Chapter 7 : Test Hypotheses

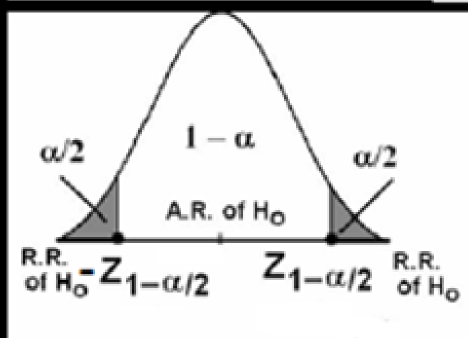
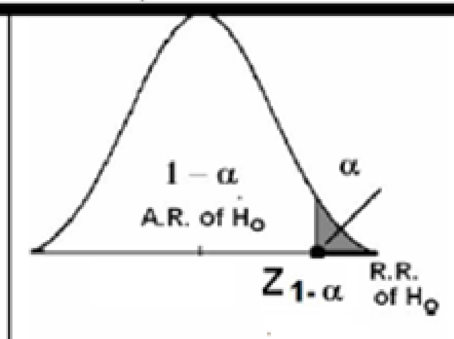
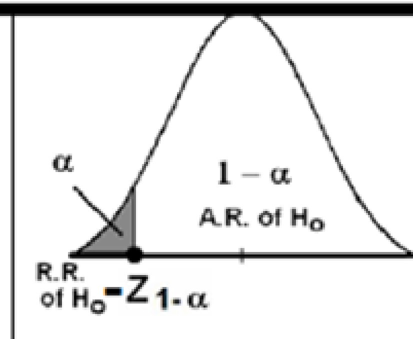
اختبار الفرضيات

أ /خلود باسالم

Testing Hypothesis about population mean (μ)

First Case: σ known , normal or non-normal , $n \geq 30$

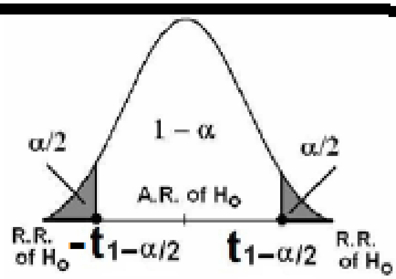
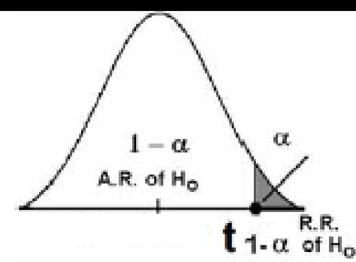
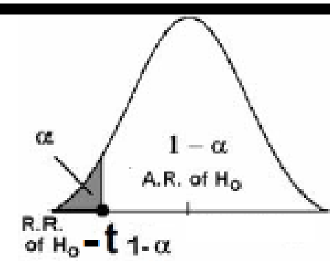
Test Procedures:

<table>
<tr><td>1-</td><td>Hypotheses</td><td>$H_o: \mu = \mu_o$
$H_A: \mu \neq \mu_o$</td><td>$H_o: \mu \leq \mu_o$
$H_A: \mu > \mu_o$</td><td>$H_o: \mu \geq \mu_o$
$H_A: \mu < \mu_o$</td></tr>
<tr><td>2-</td><td>Test Statistic (T.S.)</td><td colspan="3">$Z = \dfrac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$</td></tr>
<tr><td>3-</td><td>R.R. & A.R. of H_o
Rejection Region(R.R) & Acceptance Region(A.R)</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td></td><td>Critical value
Reliability Coefficient</td><td>$Z_{1-\alpha/2}$ or $-Z_{1-\alpha/2}$</td><td>$Z_{1-\alpha}$</td><td>Z_{α}</td></tr>
<tr><td>4-</td><td>Decision:</td><td colspan="3">We reject H_o (and accept H_A) at the significance level α if:</td></tr>
<tr><td></td><td>Reject H_o if the following condition satisfies</td><td>$Z > Z_{1-\alpha/2}$ or
$Z < -Z_{1-\alpha/2}$
Two-Sided Test</td><td>$Z > Z_{1-\alpha}$
One-Sided Test</td><td>$Z < -Z_{1-\alpha}$
One-Sided Test</td></tr>
</table>

Testing Hypothesis about population mean (μ)

