PHYS 505
1st Midterm Exam
Tuesday 20th November 2016

Instructor: Dr. V. Lempesis

Please answer all questions

1. Round the following recordings at the digit which is underlined:

Recorded Value	5.6<u>3</u>7	3<u>5</u>19	4.<u>8</u>47	0.021<u>8</u>46	<u>4</u>.501
Rounded Value	**5.64**	**3500**	**4.8**	**0.0218**	**5**

(5 marks)

2. Fill in the following table by keeping significant figures and rounding properly the recorded values taken in an experiment for a physical quantity:

	Before the selection of significant figures		After the selection of significant figures		Final Result
Recorded value	x	δx	δx	x	x
1	192.31	11	**11**	**192**	**192 ± 11**
2	136.4	32	**30**	**140**	**140 ± 30**
3	103.287	0.261	**0.26**	**103.29**	**103.29 ± 0.26**
4	7.121	0.542	**0.5**	**7.1**	**7.1 ± 0.5**
5	163	4.62	**5**	**163**	**163 ± 5**

(5 marks)

3. You are given the following recordings for the length of a rod

ℓ_i mm	$\ell_i - \bar{\ell}$ mm	$\left(\ell_i - \bar{\ell}\right)^2$ mm^2
2.36	**0.0388**	**0.150544**
2.42	**0.098**	**0.009604**
2.21	**-0.112**	**0.012544**
2.30	**-0.022**	**0.000484**
2.32	**-0.002**	**0.000004**
$\sum_{i=1}^{5} \ell_i =$ **11.61**	$\sum_{i=1}^{5} \left(\ell_i - \bar{\ell}\right) =$ **0**	$\sum_{i=1}^{5} \left(\ell_i - \bar{\ell}\right)^2 =$ **0.02408**

a) Fill in the table

(2 marks)

b) Find the average value of the length of the rod: (1 mark)

$$\overline{\ell} = \sum_{i=1}^{5} \ell_i / 5 = 2.322 \text{ mm}$$

c) Find the absolute error of the average value and round to correct number of significant digits

(2 marks)

$$\delta\ell = \sqrt{\sum_{i=1}^{5} \left(\ell_i - \overline{\ell}\right)^2 / 5(5-1)} = 0.03469 \text{ mm} = 0.03 \text{ mm}$$

d) Quote the experimental result:

(2 marks)

$$\ell = \left(2.32 \pm 0.03\right) \text{ mm}$$

e) Find the relevant error:

(1 marks)

$$\frac{\delta\ell}{\overline{\ell}} = \frac{0.03}{2.32} \times 100\% = 1.29\%$$

4. How many significant figures there are in the following recordings:

Recording	Number of significant digits
720	**2**
20.003	**5**
00102	**3**
0.003000	**4**
0.530 / 0.1010	**3**

(5 marks)

5. We are given the following probability distribution:

$$f(x) = \begin{cases} \frac{3}{4}\left(-x^2 + 4x - 3\right) & 1 \le x \le 3 \\ 0 & \textit{otherwise} \end{cases}$$

a) Find the average value μ. (2 marks)

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{1}^{3} \frac{3}{4}\left(-x^2 + 4x - 3\right) x\, dx = 2$$

b) Find the standard deviation of the distribution σ. (3 marks)

$$\sigma^2 = \int_{-\infty}^{\infty} xf(x)\,dx - \mu^2 = \int_1^3 \frac{3}{4}\left(-x^2+4x-3\right)x^2\,dx - 4 = \frac{21}{5} - 4 = 1/5$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5} \approx 0.44$$

c) Find the mode of the distribution. (3 marks)

$$f'(x) = 0 \Rightarrow \left[\frac{3}{4}\left(-x^2+4x-3\right)\right]' = 0 \Rightarrow$$

$$-2x+4 = 0 \Rightarrow x = 2$$

At $x = 2$ we have

$$f(2) = \frac{3}{4}\left(-2^2+4\cdot 2-3\right) = \frac{3}{4}.$$

MATHEMATICAL FORMULAS

- $\delta x = \sqrt{\dfrac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{N(N-1)}}$, $\mu = \int_{-\infty}^{+\infty} xf(x)\,dx$, $\int_{-\infty}^{+\infty} x^2 f(x)\,dx - \mu^2$