

X unknown pdf with μ and $\text{Var}(X) = \sigma^2$

Chebyshev's Theorem:

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2} \text{ or } P(|X - \mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}$$

↳ $\Rightarrow P(X \geq \mu + k\sigma \text{ or } X \leq \mu - k\sigma) \leq \frac{1}{k^2}$ or $P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

Q₁ a) $P(-6 < X < 26)$ $\begin{cases} \rightarrow -6 = \mu - k\sigma \Rightarrow -6 = 10 - k4 \Rightarrow k=4 \\ \text{or} \\ \rightarrow 26 = \mu + k\sigma \Rightarrow 26 = 10 + k4 \Rightarrow k=4 \end{cases}$

$$\therefore P(-6 < X < 26) = P(|X - 10| \leq 16) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{16} = \frac{15}{16}$$

b) $P(|X - 10| \leq 12) \rightarrow 12 = k\sigma \Rightarrow 12 = k4 \Rightarrow k=3$

$$\therefore P(|X - 10| \leq 12) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

c) $P(|X - 10| > 12) \rightarrow 12 = k\sigma \Rightarrow 12 = k4 \Rightarrow k=3$

$$\therefore P(|X - 10| > 12) \leq \frac{1}{k^2} = \frac{1}{9}$$

Q₂ data set with $\mu = 195$ and $\sigma = 17$ for unknown dis. for X .

$P(161 < X < 229)$ $\begin{cases} \rightarrow 229 = \mu + k\sigma \Rightarrow 229 = 195 + 17k \Rightarrow k=2 \\ \text{or} \\ \rightarrow 161 = \mu - k\sigma \Rightarrow 161 = 195 - 17k \Rightarrow k=2 \end{cases}$

$$\therefore P(161 < X < 229) = P(|X - 195| < 34) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{4} = \frac{3}{4}$$