Exercises 5

Discrete Probability Distributions

Binomial distribution

1 : In a certain city district the need for money to buy drugs is stated as the: reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,
(a) exactly 2 resulted from the need for money to buy drugs;
(b) at most 3 resulted from the need for money to buy

Solution: X: # of thefts resulted from the need for money to buy drugs of 5 theft cases

For $n = 5$ and $p = 3/4$, we have $f_X(x) = \binom{5}{x}(\frac{3}{4})^x(\frac{1}{4})^{5-x}$, $x = 0, 1, \ldots, 5$

(a) $P(X = 2) = \binom{5}{2}(3/4)^2(1/4)^3 = 0.0879$,

(b) $P(X \le 3) = \sum_{x=0}^{3} f_X(x)$ or $= 1 - P(X \geq 3)$ $1 - P(X = 4) - P(X = 5)$
$= 1 - \binom{5}{4}(3/4)^4(1/4)^1 - \binom{5}{5}(3/4)^5(1/4)^0 = 0.3672.$

2 In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that
(a) from 3 to 6 have blowouts;
(b) fewer than 4 have blowouts
(c) more than 5 have blowouts.

Solution: X: # of tires that blowout of 15 tires, $f_X(x) = \binom{15}{x}(\frac{1}{4})^x(\frac{3}{4})^{15-x}$, $x = 0, 1, \ldots, 15$

For $n = 15$ and $p = 0.25$, we have

(a) $P(3 \le X \le 6) = P(X \le 6) - P(X \le 2) = 0.9434 - 0.2361 = 0.7073.$ or $\sum_{x=3} [f_X(x)]$

(b) $P(X < 4) = P(X \le 3) = 0.4613.$ or $1 - P(X \geq 4) = 1 - \{\sum_{x=4}^{15} [f_X(x)]\}$

(c) $P(X > 5) = 1 - P(X \le 5) = 1 - 0.8516 = 0.1484.$ or $\sum_{x=6}^{15} [f_X(x)]$

3 The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?

Solution X: # of patient that recovers from heart operation of 7 patients.
$f_X(x) = \binom{7}{x}(.9)^x(.1)^{7-x}$, $x = 0, 1, 2, \ldots, 7$

$n = 7$ and $p = 0.9$, we have

$P(X = 5) = P(X \le 5) - P(X \le 4) = 0.1497 - 0.0257 = 0.1240.$ (use the cdf)
or $\binom{7}{5}(.9)^5(.1)^{7-5} = .1240$

4 It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that
(a) none contracts the disease;
(b) fewer than 2 contract the disease;
(c) more than 3 contract the disease.

X: # of mice that contract the disease of 5 mice inoculated
$f_X(x) = \binom{5}{x}(.4)^x(.6)^{5-x}$, $x = 0, 1, \ldots, 5$

Solution

$p = 0.4$ and $n = 5$.

(a) $P(X = 0) = 0.0778.$ $1-P(X\geq 2) = 1-\left\{\sum_{x=2}^{5}[f_X(x)]\right\}$

(b) $P(X < 2) \overset{or}{=} P(X \leq 1) = 0.3370 = \sum_{x=0}^{1}[f_X(x)]$

(c) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.9130 = 0.0870 = 1-\left\{\sum_{x=0}^{3}[f_X(x)]\right\}$ $\sum_{x=4}^{5}[f_X(x)]$

(5) In a study of brand recognition, 95% of consumers recognized Coke. The company randomly selects 4 consumers for a taste test.

•Let X be the number of consumers who recognize Coke of 4 consumers.

a. Write out the PMF table for this.
a. Find the probability that among the 4 consumers, 2 or more will recognize Coke
b. Find the expected number of consumers who will recognize Coke.
c. Find the variance for the number of consumers who will recognize Coke

Solution $n=4,\ p=.95,\ f_X(x) = \binom{4}{x}(.95)^x(.05)^{4-x},\ x=0,1,\dots,4$

a)

X	p(x)
0	$\binom{4}{0}(0.95)^0(0.05)^4 = 0.0000062$
1	$\binom{4}{1}(0.95)^1(0.05)^3 = 0.00047$
2	$\binom{4}{2}(0.95)^2(0.05)^2 = 0.01354$
3	$\binom{4}{3}(0.95)^3(0.05)^1 = 0.17147$
4	$\binom{4}{4}(0.95)^4(0.05)^0 = 0.81451$

b)

