1

$S = \{v_1, v_2, \dots, v_n\} \subseteq V$

then we can induce $\text{Span}(S) \blacktriangledown V$

$\downarrow$

the space generated by S

To find basis of $\text{span}(S)$:

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \xrightarrow{REF} \begin{bmatrix} 1 & & 1 & \cdots \end{bmatrix}$$

the columns with 1 leaders ~~[illegible]~~ form the basis

$\equiv$

$S = \{v_1, \dots, v_n\}$ spans V

which means S is the set of generators

$\Rightarrow \exists B$ basis where $B \subseteq S$

To find B :

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \xrightarrow{REF} \begin{bmatrix} 1 & 1 & \cdots \\ \vdots & \vdots & \end{bmatrix}$$

The columns with 1 leaders form B

② Let $\underset{n \times m}{A}$ be a matrix.

A

row space = Span(S) where S the set of rows

To find basis
$A \xrightarrow{REF} \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & \ddots \end{bmatrix}$

the rows with 1 leader

Dim = Rank(A)

Column space = Span(S') where S' the set of columns

To find basis
$A \xrightarrow{REF} \begin{bmatrix} 1 & 1 & \\ \vdots & \vdots & 0 \end{bmatrix}$

the columns with 1 leader

Dim = Rank(A)

null space = the set of solutions of $AX=0$

To find basis (disjoint of parameters)

Dim = nullity(A)

<u>Rule</u>

① nullity(A) + Rank(A) = number of columns

② Rank(A) = Rank(A^t)

③

Let $S = \{v_1, \dots, v_n\} \subseteq V$

be L. indep

$\Rightarrow \exists$ B basis where $S \subseteq B$

To find B

$$\begin{array}{cc} & \text{standered basis} \\ \left[\begin{matrix} v_1 & v_2 & \dots & v_n \end{matrix}\right. & \left.\begin{matrix} e_1 & e_2 & e_n \end{matrix}\right] \end{array}$$

$$\Big\downarrow \text{REF}$$

Column with 1 ~~leading~~ leaders form the basis

4 Let $B = \{v_1, \dots, v_n\}$ be a basis of V

$\Rightarrow$ for any $v \in V$

$$v = \lambda_1 v_1 + \dots + \lambda_n v_n$$

$$[v]_B = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}$$ Coordinat of v regarding B

(ex) $S = \{(1,2), (0,3)\}$ is the basis of $\mathbb{R}^2$. Find $[(5,0)]_S$ and if $[v]_S = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, find v ?

⑤ transition matrix

Let V be vector space with two basis $\underbrace{\{u_1, u_2\}}_{B}$, $\underbrace{\{v_1, v_2\}}_{B'}$

Then

$$\left.\begin{aligned} u_1 &= \lambda_1 v_1 + \lambda_2 v_2 \\ u_2 &= \beta_1 v_1 + \beta_2 v_2 \end{aligned}\right\} \text{for } B' \text{ basis}$$

So, $[u_1]_{B'} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$; $[u_2]_{B'} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$

Hence

$$ {}_{B'}P_{B} = \begin{bmatrix} \lambda_1 & \beta_1 \\ \lambda_2 & \beta_2 \end{bmatrix} = \text{transition matrix from } B \to B' $$

Remark

① ${}_{B'}P_{B}$ invertible

② $\left[{}_{B'}P_{B}\right]^{-1} = {}_{B}P_{B'}$

③ $[v]_{B'} = {}_{B'}P_{B}\,[v]_{B}$.