Second case: σ unknown , normal , n < 30

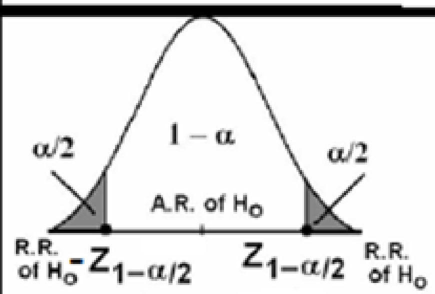
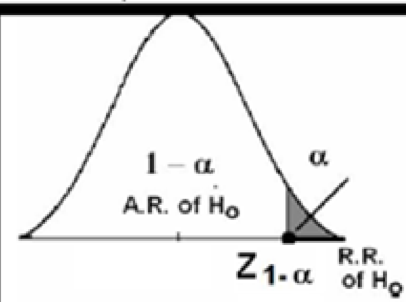
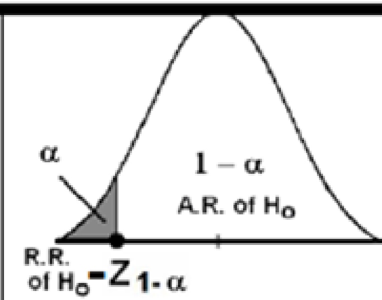
Test Procedures:

1-	Hypotheses	$H_o: \mu = \mu_o$ $H_A\ \mu \neq \mu_o$	$H_o: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_o: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
2-	Test Statistic (T.S.)	$T = \dfrac{\overline{X} - \mu_o}{S/\sqrt{n}}$ $(df = \nu = n-1)$		
3-	R.R. & A.R. of H_o Rejection Region(R.R) & Acceptance Region(A.R)			
	Critical value Reliability Coefficient	$-t_{1-\alpha/2}$ or $t_{1-\alpha/2}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
4-	Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	Reject H_o if the following condition satisfies	$T > t_{1-\alpha/2}$ or $T < -t_{1-\alpha/2}$ Two-Sided Test	$T > t_{1-\alpha}$ One-Sided Test	$T < -t_{1-\alpha}$ One-Sided Test

Testing Hypothesis about population mean (μ)

Special case: σ unknown , non-normal , n $\geq$ 30 (large)

Test Procedures:

1-	Hypotheses	$H_o: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_o: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_o: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
2-	Test Statistic (T.S.)	$Z = \dfrac{\bar{X} - \mu_o}{S / \sqrt{n}}$		
3-	R.R. & A.R. of H_o Rejection Region(R.R) & Acceptance Region(A.R)			
	Critical value Reliability Coefficient	$Z_{1-\alpha/2}$ or $-Z_{1-\alpha/2}$	$Z_{1-\alpha}$	Z_{α}
4-	Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	Reject H_o if the following condition satisfies	$Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha}$ One-Sided Test	$Z < -Z_{1-\alpha}$ One-Sided Test

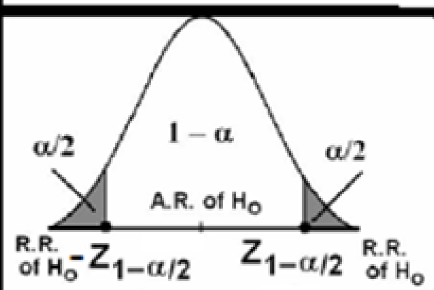
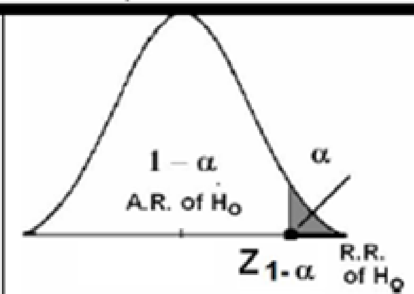
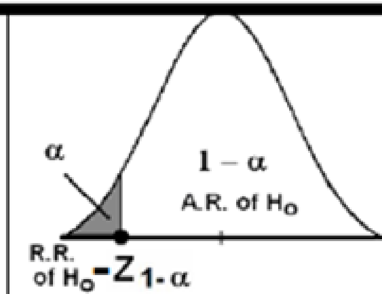
P-Value:

Alternative Hypothesis:	$H_A: \mu \neq \mu_o$	$H_A: \mu > \mu_o$	$H_A: \mu < \mu_o$
P-Value =	$2 \times P(Z > \lvert z_c \rvert)$	$P(Z > z_c)$	$P(Z > -z_c)$
Significance Level =	α		
Decision:	Reject H_o if P-value $< \alpha$.		