$1-P(X<2) = 1-\left\{\sum_{x=0}^{1}[f_X(x)]\right\}$

$$P(X \geq 2) \overset{or}{=} P(X=2)+P(X=3)+P(X=4)$$
$$= 0.01354 + 0.17147 + 0.81451 = \mathbf{0.99952} = \sum_{x=2}^{4}[f_X(x)]$$

c)

$$E(X) = 0(.0000062) + 1(.00047) + 2(.01354) + 3(0.17147) + 4(0.81451) = \mathbf{3.8} = \sum_{x=0}^{4} x f_X(x)$$

Or

$E(X) = np = 4(0.95) = \mathbf{3.8}$

d) $Var(X) = np(1-p) = 4(0.95)(0.05) = \mathbf{0.19}$

$p(H) = p(T) = \frac{1}{2}$

Geometric distribution

(6) Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 4 tosses are needed.

X: # of try toss the coin 3 times until get the try toss the coin 3 time that one of the 3 results is different — First try that we have odd man

Solution $f_X(x) = p(1-p)^{x-1}, \; x = 1, 2, \dots$

to find the p: $\{H,T\}\times\{H,T\}\times\{H,T\} = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$p = P(\text{one of 3 results is different}) = P(\{HHT, HTH, HTT, THH, THT, TTH\})$

$= \frac{6}{8} = \frac{3}{4}$ [illegible]

$= P(\{HHT\}) + P(\{HTH\}) + \dots$
$= P(H)P(H)P(T) + P(H)P(T)P(H) + \dots$
$= (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) + \dots$
$= 6(1/2)^3 = 6/8 = 3/4$

$\therefore P(X<4) = \sum_{x=1}^{3}\left[(\frac{3}{4})(\frac{1}{4})^{x-1}\right] = \frac{63}{64} = .984375 \simeq .9844$

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(7) According to a study published by a group of University of Massachusetts sociologists, about two thirds of the 20 million persons in this country who take Valium are women. Assuming this figure to be a valid estimate, find the probability that on a given day the fifth prescription written by a doctor for Valium is
(a) the first prescribing Valium for a woman;
(b) the third prescribing Valium for a woman

X: # of Prescriptions of Valium until get the first Prescription of Valium for a woman

Y: # of Prescriptions of Valium until get the third Prescription of Valium for a woman

$p = P(\text{woman take Valium}) = \frac{2}{3}$

Solution $f_X(x) = (\frac{2}{3})(\frac{1}{3})^{x-1}, x = 1, 2, \dots$ / $f_Y(y) = \binom{y-1}{3-1}(\frac{2}{3})^3(\frac{1}{3})^{y-3}, y = 3, 4, 5, \dots$

(a) Using the geometric distribution, we have $f_X(5) = (2/3)(1/3)^4 = 2/243$.

(b) Using the negative binomial distribution, we have

$$f_Y(5) = \binom{4}{2}(2/3)^3(1/3)^2 = \frac{16}{81}.$$

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(8) The probability that a student passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test
(a) on the third try;
(b) before the fourth try. (u can add after, between two points)

X: # of try written test until get first written test that will passes

$p = P(\text{passes the written test}) = .7$

$f_X(x) = (.7)(.3)^{x-1}, x = 1, 2, \dots$

Using the geometric distribution

(a) $P(X=3) = f_X(3) = (0.7)(0.3)^2 = 0.0630.$

(b) $P(X<4) = \sum_{x=1}^{3} f_X(x) = \sum_{x=1}^{3}(0.7)(0.3)^{x-1} = 0.9730.$

Hyper geometric distribution

(9) From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that
(a) all 4 will fire?
(b) at most 2 will not. fire?

3 not fire
7 fire
10 missiles
4 missiles that will be fired

Solution

(a) Probability that all 4 fire $= \dfrac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \frac{1}{6}$.

