Solution 1
Vectors

1. The polar coordinates of a point are $r = 5.50$ m and $\theta = 240°$. What are the Cartesian coordinates of this point?

P3.1
$$x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$
$$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

19. A vector has an x component of -25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.

P3.19 $A_x = -25.0$

$A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2\text{ units}}$$

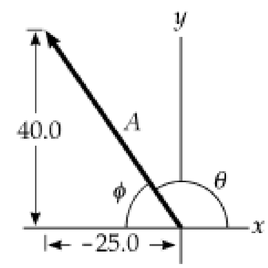

FIG. P3.19

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

21. Obtain expressions in component form for the position vectors having the following polar coordinates: (a) 12.8 m, 150° (b) 3.30 cm, 60.0° (c) 22.0 in., 215°.

P3.21 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x,\ y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x,\ y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x,\ y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

27. Given the vectors $\mathbf{A} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\mathbf{B} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$, (a) draw the vector sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and the vector difference $\mathbf{D} = \mathbf{A} - \mathbf{B}$. (b) Calculate $\mathbf{C}$ and $\mathbf{D}$, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the $+x$ axis.

P3.27 (a) See figure to the right.

(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$\mathbf{C} = \sqrt{25.0 + 16.0}$ at $\tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$\mathbf{D} = 8.06$ at $(180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

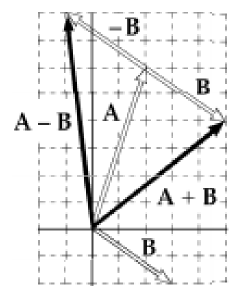

FIG. P3.27

30. Vector **A** has x and y components of -8.70 cm and 15.0 cm, respectively; vector **B** has x and y components of 13.2 cm and -6.60 cm, respectively. If $\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$, what are the components of **C**?

P3.30 $\mathbf{A} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\mathbf{B} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$:

$$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\mathbf{C} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or

$$C_x = \boxed{7.30\text{ cm}};\ C_y = \boxed{-7.20\text{ cm}}$$

31. Consider the two vectors $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\mathbf{B} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$. Calculate (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $|\mathbf{A} + \mathbf{B}|$, (d) $|\mathbf{A} - \mathbf{B}|$, and (e) the directions of $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

P3.31 (a) $(\mathbf{A} + \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

33. A particle undergoes the following consecutive displacements: 3.50 m south, 8.20 m northeast, and 15.0 m west. What is the resultant displacement?

P3.33 $d_1 = \left(-3.50\hat{\mathbf{j}}\right)$ m

$d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right)$ m

$d_3 = \left(-15.0\hat{\mathbf{i}}\right)$ m

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}.$$

The direction is $\theta = \arctan\left(\frac{2.30}{-9.20}\right) = \boxed{166°}$.

41. The vector **A** has *x*, *y*, and *z* components of 8.00, 12.0, and − 4.00 units, respectively. (a) Write a vector expression for **A** in unit–vector notation. (b) Obtain a unit–vector expression for a vector **B** one fourth the length of **A** pointing in the same direction as **A**. (c) Obtain a unit–vector expression for a vector **C** three times the length of **A** pointing in the direction opposite the direction of **A**.

P3.41 (a) $\mathbf{A} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\mathbf{B} = \frac{\mathbf{A}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

49. Three displacement vectors of a croquet ball are shown in Figure P3.49, where $|\mathbf{A}| = 20.0$ units, $|\mathbf{B}| = 40.0$ units, and $|\mathbf{C}| = 30.0$ units. Find (a) the resultant in unit–vector notation and (b) the magnitude and direction of the resultant displacement.

P3.49 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$

$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\mathbf{R} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $|\mathbf{R}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$

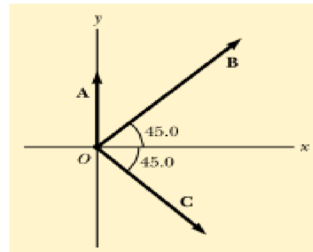

Figure P3.49

50. If $\mathbf{A} = (6.00\hat{\mathbf{i}} - 8.00\hat{\mathbf{j}})$ units, $\mathbf{B} = (-8.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ units, and $\mathbf{C} = (26.0\hat{\mathbf{i}} + 19.0\hat{\mathbf{j}})$ units, determine a and b such that $a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$.

P3.50 Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0.$$

Solving simultaneously,

$$\boxed{a = 5.00,\ b = 7.00}.$$

Therefore,

$$5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0.$$

59. A person going for a walk follows the path shown in Fig. P3.59. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

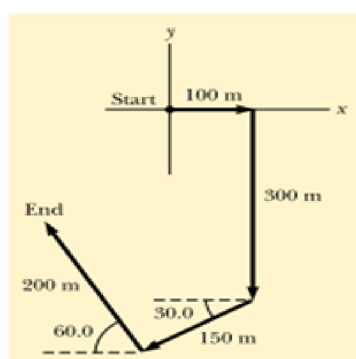

Figure P3.59

P3.59

$$\mathbf{d}_1 = 100\hat{\mathbf{i}}$$

$$\mathbf{d}_2 = -300\hat{\mathbf{j}}$$

$$\mathbf{d}_3 = -150\cos(30.0°)\hat{\mathbf{i}} - 150\sin(30.0°)\hat{\mathbf{j}} = -130\hat{\mathbf{i}} - 75.0\hat{\mathbf{j}}$$

$$\mathbf{d}_4 = -200\cos(60.0°)\hat{\mathbf{i}} + 200\sin(60.0°)\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 173\hat{\mathbf{j}}$$

$$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{ m}}$$

$$|\mathbf{R}| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240\text{ m}}$$

$$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$$

$$\theta = 180 + \phi = \boxed{237°}$$

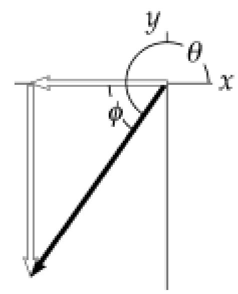

FIG. P3.59