Some Continuous Probability Distributions

6.1 Continuous Uniform Distribution

One of the simplest continuous distributions in all of statistics is the **continuous uniform distribution**. This distribution is characterized by a density function that is "flat," and thus the probability is uniform in a closed interval, say [*A, B*]. Although applications of the continuous uniform distribution are not as abundant as those for other distributions discussed in this chapter, it is appropriate for the novice to begin this introduction to continuous distributions with the uniform distribution.

Uniform Distribution:

The density function of the continuous uniform random variable X on the interval $[A, B]$ is

$$f(x) = \begin{cases} \frac{1}{B-A}\,; & A \leq x \leq B \\ 0\,; & \text{elsewhere.} \end{cases}$$

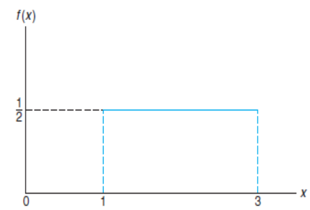

Figure 6.1: The density function for a random variable on the interval [1, 3].

The mean and variance of the uniform distribution are

$$\mu = \frac{A+B}{2}, \quad \sigma^2 = \frac{(B-A)^2}{12}.$$

Example 6.1:

Suppose that a large conference room at a certain company can be reserved for no more than 4 hours. Both long and short conferences occur quite often. In fact, it can be assumed that the length X of a conference has a uniform distribution on the interval $[0, 4]$.

(a) What is the probability density function?

(b) What is the probability that any given conference lasts at least 3 hours?

(c)What is the mean and the variance?

Solution :

(a) The appropriate density function for the uniformly distributed random variable X in this situation is

$$f(x) = \begin{cases} \frac{1}{4}; 0 \leq x \leq 4 \\ 0\ ; \quad \text{elsewhere.} \end{cases}$$

(b) $P(X \geq 3) = \int_3^4 \frac{1}{4} dx = \frac{1}{4}$

(c) $\mu = \frac{4}{2} = 2, \quad \sigma^2 = \frac{(4)^2}{12}$=1.3333

6.6 Exponential Distribution

Exponential Distribution:

The continuous random variable X has an **exponential distribution**, with parameter β, if its density function is given by

$$f(x) = \begin{cases} \frac{1}{\beta} e^{\frac{-x}{\beta}}; x > 0 \\ 0\ ; \text{elsewhere.} \end{cases}$$

where $\beta > 0$.

The mean and variance of the exponential distribution are

$\mu = \beta,\ \sigma^2 = \beta^2$

Example 6.17:

Suppose that a system contains a certain type of component whose time, in years, to failure is given by *T*. The random variable *T* is modeled nicely by the exponential distribution with mean time to failure $\beta = 5$. what is the probability that a component is still functioning after 8 years?

Solution **:** The probability that a given component is still functioning after 8 years is given by

$P(T > 8) = \frac{1}{5}\int_8^\infty e^{-t/5}dt = e^{-8/5} \approx 0.2$

Example 6.21:

Consider Exercise 3.31 on page 94. Based on extensive testing, it is determined that the time Y in years before a major repair is required for a certain washing machine is characterized by the density function

$$f(y) = \begin{cases} \frac{1}{4} e^{\frac{-y}{4}} & ; y > 0 \\ 0 & ; \text{elsewhere.} \end{cases}$$

Note that Y is an exponential random variable with $\mu = 4$ years. The machine is considered a bargain if it is unlikely to require a major repair before the sixth year.

What is the probability $P(Y > 6)$?

What is the probability that a major repair is required in the first year?

Solution **:** Consider the cumulative distribution function $F(y)$ for the exponential distribution,

$$F(y) = \frac{1}{\beta} \int_0^y e^{\frac{-t}{\beta}} dt = 1 - e^{\frac{-y}{\beta}}$$

Then

$$P(y > 6) = 1 - F(6) = e^{-\frac{3}{2}} = 0.223$$

Thus, the probability that the washing machine will require major repair after year six is 0.223. Of course, it will require repair before year six with probability 0.777.

Thus, one might conclude the machine is not really a bargain. The probability that a major repair is necessary in the first year is

$$P(Y < 1) = 1 - e^{-\frac{1}{4}} = 1 - 0.779 = 0.221$$