(b) Probability that at most 2 will not fire $= \dfrac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}} + \dfrac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \dfrac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \dfrac{29}{30}$

2 or 1 or 0

(10)

A random committee of size 3 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$

4 doctors
2 nurses
3
6

Solution

$$P(2 \le X \le 3) = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} + \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = \frac{4}{5}.$$

(11)

A manufacturing company uses an acceptance scheme on production items before they are shipped. The plan is a two-stage one. Boxes of 25 are readied for shipment and a sample of 3 is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.

(a) What is the probability that a box containing 3 defectives will be shipped?

(b) What is the probability that a box containing only 1 defective will be sent back for screening?

3
25

Solution

(a) $P(X = 0) = \frac{77}{115} = \dfrac{\binom{3}{0}\binom{22}{3}}{\binom{25}{3}}$

3 defectives
22 o.w
25
3 from o.w

(b) $P(X = 1) = \frac{3}{25} = \dfrac{\binom{1}{1}\binom{24}{2}}{\binom{25}{3}}$

1 defectives
24 o.w
25
3
1 from defectives
2 o.w

Poisson distribution

(12)

On average a certain intersection results in 3 traffic accidents per month.

For any given month at this intersection. What is the probability that:

(a) exactly 5 accidents will occur?

(b) less than 3 accidents will occur?

(c) at least 2 accidents will occur?

$\lambda = 3$ traffic accidents / month

X: # of traffic accidents in a month

$f_X(x) = \dfrac{e^{-3}3^x}{x!}, \ x = 0, 1, \dots$

Solution

(a) Using the Poisson distribution with $x = 5$ and $\mu = 3$, we find from Table A.2 that

$$\frac{e^{-3}3^5}{5!} = .1008$$

or

$$f_X(5) = P(X=5) = \sum_{x=0}^{5} f_X(x) - \sum_{x=0}^{4} f_X(x) = 0.1008. \longrightarrow \text{use the cdf}$$

(b) $P(X < 3) = P(X \le 2) = 0.4232 = \sum_{x=0}^{2}[f_X(x)]$

(c) $P(X \ge 2) = 1 - P(X \le 1) = 0.8009 = 1 - \left\{\sum_{x=0}^{1}[f_X(x)]\right\}$

1 month $\longrightarrow \lambda = 3$

1 year = 12 month $\longrightarrow \lambda' = ?$

$\Rightarrow \lambda'(1) = (3)(12)$

$\Rightarrow \lambda' = 36$ traffic accidents / year

For any given year at this intersection. What is the probability that:

(a) exactly 5 accidents will occur? $P(Y=5) = f_Y(5) = 1.16877 \times 10^{-10}$

(b) less than 3 accidents will occur? $P(Y<3) = P(Y \le 2) = 1.589 \times 10^{-10}$

(c) at least 2 accidents will occur? $P(Y \ge 2) = 1 - P(Y < 2) = 1 - P(Y \le 1) \simeq 1$

Y: # of traffic accidents in a year

$f_Y(y) = \dfrac{e^{-36}36^y}{y!}, \ y = 0, 1, \dots$

(13)

A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make

(a) 4 or more errors?
(b) no errors?

$\lambda = 2$ error / Page
X: # of errors in a Page
$f_X(x) = \dfrac{e^{-2}\,2^x}{x!}, \; x = 0, 1, \dots$

Solution

(a) $P(X \geq 4) = 1 - P(X \leq 3) = 0.1429 = 1 - \left\{ \sum_{x=0}^{3} \left[f_X(x) \right] \right\}$

(b) $P(X = 0) = f_X(0) = 0.1353 = \dfrac{e^{-2}\,2^0}{0!}$

14

A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probability that for a given year that area will be hit by
(a) fewer than 4 hurricanes;
(b) anywhere from 6 to 8 hurricanes.

$\lambda = 6$ hit by hurricanes / year
X: # of hit by hurricanes in a year
$f_X(x) = \dfrac{e^{-6}\,6^x}{x!}, \; x = 0, 1, 2$

Solution

(a) $P(X < 4) = P(X \leq 3) = 0.1512 = \sum_{x=0}^{3} \left[f_X(x) \right]$

(b) $P(6 \leq X \leq 8) = P(X \leq 8) - P(X \leq 5) = 0.4015$. use cdf

or $\sum_{x=6}^{8} \left[f_X(x) \right]$

c) Find the probability that for a given **3 months** that area will be hit by fewer than 4 hurricanes.

