	Topics	Problem Set
Ch 16	**16.1** Functions of Several Variables.	**16.1:** 1, 3, 5, 8, 9, 14, 15, 17, 23, 37, 40, 41, 47.
	16.2 Limits and Continuity.	**16.2:** 3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 28, 29, 32, 36, 38, 42. + ***Sheet 1.***
	16.3 Partial Derivatives.	**16.3:** 4, 6, 8, 12, 13, 16, 17, 21, 23,27, 29, 32, 34, 36, 39, 42, 44, 47 + ***Sheet 2.***
	16.4 Increments and Differentials.	**16.4:** 2, 9, 11, 12, 16, 20, 24(b), 41, 42 + ***Sheet 3.***
	16.5 Chain Rules.	**16.5:** 4, 6, 10, 12, 14, 18, 19, 22, 26, 38, 40, 42 + ***Sheet 4.***
	16.8 Extrema of Functions of Several Variables.	**16.8:** 5,11, 15, 20, 23, 24, 26, 30, 31, 32 + ***Sheet 5.***
	16.9 Lagrange Multipliers .	**16.9:** 1, 2, 3, 11.
Ch 17	**17.1** Double Integrals.	**17.1:** 1 to10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50 + ***Sheet 6.***
	17.2 Area and Volume.	**17.2:** 2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32 + ***Sheet 7***
	17.3 Double Integrals in Polar Coordinates.	**17.3:** 1 to 12, 13, 15, 17, 18, 19, 21, 23, 24 + ***Sheet 8.***
	17.5 Triple Integrals.	**17.5:** 2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28 + ***Sheet 9.***
Ch 11	**11.1** Sequences.	**11.1:** 3,5,7,11,12,13,16,17,18,23,24,28,29,30,31,32,33,34,36, 37,39,41,42.
	11.2 Convergent or Divergent Series.	**11.2:** 2,4,5,6,8,10,14,15,18,20,25,28,30,34,37,39,42,43,45,46.
	11.3 Positive-Term Series.	**11.3:** 2,3,5,7,9,11,14,15,16,18,20,22,24,25,30,33,34,35,39,40, 42,43,45,46,51,52,57,58.
	11.4 The Ratio and Root Tests.	**11.4:** 2,4,6,8,10,11,14,15,18,20,21,23,25,27,28,29,31,33,35,38
	11.5 Alternating Series and Absolute Convergence.	**11.5:** 2,3,5,7,9,10,12,13,16,18,20,21,22,27,29,32,33,35,38,41, 43,45,46.
	11.6 Power Series.	**11.6:** 5,6,7,14,15,19,23,25,27,30,35,36,41,42.
	11.7 Power Series Representations of Functions.	**11.7:** 2,4,6,7,10,13,14,16,19,22,25,29,30,32,33,34,37.
	11.8 Maclaurin and Taylor Series.	**11.8:** 2,4,8,10,13,15,18,19,21,26,29,32,34,36,38,39,42.

Sheet 1:

1. Find the following limits, if they exist:

1) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy^2}{x^2+y^2+z^2}$,

2) $\lim_{(x,y)\to(2,1)} \dfrac{(y-1)(x-2)^2}{(y-1)^3+(x-2)^3}$.

3) $\lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^6}$,

4) $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^4+y^2}$,

5) $\lim_{(x,y)\to(0,0)} \dfrac{10xy}{5x^3+2y^3}$

6) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{y^3+x^3\sin z^3}{x^2+y^2+z^2}$,

7) $\lim_{(x,y)\to(0,0)} \dfrac{x^3-x^2y+xy^2-y^3}{x^2+y^2}$

8) $\lim_{(x,y)\to(0,0)} \left[\dfrac{4x^2y}{x^4+y^2}+\dfrac{y^4}{x^2+y^2}\right]$,

9) $\lim_{(x,y)\to(1,-1)} \dfrac{2x-y}{x^2+y^2}$

$= \dfrac{2(1)+1}{1+1} = \dfrac{3}{2}$

2. Discuss the continuity of the following functions on their domain:

1. $f(x,y) = \begin{cases} \dfrac{x^2y}{x^4+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

2. $f(x,y,z) = \begin{cases} \dfrac{x^3+y^3+z^3}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

3. $f(x,y,z) = \begin{cases} \dfrac{xz-y^2}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

4. $f(x,y) = e^{x^2+5xy+y^3}$.

5. $h(x,y) = \sin(\sqrt{y-4x^2})$.

6. $k(x,y,z) = \ln(36-4x^2-y^2-9z^2)$.

Sheet 2:

1. Using the definition, find f_x, f_y of the function

$$f(x,y) = 3x^2 - 2xy + y^2.$$

2. Discuss the continuity of the function f at (0,0), where

$$f(x,y) = \begin{cases} \frac{\sin xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Does f_x and f_y exist at $(0,0)$.

