Some Continuous Distributions

Normal Distribution: $X \sim N(\mu, \sigma)$

Probability density function (pdf)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \; ; \; -\infty < x < \infty$$

Mean:

$$E(X) = \mu$$

Variance:

$$V(X) = \sigma^2$$

Moment Generating function (MGF):

$$M_X(t) = exp(\mu t + \sigma^2 t^2/2)$$

Uniform Distribution: $X \sim U(a, b)$

Probability density function (pdf)

$$f(x) = \frac{1}{b-a} \; ; \quad a \leq x \leq b$$

Cumulative distribution function (CDF):

$$F(x) = \begin{cases} 0 \; ; & x < a \\ \frac{x-a}{b-a} \; ; & a \leq x \leq b \\ 1 \; ; & x > b \end{cases}$$

Mean:

$$E(X) = \frac{a+b}{2}$$

Variance:

$$V(X) = \frac{(b-a)^2}{12}$$

Moment Generating function (MGF):

$$M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)} ; t \neq 0$$

Exponential Distribution: $X \sim exp(\theta)$

Probability density function (pdf)

$$f(x) = \theta e^{-\theta x}\,; \quad x > 0, \theta > 0$$

Cumulative distribution function (CDF):

$$F(x) = 1 - e^{-\theta x}\,; \quad x > 0$$

Mean:

$$E(X) = \frac{1}{\theta}$$

Variance:

$$V(X) = \frac{1}{\theta^2}$$

Moment Generating function (MGF):

$$M_X(t) = \frac{\theta}{\theta - t}\,; t < \theta$$

Gamma Distribution: $X \sim Gamma(\alpha, \theta)$

Probability density function (pdf)

$$f(x) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\theta x}\,; \quad x > 0, \alpha > 0, \theta > 0$$

where $\Gamma(n) = (n-1)!$

Mean:

$$E(X) = \frac{\alpha}{\theta}$$

Variance:

$$V(X) = \frac{\alpha}{\theta^2}$$

Moment Generating function (MGF):

$$M_X(t) = \left(\frac{\theta}{\theta - t}\right)^\alpha \,; t < \theta$$

Chi square Distribution: $X \sim \chi_k^2$

Probability density function (pdf)

$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}\ ; \quad x > 0$$

Mean:

$$E(X) = k$$

Variance:

$$V(X) = 2k$$

Moment Generating function (MGF):

$$M_X(t) = \left(\frac{1}{1-2t}\right)^{k/2}\ ; t < \frac{1}{2}$$

t Distribution: $X \sim t_v$

Probability density function (pdf)

$$f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}}\ ; -\infty < x < \infty$$

Mean:

$$E(X) = 0; v > 1$$

Variance:

$$V(X) = \frac{v}{v-2}; v > 2$$

F Distribution: $X \sim F(n, m)$

Probability density function (pdf)

$$f(x) = \frac{(nx)^n m^m}{(nx+m)^{n+m}} \;; \quad x > 0$$

Mean:

$$E(X) = \frac{m}{m-2}; m > 2$$

Variance:

$$V(X) = \frac{2m^2(n+m-2)}{n(m-2)^2(m-4)}; m > 4$$

Beta Distribution: $X \sim Beta(\alpha, \beta)$

Probability density function (pdf)

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)} \;; \quad 0 < x < 1$$

where $\mathrm{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Mean:

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

Variance:

$$V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Some Discrete Distributions

Uniform Distribution:

Probability density function (pdf)

$$f(x) = \frac{1}{b-a+1} \;; \quad x \in \{a, a+1, \dots, b-1, b\}$$

Mean:

$$E(X) = \frac{a+b}{2}$$

Variance:

$$V(X) = \frac{(b-a+1)^2 - 1}{12}$$

Moment Generating function (MGF):

$$M_X(t) = \frac{e^{at} - e^{(b+1)t}}{(b-a+1)(1-e^t)}$$

Poisson Distribution:

Probability density function (pdf)

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}; x \in \{0,1,\dots\}, \lambda > 0$$

Mean:

$$E(X) = \lambda$$

Variance:

$$V(X) = \lambda$$

Moment Generating function (MGF):

$$M_X(t) = exp[\lambda(e^t - 1)]$$

Bernoulli Distribution:

Probability density function (pdf)

$$f(x) = \begin{cases} 1-p; & x = 0 \\ p; & x = 1 \end{cases}; 0 < p < 1$$

Mean:

$$E(X) = p$$

Variance:

$$V(X) = p(1-p)$$

Moment Generating function (MGF):

$$M_X(t) = (1-p) + pe^t$$

Binomial Distribution:

Probability density function (pdf)

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}; x \in \{0,1,\dots\}, 0 < p < 1$$

Mean:

$$E(X) = np$$

Variance:

$$V(X) = np(1-p)$$

Moment Generating function (MGF):

$$M_X(t) = [(1-p) + pe^t]^n$$

Negative Binomial Distribution:

Probability density function (pdf)

$$f(x) = \binom{x+r-1}{x} p^x (1-p)^r; x \in \{0,1,\ldots\}, 0 < p < 1$$

Mean:

$$E(X) = \frac{rp}{1-p}$$

Variance:

$$V(X) = \frac{rp}{(1-p)^2}$$

Moment Generating function (MGF):

$$M_X(t) = \left[\frac{1-p}{1-pe^t}\right]^r$$

Geometric Distribution:

Probability density function (pdf)

$$f(x) = p(1-p)^x; x \in \{0,1,\ldots\}, 0 < p < 1$$

Mean:

$$E(X) = \frac{1-p}{p}$$

Variance:

$$V(X) = \frac{1-p}{p^2}$$

Moment Generating function (MGF):

$$M_X(t) = \frac{p}{1-(1-p)e^t}$$