1 year = 12 months $\rightarrow \lambda = 6$
3 month $\longrightarrow \lambda' = ?$
$\Rightarrow \lambda' = \dfrac{6 \times 3}{12} = \dfrac{3}{2} = 1.5$ hit by hurricanes / 3 month, Y: # of hit by hurricanes in 3 month

$f_Y(y) = \dfrac{e^{-1.5}\,(1.5)^y}{y!}, \; y = 0, 1, \dots \quad \therefore P(Y < 4) = \sum_{y=0}^{3} \left[f_Y(y) \right] =$

Uniform Distribution

15 When a die is tossed once, each element of the sample space occurs with probability 1/6. Therefore we have a uniform distribution. $f_X(x) = \dfrac{1}{n} = \dfrac{1}{6}, \; x = 1, 2, \dots, 6$

Find:

$$P(1 \leq x < 4) = \sum_{x=1}^{3} \left[f_X(x) \right] = \frac{1}{2}$$
$$P(x < 3) = \sum_{x=1}^{2} \left[f_X(x) \right] = 1/3$$
$$P(3 < x < 6) = \sum_{x=4}^{5} \left[f_X(x) \right] = 1/3$$

Find also the mean and variance. $\longrightarrow$ $E(X) = 3.5$, $Var(X) = 2.917$

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16 X has is uniformly distributed on the set $\{1,2,3,\dots,N\}$, and

Y is uniformly distributed on the set $\{a, a+k, a+2k, \dots, b\}$, then find

- P(X) and P(Y)
- M(t) for X and for Y
- E(X) and E(Y)
- V(X) and Var(Y)

Solution (Prove this)

$$P(x) = \frac{1}{N} \qquad\qquad P(y) = \frac{k}{b-a+k}$$

$$M(t) = \frac{e^t(1-e^{tN})}{N(1-e^t)} = \frac{e^t(e^{tN}-1)}{N(e^t-1)} \qquad\qquad M(t) = \frac{e^{at}(1-e^{ktN})}{N(1-e^{kt})}$$

$$E(X) = \frac{N+1}{2} = \frac{\Sigma x}{N} = \frac{1+2+\dots+N}{N} \qquad\qquad E(Y) = \frac{a+b}{2}$$

$$Var(X) = \frac{N^2-1}{12} \qquad\qquad Var(Y) = k^2\left(\frac{N^2-1}{12}\right)$$

X: First case:

$$M(t) = E(e^{tx}) = \sum_{x=1}^{N} e^{tx} \frac{1}{N}$$

$$= \frac{1}{N} \sum_{x=1}^{N} (e^t)^x$$

$$= \frac{1}{N} e^t \frac{1-e^{tN}}{1-e^t}$$

$$= \frac{e^t(1-e^{tN}}{N(1-e^t)}$$

$$E(X) = \sum xP(x)$$

$$= \sum_{x=1}^{N} x \frac{1}{N}$$

$$= \frac{1}{N}(1+2+3+\cdots+N)$$

$$= \frac{1}{N}\left[\frac{N(N+1)}{2}\right]$$

$$= \frac{N+1}{2}$$

$$E(X^2) = \sum x^2 P(x)$$

$$= \sum_{x=1}^{N} x^2 \frac{1}{N}$$

$$= \frac{1}{N}(1^2+2^2+3^2+\cdots+N^2)$$

$$= \frac{1}{N}\left[\frac{N(N+1)(2N+1)}{6}\right]$$

$$= \frac{(N+1)(2N+1)}{6}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2}\right)^2$$

$$= \frac{2N^2+3N+1}{6} - \frac{N^2+2N+1}{4}$$

$$= \frac{N^2-1}{12}$$

Y:,Second case (generalization of the first case):

$$P(y) = \frac{k}{b-a+k}.$$

Y= kX+(a-k)

$$\begin{aligned} M_Y(t) &= M_{kX+(a-k)}(t) \\ &= e^{t(a-k)} M_X(kt) \\ &= e^{t(a-k)} \frac{e^{kt}(1-e^{ktN})}{N(1-e^{kt})} \\ &= \frac{e^{at}(1-e^{ktN})}{N(1-e^{kt})} \end{aligned}$$

$$\begin{aligned} E(Y) &= E(kX+(a-k)) \\ &= kE(X)+(a-k) \\ &= k\left(\frac{N+1}{2}\right)+(a-k) \\ &= \frac{k}{2}\left(\frac{b-a+k}{k}+1\right)+a-k \\ &= \frac{a+b}{2} \end{aligned}$$

$$\begin{aligned} Var(Y) &= Var(kX+(a-k)) \\ &= k^2 Var(X) \\ &= k^2\left(\frac{N^2-1}{12}\right) \end{aligned}$$