3. Find f_x, f_y at (0,0), if they exist:

$$\text{Let } f(x,y) = \begin{cases} \frac{x^2 y}{x^3+y^3}, & x^3+y^3 \neq 0 \\ 0, & x^3+y^3 = 0 \end{cases}$$

4. Find f_x, f_y at (1,2), if they exist:

$$\text{Let } f(x,y) = \begin{cases} \frac{3x^2}{2x-y} + \frac{y^3}{y-2x}, & y \neq 2x \\ 12, & y = 2x \end{cases}$$

5. Find f_x, f_y at (1,1) and (0,0), if they exist:

$$\text{Let } f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$$

6. Let $f(x,y) = e^{x-y}\sin(x+y)$. Show that

$$(f_x)^2 + (f_y)^2 = \frac{2(f(x,y))^2}{\sin^2(x+y)}.$$

Sheet 3:

1. Use the differential to approximate the change in the function

$$w = f(x, y, z) = x^2 \ln (z^2 + y^2)$$

dw

as (x, y, z) changes from $(1, 2, 3)$ to $(0.9, 1.9, 3.1)$.

2. Use the differential to approximate the change in the function

$$w = f(x, y) = yx^{\frac{2}{5}} + x\sqrt{y}$$

dw

as (x,y) changes from $(52,16)$ to $(35,18)$.

3. Discuss the continuity and the differentiability of the functions in problems 3 and 4 in section 16.3 (part II) as the indicated points.

4. Discuss the differentiability of the function f at the points $(0,0)$ and $(-1,1)$, where

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

5. Let $$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Show that f is continuous but not differentiable at the point $(0,0)$.

6. Let $$f(x,y,z) = \begin{cases} \frac{xy^2z}{x^4+y^4+z^4}, & (x,y,z) \neq (0,0,0) \\ 0, & (x,y,z) = (0,0,0) \end{cases}$$

$$\frac{ab^2c\,t^4}{(a^4+b^4+c^4)\,t^4} = \frac{ab^2c}{a^4+b^4+c^4}$$

* Show that $f_x(0,0,0)$, $f_y(0,0,0)$ and $f_z(0,0,0)$ exist.

* Discuss the differentiability of f at $(0,0,0)$. not continuous $\Rightarrow$ not d.

7. Let $$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

$$f_y = \frac{3xy^2(x^2+y^4) - xy^3(4y^3)}{(x^2+y^4)^2} =$$

* Show that f_x and f_y exist for all $(x,y) \in \mathbb{R}^2$.

* Show that f_x and f_y are not continuous at point $(0,0)$.

* Show that f is not differentiable at $(0,0)$.

Theorem 16.18
If a function ...
variables is di...
at (x_0, y_0) the...

Sheet 4:

1. If $w = f(x, y)$ such that $x = r\cos\theta$, and $y = r\sin\theta$. Find $g(r, \theta)$ such that the equation below holds

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + g(r, \theta)\left(\frac{\partial w}{\partial \theta}\right)^2.$$

2. If $u = f(x, y)$ where $y = e^t$ and $x = e^s$. Find

$$\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}.$$

3. If $w = x^2 + y^2 + z^2$, where $x = r\cos\theta$, $y = r\sin\theta$ and $z = r$. Use the differential to show that $dw = 4r\,dr$.

4. Let $z = f(x, y)$ be determined implicitly by $yx^2 + z^2 + \cos(xyz) - 4 = 0$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. Then show that

$$2y\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x} = \frac{xyz\sin(xyz)}{2z - xy\sin(xyz)}.$$

Sheet 5: 16.8

1. Find the extrema of the function $f(x, y) = (x-4)^2 + y^2$, on the region R bounded by $y = 4\sqrt{x}$ and $y = 4x$.

2. Let $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$, where $x \neq 0$ and $y \neq 0$. Find the local extrema and the saddle points if they exist.

3. Find the maximum and the minimum of the function $f(x, y) = x^3 - 3x + y^2$ on the region bounded by $x^2 - 2x + y^2 = 0$.

$(x-1)^2 + y^2 = 1$

Sheet 6:

1. Sketch the region bounded by the graphs of the given equations, then evaluate the given integrals

$$a)\ y = x, y = \sqrt{x}, x = 0; \quad \iint_R \sin y^2 \, dA.$$

$$b)\ y = x^{3/2}, y = 0, x = 1; \quad \iint_R y e^{x^2} \, dA.$$

2. Evaluate the double integral

$$\int_0^2 \int_{y/2}^1 e^{x^2} \, dx \, dy.$$

Sheet 7:

1. Sketch the region bounded by the graphs $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$ and use the double integral to compute its area.

2. Sketch the region bounded by the graphs $x = -\sqrt{9 - y^2}$, $y = -2x + 9$,

$y = -3$, $y = 3$ and use the double integral to compute its area.

Sheet 8:

Use polar coordinate to evaluate the double integral

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} (x^2+y^2)^{3/2}\,dy\,dx.$$

Sheet 9:

1. Sketch the region bounded by the graphs of the equations

a) $z = x^2 + y^2,\ y + z = 2.$

b) $z + y^2 = 4,\ x + z = 4,\ x = 0,\ z = 0.$

c) $z = 9 - y^2,\ z = 0, x = -1,\ x = 2.$

2. Set up a triple integral for the volume of the region in the first octant, bounded above by the cylinder $z = 1 - y^2$ and lying between the vertical planes $x + y = 1$ and $x + y = 3$.