Testing Hypothesis difference between two population means ($\mu_1 - \mu_2$)

First case: σ_1^2 and σ_2^2 known

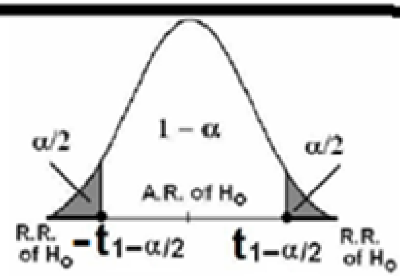
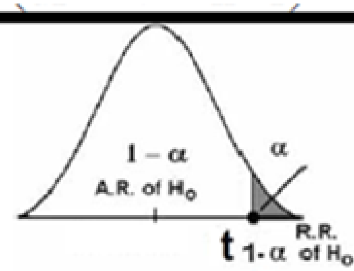
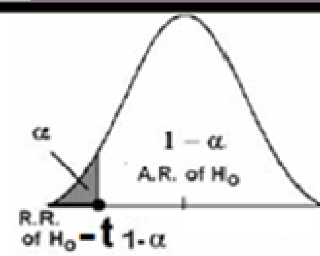
Test Procedures:

1-	Hypotheses	$H_o: \mu_1 - \mu_2 = 0$ $H_A: \mu_1 - \mu_2 \neq 0$	$H_o: \mu_1 - \mu_2 \leq 0$ $H_A: \mu_1 - \mu_2 > 0$	$H_o: \mu_1 - \mu_2 \geq 0$ $H_A: \mu_1 - \mu_2 < 0$
2-	Test Statistic (T.S.)	$Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$		
3-	R.R. & A.R. of H_o Rejection Region(R.R) & Acceptance Region(A.R)			
	Critical value Reliability Coefficient	$Z_{1-\alpha/2}$ or $-Z_{1-\alpha/2}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
4-	Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	Reject H_o if the following condition satisfies	$Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha}$ One-Sided Test	$Z < -Z_{1-\alpha}$ One-Sided Test

أ /خلود باسالم

Testing Hypothesis difference between two population means ($\mu_1 - \mu_2$)
First case: Normal + Unknown but equal $\sigma_1^2 = \sigma_2^2 = \sigma$

Test Procedures:

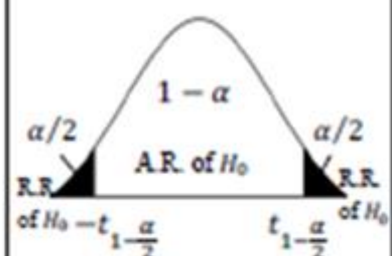
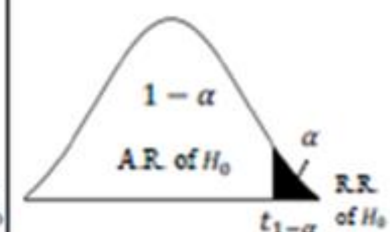
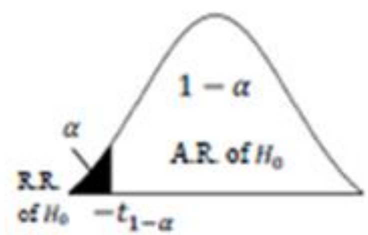
1-	Hypotheses	$H_o: \mu_1 - \mu_2 = 0$ $H_A: \mu_1 - \mu_2 \neq 0$	$H_o: \mu_1 - \mu_2 \leq 0$ $H_A: \mu_1 - \mu_2 > 0$	$H_o: \mu_1 - \mu_2 \geq 0$ $H_A: \mu_1 - \mu_2 < 0$
2-	Test Statistic (T.S.)	$T = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_p^2}{n_1} + \dfrac{S_p^2}{n_2}}}$, $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$ df= $\nu = n_1+n_2-2$ The pooled estimate of variance S_p^2		
3-	R.R. & A.R. of H_o Rejection Region(R.R) & Acceptance Region(A.R)			
	Critical value Reliability Coefficient	$-t_{1-\alpha/2}$ or $t_{1-\alpha/2}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
4-	Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	Reject H_o if the following condition satisfies	$T > t_{1-\alpha/2}$ or $T < -t_{1-\alpha/2}$ Two-Sided Test	$T > t_{1-\alpha}$ One-Sided Test	$T < -t_{1-\alpha}$ One-Sided Test

Confidence Interval for the Difference Between Two Population Means ($\mu_1 - \mu_2 = \mu_D$) for Dependent (Related) Populations: Paired t-Test:

Calculate the Quantities	• The differences (D-observations): $D_i = X_i - Y_i$, $i = 1,2, \dots, n$ • Sample Mean of the D-observations: $\bar{D} = \frac{\sum_{i=1}^{n} D_i}{n}$ • Sample Variance of the D-observations: $S_D^2 = \frac{\sum_{i=1}^{n}(D_i - \bar{D})^2}{n-1}$ • Sample Standard Deviation of the D-observations: $S_D = \sqrt{S_D^2}$
$100(1-\alpha)\%$ Confidence Interval for μ_D	$\bar{D} \pm t_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$, $df = v = n - 1$

Hypotheses Testing for the Difference Between Two Population Means ($\mu_1 - \mu_2 = \mu_D$)

Dependent (Related) Populations: Paired t-Test:

1-	Hypotheses	$H_0: \mu_1 = \mu_2$ vs $H_A: \mu_1 \neq \mu_2$ or $H_0: \mu_D = 0$ vs $H_A: \mu_D \neq 0$	$H_0: \mu_1 \leq \mu_2$ vs $H_A: \mu_1 > \mu_2$ or $H_0: \mu_D \leq 0$ vs $H_A: \mu_D > 0$	$H_0: \mu_1 \geq \mu_2$ vs $H_A: \mu_1 < \mu_2$ or $H_0: \mu_D \geq 0$ vs $H_A: \mu_D < 0$
2-	Test Statistic (T.S.)	$T = \dfrac{\overline{D}}{S_D/\sqrt{n}} \sim t(n-1), \quad df = v = n-1$		
3-	Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
	Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
4-	Decision	We reject H_0 (and accept H_A) at the significance level α if:		
		$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

Testing Hypothesis about population Proportion (P)

$n \geq 30,\ np > 5$

Test Procedures:

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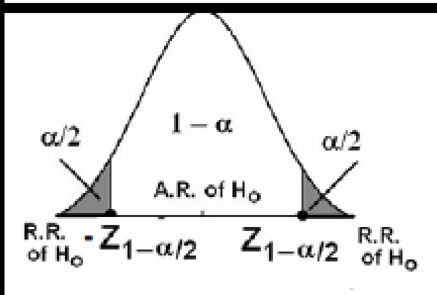
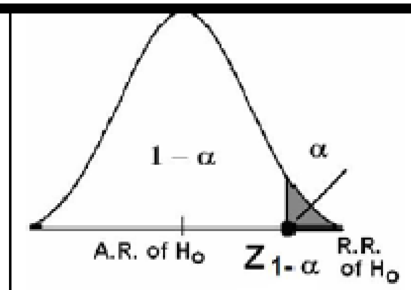
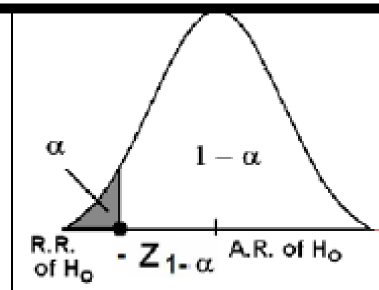
$H_A: p \neq p_o$</td><td>$H_o: p \leq p_o$
$H_A: p > p_o$</td><td>$H_o: p \geq p_o$
$H_A: p < p_o$</td></tr>
<tr><td>2-</td><td>Test Statistic (T.S.)</td><td colspan="3">$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$</td></tr>
<tr><td rowspan="2">3-</td><td>R.R. & A.R. of H_o</td><td>
</td><td>
</td><td>
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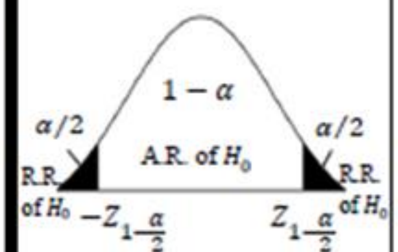
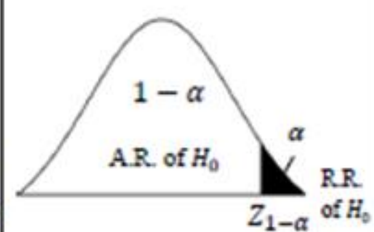
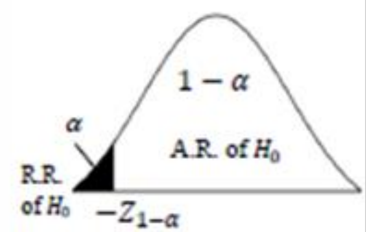
Two-Sided Test</td><td>$Z > Z_{1-\alpha}$
One-Sided Test</td><td>$Z < -Z_{1-\alpha}$
One-Sided Test</td></tr>
</table>

Hypotheses Testing for the Difference Between Two Population Proportions $(p_1 - p_2)$:

Test Procedures:

1-	Hypotheses	$H_0: p_1 = p_2$ vs $H_A: p_1 \neq p_2$	$H_0: p_1 \leq p_2$ vs $H_A: p_1 > p_2$	$H_0: p_1 \geq p_2$ vs $H_A: p_1 < p_2$
2-	Test Statistic (T.S.)	$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_1} + \dfrac{\bar{p}(1-\bar{p})}{n_2}}} \sim N(0,1)$ $\hat{p}_1 = \dfrac{X_1}{n_1}, \quad \hat{p}_2 = \dfrac{X_2}{n_2}, \quad \bar{p} = \dfrac{X_1 + X_2}{n_1 + n_2}$		$\bar{p}$ = the pooled estimate of the common proportion p
3-	Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
	Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
4-	Decision	We reject H_0 (and accept H_A) at the significance level α if: $Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$