Bernoulli Distribution

X7 . Suppose our class passed (C or better) the last exam with probability 0.75.

a. Find the probability that someone passes the exam.
b. Find the mean value of the random variable
c. Find the standard deviation value of the random variable
d. Find the moment generating function of the random variable

X: # of students pass the class
1 student
$P=.75,\ n=1.$
$P_X(x)=\binom{1}{x}(.75)^x(.25)^{1-x},\ x=0,1$
$=(.75)^x(.25)^{1-x},\ x=0,1$

Solution:

The r.v X has Bernoulli (0.75) where the success (X=1) is passing the exam, then
P(X=1)=0.75
E(X)=0.75, Var(x)= $\underbrace{0.75 * 0.25}_{=.1875}$, $M_X(t)=(.25)+(.75)e^t$

X: # of Pearsons have disease of 1 Pearson
$P=.2$
$P_X(x)=(.2)^x(.8)^{1-x},\ x=0,1$

....

8 **20% from a population have a particular disease. In testing process for infection by this disease.**

b. Find the probability that someone infected by this disease. $P_X(1)=.2$
c. Find the mean value of the random variable $\mu=.2$
d. Find the standard deviation value of the random variable $\sigma=\sqrt{\sigma^2}=\sqrt{(.2)(.8)}=\sqrt{.16}$
e. Find the moment generating function of the random variable

$M_X(t)=(.8)+(.2)e^t$

(19) Suppose X has a geometric distribution with p=0.8. Compute the probability of the following events. $f_X(x) = p q^{x-1}, x=1,2,\dots ; p=.8 ; q=.2$

$3 \le X \le 5 \cap 7 \le X \le 10 = \phi$

$a) X > 3 \Rightarrow P(X>3) = 1 - P(X \le 3) = 1 - \left\{ \sum_{x=1}^{3} \left[(.8)(.2)^{(x-1)} \right] \right\} = .008$

$b) 4 \le X \le 7 \Rightarrow P(4 \le X \le 7) = \sum_{x=4}^{7} \left[(.8)(.2)^{(x-1)} \right] = .007987200$

$c) 3 \le X \le 5 \quad or \quad 7 \le X \le 10 \Rightarrow P(3 \le X \le 5 \text{ or } 7 \le X \le 10) = P(3 \le X \le 5) + P(7 \le X \le 10) - P(\phi)$
$= .0397438976$

(20) If the probability is 0.75 that an application for a driver's license will pass the road test on any given try, what is the probability that an application will finally pass the test on the fourth try ∴ geometric dis. where $f_X(x) = P(X=x) = p q^{x-1}; x=1,2,\dots ; p=.75 ; q=.25$
where X: # of try the test to have the pass (get the driver's license)

$$f_X(4) = p(X=4) = (0.75)(0.25)^3 = 0.0117$$

(21) Suppose that 30% of the application for a certain industrial job have advanced training in computer programming. Application are interviewed sequentially and are selected at random from the pool. Find the probability that the first application having advanced in programming is found on the fifth interview. ∴ geometric dis.
$f_X(x) = p q^{x-1}, x=1,2,\dots ; p=.3 ; q=.7$
X: # of the interview to get the job

$$f_X(5) = p(x=5) = (0.3)\ (0.7)^4 = 0.07203$$

(22) Let X be uniformly distributed on 0,1,…,99. Calculate ∴ uniform dis.
$f_X(x) = \frac{1}{n}, n=100, x=0,1,\dots,99$
X: the values with equal probabilities

$a) P(X \ge 25) = \sum_{x=25}^{99} \frac{1}{100} = \frac{1}{100} \sum_{x=25}^{99} (1) = .75$

