



FIRST MID TERM EXAM

Name (in Arabic):

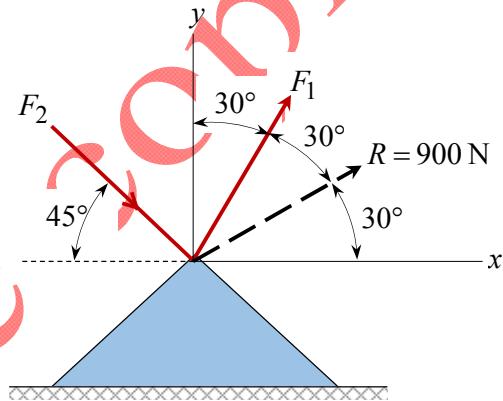
Student No.:

Section / Instructor:

Q. No.	Max. Marks	Marks Obtained
1	25	
2	20	
Total	45	

Question # 1(a) (5 points)

The resultant R of the forces F_1 and F_2 is 900 N and its line of action has an angle of 30° with x -axis as shown in the figure. Determine the magnitude of F_1 and F_2 forces.



Solution

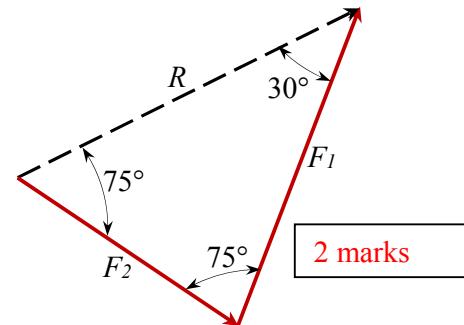
Applying law of sines:

$$\frac{F_1}{\sin 75^\circ} = \frac{R}{\sin 75^\circ} \Rightarrow F_1 = R = 900 \text{ N} \quad \text{Ans.}$$

1.5 marks

$$\frac{F_2}{\sin 30^\circ} = \frac{R}{\sin 75^\circ} \Rightarrow F_2 = R \times \frac{\sin 30^\circ}{\sin 75^\circ} = 465.9 \text{ N} \quad \text{Ans.}$$

1.5 marks



2 marks

Alternative Solution

$$\begin{aligned} \rightarrow R_x &= F_1 \cos 60^\circ + F_2 \cos 45^\circ = 900 \times \cos 30^\circ \\ \Rightarrow 0.5F_1 + 0.71F_2 &= 779.4 \end{aligned} \quad \dots(1)$$

1.5 marks

$$\begin{aligned} \uparrow R_y &= F_1 \cos 30^\circ - F_2 \cos 45^\circ = 900 \times \sin 30^\circ \\ \Rightarrow 0.87F_1 - 0.71F_2 &= 450.0 \end{aligned} \quad \dots(2)$$

1.5 marks

Solving Eqs. (1) and (2) simultaneously, we have

$$F_1 = 900 \text{ N} \quad \text{Ans.}$$

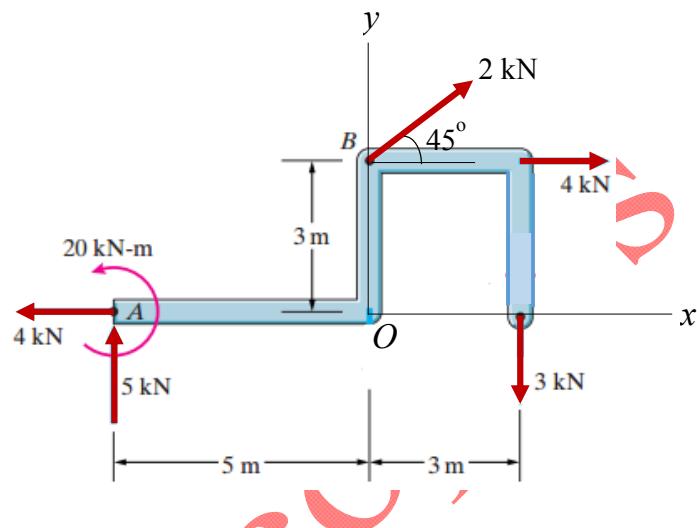
2 marks

$$F_2 = 465.9 \text{ N} \quad \text{Ans.}$$

Question #1(b) (15 points)

For the force system shown in the figure:

- Replace the given system of forces by a single resultant force R ,
- Find the intersections of R with x and y axes measured from point O , and
- Sketch R in x - y plane.

