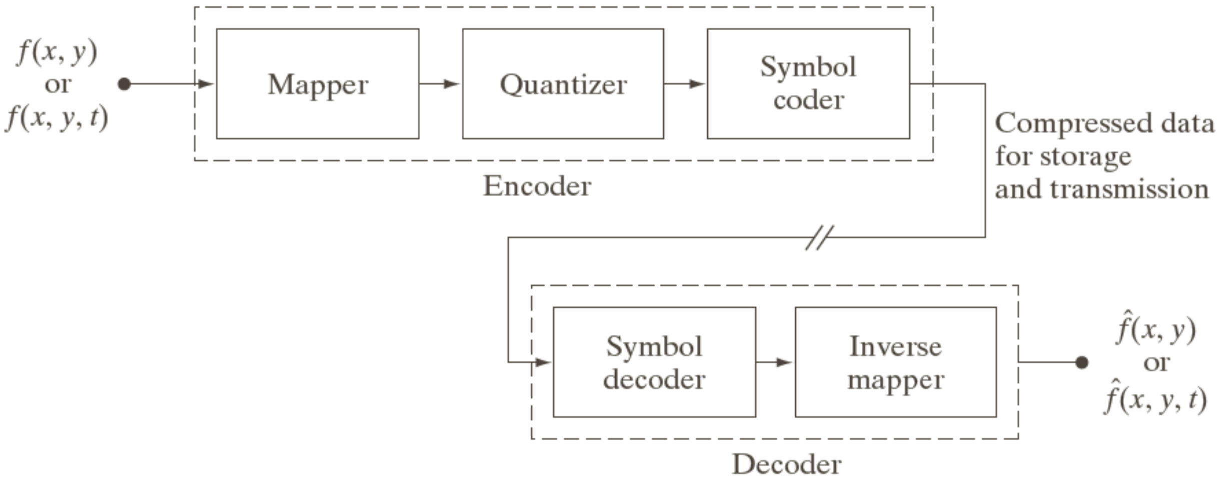


FIGURE 8.5 Functional block diagram of a general image compression system.

Quantizer: irreversible.

Some Compression Standards

Image Compression Standards, Formats, and Containers

- **Still Image**
  - **Binary**
    - CCITT Group 3
    - CCITT Group 4
    - JBIG (or JBIG1)
    - JBIG2
    - TIFF
  - **Continuous Tone**
    - JPEG
    - JPEG-LS
    - JPEG-2000
    - BMP
    - GIF
    - PDF
    - PNG
    - TIFF
- **Video**
  - DV
  - H.261
  - H.262
  - H.263
  - H.264
  - MPEG-1
  - MPEG-2
  - MPEG-4
  - MPEG-4 AVC
  - AVS
  - HDV
  - M-JPEG
  - QuickTime
  - VC-1 (or WMV9)

Huffman Coding

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

Encoded: 010100111100

Decoded: $a_3 a_1 a_2 a_2 a_6$

Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

Average length of this code:

$$L_{Avg} = (0.4)(1) + (0.3)(2) + (0.1)(3) + (0.06)(5) + (0.04)(5)$$

$$= 2.2 \text{ bits/pixel}$$

Golomb Coding

$$G_m(n)$$

Step 1: Form the unary code of quotient $\lfloor n/m \rfloor$.

Step 2: Let $k = \lceil \log_2 m \rceil$, $c = 2^k - m$, $r = n \bmod m$, compute r'

$$r' = \begin{cases} r & \text{truncated to } k-1 \text{ bits} \quad 0 \le r < c \\ r+c & \text{truncated to } k \text{ bits} \quad \text{otherwise} \end{cases}$$

Step 3: Concatenated the results of steps 1 and 2.

Example: Compute $G_4(9)$.

$\lfloor 9/4 \rfloor = \lfloor 2.25 \rfloor = 2$ ⟶ Unary code: 110

$k = 2$, $c = 2^2 - 4 = 0$, $r = 1$ (0001). ⟶ r' = 01 (truncated to 2 bits).

After concatenating: $G_4(9) = 11001$

Exponential Golomb Coding

$$G^{k}_{\exp}(n)$$

Step 1: Find an integer i >= 0 such that $\sum_{j=0}^{i-1} 2^{j+k} \le n < \sum_{j=0}^{i} 2^{j+k}$

And form the unary code of i.

If k = 0, i = $\lfloor \log_2(n+1) \rfloor$

Step 2: Truncate the binary representation of

$$n - \sum_{j=0}^{i-1} 2^{j+k} \quad \text{for } k+i \text{ least significant bits}$$

Step 3: Concatenated the results of steps 1 and 2.

Example: Compute $G^{0}_{\exp}(8)$.

i = 3, because k = 0. → 1110

Check for the equation in Step 1.

$$8 - \sum_{j=0}^{3-1} 2^{j+0} = 8 - 7 = 1 = 0001 \xrightarrow{\text{Truncate}} 001$$

After concatenating: $G^{0}_{\exp}(8)$ = 1110001