$b) P(2.6 < X < 12.2) = \sum_{x=3}^{12} \frac{1}{100} = .1$

$c) P(8 < X \le 10 \quad or \quad 2 < X \le 32)$ $= P(8 < X \le 10) + P(2 < X \le 32) - P(8 < X \le 10) = .3$

$d) P(25 \le X \le 30) = \sum_{x=25}^{30} \left(\frac{1}{100}\right) = .06$

(23) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it. ∴ negative Binomail dis. $\Rightarrow f_X(x) = P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x=r, r+1, r+2, \dots$
$p=.4, q=1-p=.6, r=3$
X: # of children that they exposed to the disease until the disease catch the third child.

$$P(X=10) = f_X(10) = \binom{9}{2}(0.4)^3(0.6)^7 = 0.0645$$

(24) In an assembly process, the finished items are inspected by a vision sensor, the image data is processed , and a determination is made by computer as to whether or not a unit is satisfactory. If it is assumed that 2% of the units will be rejected, then what is the probability that the thirtieth unit observed will be second rejected unit?
∴ negative Binomial dis. $\Rightarrow f_X(x) = P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x=r, r+1, \dots ; p=.02, 1-p=.98, r=2$
X: # of units that observed until get the second rejected unit.

$$f_X(30) = P(X=30) = C_1^{29} (0.02)^2 (0.98)^{28} = 0.0066$$

Hypergeometric

Hypergeometric

(25) If 2 balls are randomly drawn from a bowl containing 6 white and 5 black balls, what is the probability that one of the drawn balls is white and the other black?

6 white, 5 black, 11 balls; 2 balls

$$\frac{\binom{6}{1}\binom{5}{1}}{\binom{11}{2}} = .5455$$

(26) Of 10 girls in a class, 3 have blue eyes. If two of the girls are chosen at random, what is the probability that

a)both have blue eyes.

b) Neither have blue eyes.

c)at least one has blue eyes.

3 blue, 7 o.w, 10 girls; 2 girls

$$a)\ \frac{\binom{3}{2}\binom{7}{0}}{\binom{10}{2}} = .06,$$

$$b)\ \frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}} = .4667$$

$$c)\ \frac{\left[\binom{3}{1}\binom{7}{1}+\binom{3}{2}\binom{7}{0}\right]}{\binom{10}{2}} = .533$$

Binomial

(27) . A company installs new central heating furnaces, and has found that for 15% of all installations a return visit is needed to make some modifications. Six installations were made in a particular week. Assume independence of outcomes for these installations.

- **What is the probability that a return visit was needed in all of these cases?**

∴ Binomial dis.

$f_X(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, \dots, n$

$p = .15, \ q = .85, \ n = 6$

X : # of installations need to make some modifications from 6 installations.

$$f_X(6) = P(X=6) = \binom{6}{6}(0.15)^6(0.85)^0$$
$$= 1(0.0000114)(1)$$
$$= 0.0000114$$

- **What is the probability that a return visit was needed in none of these cases?**

$$f_X(0) = P(X=0) = \binom{6}{0}(0.15)^0(0.85)^6$$
$$= 1(1)(0.3771)$$
$$= 0.3771$$

- **What is the probability that a return visit was needed in more than one of these cases?**

$= \sum_{x=2}^{6}\left[\binom{6}{x} .15^{x} .85^{(6-x)}\right]$

or

$P(X>1) = 1-[P(X=0)+P(X=1)]$ ($P(X\le 1)$) or $1-\left\{\sum_{x=0}^{1}\left[\binom{6}{x} .15^{x} .85^{(6-x)}\right]\right\}$

$$=1-\left[0.3771+\binom{6}{1}(0.15)^1(0.85)^5\right]$$

$$=1-[0.3771+(6)(0.15)(.4437)]$$

$$=1-(0.3771+0.3993)$$

$$=0.2236$$

28) A fair die is rolled 4 times. Find

a) The probability of obtaining exactly one 6.

b)The probability of obtaining no 6.

c)The probability of obtaining at least one 6.

Binomal dis.

