KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1431/1432)

Solution Badeel First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	a	b	c	d	a	b	c	d	a	b

Q. No: 1 $\lim_{n\to\infty} \frac{1}{n^4} \sum_{k=1}^{n} (k-1)(k+2)$, is equal to:

(a) 0 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) ∞

Q. No: 2 The integral $\int_0^4 \sinh(x)\, dx$ is equal to:

(a) $\int_0^4 \sinh(4+x)\, dx$ (b) $\int_0^4 \sinh(4-x)\, dx$ (c) $-\int_0^4 \sinh(4-x)\, dx$

(d) $-\int_0^4 \sinh(4+x)\, dx$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x) = ax + b$ $(a \neq 0)$ on $[\alpha, \beta]$ is:

(a) α (b) $\frac{\alpha-\beta}{2}$ (c) $\frac{\alpha+\beta}{2}$ (d) β

Q. No: 4 If $\log(x+3) - \log(x) = 1$, then x is equal to:

(a) $\frac{-1}{2}$ (b) $\frac{-1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Q. No: 5 If $\int_0^{x^2} f(t)\, dt = x\cos(\pi x)$, then $f(4)$ is equal to:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

Q. No: 6 The derivative of the function $f(x) = \tanh^{-1}(\sin(3x))$ is equal to:

(a) $\frac{3\cos(3x)}{1+\sin^2(3x)}$ (b) $\frac{3\cos(3x)}{1-\sin^2(3x)}$ (c) $\frac{\cos(3x)}{1+\sin^2(3x)}$ (d) $\frac{\cos(3x)}{1-\sin^2(3x)}$

Q. No: 7 The value of the integral $\int_0^1 4^x dx$ is equal to:

(a) $\frac{2}{3\ln 2}$ (b) $\frac{3}{2\log 2}$ (c) $\frac{3}{2\ln 2}$ (d) $\frac{2}{3\log 2}$

Q. No: 8 The value of the integral $\int \left(1+\frac{1}{x}\right)^{-3} \left(\frac{1}{x^2}\right) dx$ is equal to:

(a) $\frac{-1}{2}\left(\frac{x+1}{x}\right)^{-2} + c$ (b) $\left(\frac{x+1}{x}\right)^{-2} + c$ (c) $-\left(\frac{x+1}{x}\right)^{-2} + c$ (d) $\frac{1}{2}\left(\frac{x+1}{x}\right)^{-2} + c$

Q. No: 9 The value of the integral $\int \frac{\sin(2x)}{\sin^2 x+1} dx$ is equal to:

(a) $\ln\left|\sin^2 x+1\right|+c$ (b) $\ln\left|\sin(2x)+1\right|+c$ (c) $-\ln\left|\sin(2x)+1\right|+c$
(d) $-\ln\left|\sin^2 x+1\right|+c$

Q. No: 10 The value of the integral $\int \frac{\sinh(\sqrt{x})}{\sqrt{x}} dx$ is equal to:

(a) $\cosh(\sqrt{x})+c$ (b) $2\cosh(\sqrt{x})+c$ (c) $\frac{1}{2}\cosh(\sqrt{x})+c$ (d) $2\cos(\sqrt{x})+c$

Full Questions

Question No: 11 Approximate the integral $\int_0^{\pi} \sqrt{\sin x} dx$ using the **Simpson's rule** with $n = 4$. [**3**]

Solution:

Let $f(x) = \sqrt{\sin x}$

$\Delta x = \frac{\pi}{4}$

So $x_0 = 0$, $x_1 = \frac{\pi}{4}$, $x_2 = \frac{\pi}{2}$, $x_3 = \frac{3\pi}{4}$ and $x_4 = \pi$.

$$\begin{aligned}
\int_0^{\pi} \sqrt{\sin x} dx &\approx \tfrac{\pi-0}{3\times 4}\left\{f(0)+4f\left(\tfrac{\pi}{4}\right)+2f\left(\tfrac{\pi}{2}\right)+4f\left(\tfrac{3\pi}{4}\right)+f(\pi)\right\} \\
&\approx \tfrac{\pi}{12}\{0+(4\times 0.840\,90)+(2\times 1)+(4\times 0.840\,90)+0\} \\
&\approx \tfrac{\pi}{12}\{3.\,363\,6+2+3.\,363\,6\} \\
&\approx \tfrac{\pi}{12}\{8.\,727\,2\} \\
&\approx \{2.\,284\,8\}
\end{aligned}$$

Question No: 12 If $y = (\tan x)^{\tan^{-1} x}$, then find y'. [**2**]

Solution:

$\ln y = (\tan^{-1} x)\ln(\tan x)$

$$\Rightarrow \frac{y'}{y} = \left(\frac{1}{1+x^2}\right)\ln(\tan x) + (\tan^{-1} x)\frac{\sec^2 x}{\tan x}$$

$$\Rightarrow y' = \left[\left(\frac{1}{1+x^2}\right)\ln(\tan x) + (\tan^{-1} x)\frac{\sec^2 x}{\tan x}\right](\tan x)^{\tan^{-1} x}$$

Question No: 13 Evaluate the integral $\int \frac{1}{\sqrt{9x^2+25}}dx.$ [**3**]

Solution:

Let $u = 3x \Rightarrow du = 3dx$

$$\int \frac{1}{\sqrt{9x^2+25}}dx = \frac{1}{3}\int \frac{1}{\sqrt{u^2+25}}du = \frac{1}{3}\sinh^{-1}\left(\frac{u}{5}\right) + c = \frac{1}{3}\sinh^{-1}\left(\frac{3x}{5}\right) + c$$

Question No: 14 Evaluate the integral $\int \frac{1}{\sqrt{1-e^{2x}}}dx.$ [**2**]

Solution:

Let $u = e^x \Rightarrow du = e^x dx$

$$\int \frac{1}{\sqrt{1-e^{2x}}}dx = \int \frac{1}{u\sqrt{1-u^2}}du = -sech^{-1}\left(\frac{|u|}{1}\right) + c = -sech^{-1}\left(e^x\right) + c$$