1. **[5 Marks]**

a) Find $\frac{dy(x)}{dx}$ where $y(x) = \frac{(x^2+1)^{2/3}(3x-4)^4}{\sqrt{x}}$.

b) Let $F(x) = x^3 \int_1^{x^3} (t^3+10)^{10}\, dt$. Find $F'(1)$.

a/ we have $\ln(y(x)) = \ln\left[\frac{(x^2+1)^{2/3}(3x-4)^4}{\sqrt{x}}\right]$ [1]

$= \frac{2}{3}\ln(x^2+1) + 4\ln(3x-4) - \frac{1}{2}\ln(x)$ [1]

$\frac{1}{y(x)}\frac{dy(x)}{dx} = \frac{2}{3}\frac{2x}{x^2+3} + \frac{12}{3x-4} - \frac{1}{2x}$ [0,5]

$\frac{dy(x)}{dx} = \frac{(x^2+1)^{2/3}(3x-4)^4}{\sqrt{x}}\left[\frac{4x}{3(x^2+3)} + \frac{12}{3x-4} - \frac{1}{2x}\right]$ [0,5]

b/ $F'(x) = 3x^2 \int_1^{x^3} (t^3+10)^{10}\, dt + x^3(3x^2)(x^9+10)^{10}$ [1]

$\Rightarrow F'(1) = 1^3(3 \cdot 1^2)(1^9+10)^{10}$ [1]

$= 3 \cdot 11^{10}$

2. **[5 Marks]**

a) **Approximate** the definite integral $\int_0^1 (x^3 + e^{-x})dx$, for $n = 4$, by using **Simpson's** rule, and estimate the maximum **error** in the approximation.

$n = 4$, $\Delta x = \frac{1-0}{4} = 0.25$

$x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $x_4 = 1$ [1]

[1] $\int_0^1 (x^3 + e^{-x})dx \simeq \frac{1-0}{3(4)} \{ f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1) \}$

$\simeq \frac{1}{12} \{ 10.58561 \}$

[1] $\simeq 0.8821341753$

$|\text{Error}| \leq \frac{(b-a)^5}{180 n^4} M$, where $|f^{(4)}(x)| \leq M$, $\forall x \in [0,1]$ [0.5]

$a = 0$, $b = 1$, $n = 4$, $f(x) = x^3 + e^{-x}$

$\Rightarrow f^{(4)}(x) = e^{-x}$ $\Rightarrow |f^{(4)}(x)| \leq 1$, $\forall x \in [0,1]$ = [0.5]

$\Rightarrow |\text{Error}| \leq \frac{1^5}{180 \cdot 4^4} e^{-0} = \frac{1}{180 \cdot 4^4}$ [1]

$= \frac{1}{35840} \simeq 0.0000279$

3. **[7 Marks]**

a) Find a number z that satisfies the conclusion of the mean value theorem for $f(x) = 2^{x+1},\ x \in [1,2]$.

b) Solve $f'(x) = 6x^5 - 3\csc^2(3x)$ subject to the initial condition $f\left(\frac{\pi}{2}\right) = 0$.

c) Evaluate the integral $\int \frac{e^{1-\tan x}}{\cos^2(x)}dx$.

a)/ $f(x) = 2^{x+1}$

$\int_1^2 2^{x+1}dx = f(z)(2-1)$ [1]

$2\int_1^2 2^x dx = 2^{z+1}(1) \Rightarrow \int_1^2 2^x dx = 2^z$

then

$\frac{1}{\ln(2)}\left[2^x\right]_1^2 = 2^z \Rightarrow \frac{1}{\ln(2)}\left[2^2-1\right] = 2^z$ [1]

and then $2^z = \frac{3}{\ln(2)} \Rightarrow z = \frac{1}{\ln(2)}\ln\left[\frac{3}{\ln(2)}\right]$ [1]

b)/ we have $f(x) = \int 6x^5dx - 3\csc^2(3x)dx$

$= x^6 + 3\int(-\csc^2(3x))dx$ [0.5]

$= x^6 + \cot(3x) + C$ [0.5]+[0.5]

$F(\pi/2) = 0 \Rightarrow C = -\left(\frac{\pi}{2}\right)^6$ [0.5]

c)/ let $u = 1-\tan(x) \Rightarrow du = \frac{-dx}{\cos^2(x)}$ [1]

$\int \frac{e^{1-\tan(x)}}{\cos^2(x)}dx = -\int e^u du = -e^{1-\tan(x)} + C$ [1]

4. [8 Marks]

a) Evaluate the integral $\int \frac{1}{\sqrt{x}(4+x)} dx$.

b) Evaluate the integral $\int \frac{tan(x)}{\sqrt{16cos^2(x)-2}} dx$.

a) let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$ [1]

then

$\int \frac{dx}{\sqrt{x}(4+x)} = 2\int \frac{du}{4+u^2} = 2 \cdot \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) + C$ [2]

$= \tan^{-1}\left(\frac{u}{2}\right) + C$

$= \tan^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$ [1]

b) $u = 4\cos(x) \Rightarrow du = -4\sin(x)\,dx$ [1]

$\int \frac{\tan(x)}{\sqrt{16\cos^2(x)-2}} dx = \int \frac{\sin(x)}{\cos(x)\sqrt{16\cos^2(x)-2}} dx$ [1]

[2] $= -\int \frac{du}{u\sqrt{u^2-2}} = \frac{1}{\sqrt{2}} \sec^{-1}\left(\frac{4\cos(x)}{\sqrt{2}}\right) + C$

$= \frac{1}{\sqrt{2}} \sec^{-1}\left(2\sqrt{2}\cos(x)\right) + C$

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