$f_X(x) = \binom{4}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{4-x}, x=0,1,\dots,4$

X: # of time that 6 is showen from 4 times.

$$a)\, p(x=1) = C_1^4\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)^3 = 0.386$$

$$b)\, p(x=0) = C_0^4\left(\frac{1}{6}\right)^0\times\left(\frac{5}{6}\right)^4 = 0.482$$

$$c)\, p(x=1 \;or\; 2 \;or\; 3 \;or\; 4) = 1-p(x=0) = 0.518$$

or $P(X\ge 1) = \sum_{x=1}^{4}\left[\binom{4}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{4-x}\right]$

√Poisson

29) In a study of a drug -induced anaphylaxis among patients taking rocuronium bromide as part of their anesthesia, Laake and Rottingen found that the occurrence of anaphylaxis followed a Poisson model with λ=12 incidents per year in Norway .Find

1- The probability that in the next year, among patients receiving rocuronium, exactly three will experience anaphylaxis?

∴ Poisson dis. ⇒ X: # of patients that have anaphylaxis from medication per year

λ = averge of patients that have anaphylaxis from medication per year = 12/year

$f_X(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, x=0,1,2,\dots$

1) $P(X=3) = f_X(3) = \frac{e^{-12}(12)^3}{3!} = .001769533$

2- The probability that less than two patients receiving rocuronium, in the next year will experience anaphylaxis?

3- The probability that more than two patients receiving rocuronium, in the next year will experience anaphylaxis?

4- The expected value of patients receiving rocuronium, in the next ~~year~~ 6 months who will experience anaphylaxis.

$$2)\, P(X<2) = P(X\le 1) = P(X=0)+P(X=1) = e^{-12}\left(\frac{12^0}{0!}+\frac{12^1}{1!}\right) = 0.0000799$$

or $\sum_{x=0}^{1}\left[\frac{e^{(-12)}(12^x)}{x!}\right]$

5- The variance of patients receiving rocuronium, in the next year who will experience anaphylaxis

$1-\left\{\sum_{x=0}^{2}\left[\frac{e^{(-12)}(12)^x}{x!}\right]\right\}$

3) $P(X > 2) = 1 - P(X \le 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$$= 1 - 0.0000799 - e^{-12}\left(\frac{12^2}{2!}\right) =$$

6- The standard deviation of patients receiving rocuronium, in the next year who will experience anaphylaxis

4) $\mu = 6 = E(X) = \lambda$

5) $\sigma^2 = 12 = Var(X) = \lambda$

6) $\sigma = \sqrt{12} = \sqrt{Var(X)}$

Poisson Approximation

30 If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will suffer a bad reaction.

∴ Binomial dis. $\Rightarrow f_X(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x=0,1,\dots,n$; $p=.001$, $n=2000$

X: # of individual that suffer a bad reaction from injection from 2000 individuals.

but as $p \to 0$ and $n \to \infty$ we will use the Poisson dis. where $f_X(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x=0,1,\dots$

and $\lambda = np = (2000)(.001) = 2$

(a) $$P(X = 3) = \frac{2^3 e^{-2}}{3!} = 0.180 \quad \xleftrightarrow{\text{or}} \quad P(X=3) = \binom{2000}{3}(.001)^3(1-.001)^{2000-3} = .180537328$$

(b) $$P(X > 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \quad \xleftrightarrow{\text{or}}$$

$$= 1 - \left[\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right]$$

$$= 1 - 5e^{-2} = 0.323$$

$$P(X>2) = \begin{cases} 1 - P(X \le 2) = 1 - \left\{\sum_{x=0}^{2}\left[\binom{2000}{x}(.001)^x(1-.001)^{(2000-x)}\right]\right\} \\ \sum_{x=3}^{2000}\left[\binom{2000}{x}(.001)^x(1-.001)^{(2000-x)}\right] \end{cases}$$

$= .323323561$

31 **Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items.**

∴ Binomial dis. $\Rightarrow f_X(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x=0,1,\dots,n$; $p=.02$, $n=100$

X: # of the items that are defective from 100 items.

but as $p \to 0$ and $n \to \infty$ we will use the Poisson dis. where $f_X(x) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

and $\lambda = np = (100)(.02) = 2$

$$P(X=3) = \frac{e^{-2} 2^3}{3!} = .180 \quad \xleftrightarrow{\text{or}} \quad P(X=3) = \binom{100}{3}(.02)^3(1-.02)^{100-3} = .182275941$$