College of Sciences

Department of

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قسم الإحصاء وبحوث العمليات

جـــامــعــة
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Assignment
First Semester 1442
Stat 436 – Time Series Analysis

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1- Explain briefly meaning of each component of a time series.

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2- Compute and plot the values of $\hat{\gamma}_y(k), \hat{\rho}_y(k)$ and $\hat{\varphi}_y(kk)$ for lag $k = 0; 1; 2; 3$ and 4. Check your results using R.

t	1	2	3	4	5	6	7	8	9	10	11	12	13
y_t	2	1	4	3	3	5	2	1	0	3	1	1	4

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3- Which of the following time sequence is stationary (Weak Stationary)?

i- $y_t = t + \epsilon_t$, where $\epsilon_t \sim i.i.d.(0,1), \forall t \in R$.

ii- $y_t = a + bt + \epsilon_t$ where a and b are constants and $\epsilon_t \sim i.i.d.(0,1), \forall t \in R$.

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4- For the following data.

t	1	2	3	4
y_t	27	32	29	36

i- Calculate all the coefficients of the sample autocorrelation function and calculate the variance for these estimates.

ii- Calculate the first 3 PACF and calculate the variance of these estimates.

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5- In the $\pi - weights$ formula:

$$y_t = \varepsilon_t + \pi_1 y_{t-1} + \pi_2 y_{t-2} + \cdots$$

i- What are the conditions that should be imposed on the weights π_j, so that the process $\{y_t\}$ be invertible?

ii- If we choose $\pi_1 = -0.5, \pi_2 = 0.2, \pi_j = 0, j \geq 3$, what is the resulting process called?

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6- Classify the following AR models (that is, state if they are AR (1), or AR (2), etc.), determine the mean of each model and rewrite each one using the backshift operator.

i- $y_t - 0.17y_{t-1} + 0.19y_{t-2} = \epsilon_t$.

ii- $Z_{t+1} = 77 - 0.55Z_t - 0.24Z_{t-1} + 0.19Z_{t-2} + \epsilon_{t+1}$.

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7- Classify the following MA models (that is, state if they are MA (1), or MA (2), etc.), determine the mean of each model and rewrite each one using the backshift operator.

iii- $y_{t+1} = 10 + \epsilon_{t-1} - 0.06\epsilon_t + 0.35\epsilon_{t+1}$.

iv- $X_t = \epsilon_t + \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} + 0.35\epsilon_{t+1}$

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8- For the following models, state its type, and check its stationarity and/or invertibility:

i- $y_t = 1.5\,y_{t-1} + \varepsilon_t + 0.25\,\varepsilon_{t-1}$

ii- $y_t = \phi_1\,y_{t-1} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\,\varepsilon_{t-2}$

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9- For the AR(1) model with parameter $\phi_1 = 0.5$ and $\mu = 0$, and assuming that $\varepsilon_t \sim i.i.d\ N(0, \sigma_\varepsilon^2 = 1)$:

i-   Write down the mathematical form of this model.

ii-   Derive the mathematical form of the autocorrelation function for this model.

iii- Explain briefly the shape of the partial autocorrelation function of this model.

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