**Solution**

(a)

$$\rightarrow R_x = \sum F_x = 2 \cos 45^\circ + 4 - 4 = 1.414 \text{ kN} \rightarrow$$

2 marks

$$\uparrow R_y = \sum F_y = 2 \sin 45^\circ + 5 - 3 = 3.414 \text{ kN} \uparrow$$

2 marks

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{1.414^2 + 3.414^2} = 3.7 \text{ kN} \quad \text{Ans.}$$

2 marks

$$\text{Direction of Resultant } R: \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{3.414}{1.414}\right) = 67.5^\circ$$

$$\text{CCW (+)} M_0 = \sum Fd = +20 - 2 \cos 45^\circ (3) - 4(3) - 3(3) - 5(5) = -30.24 \text{ kN.m (CW)} \quad \text{Ans.}$$

3 marks

(b) x and y intercepts

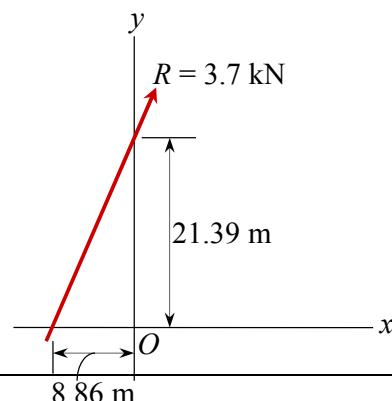
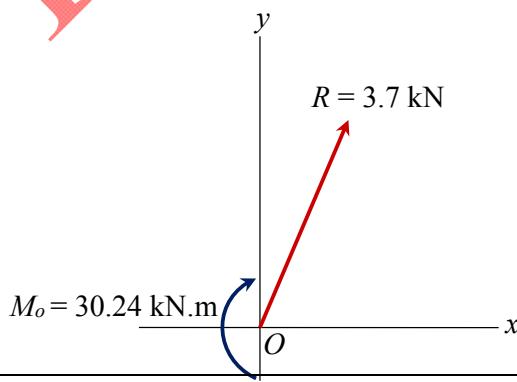
$$x = \frac{M_o}{R_y} = \frac{-30.24}{3.414} = -8.86 \text{ m} \quad \text{Ans.}$$

2 marks

$$y = -\frac{M_o}{R_x} = -\frac{-30.24}{1.414} = 21.39 \text{ m} \quad \text{Ans.}$$

2 marks

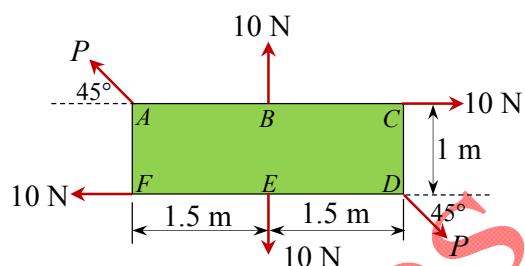
(c) Sketch of Force-couple system at point O and then sketch of single R (in x - y plane)



2 marks

Question # 1(c) (5 points)

A rectangular plate with $1 \text{ m} \times 3 \text{ m}$ sides is subjected to six forces as shown in the figure. If the moment of these forces about point A is -50 N.m (i.e. Clockwise), calculate the magnitude of the force P .

**Solution**

$$\text{Given: } M_A = -50 \text{ N.m}$$

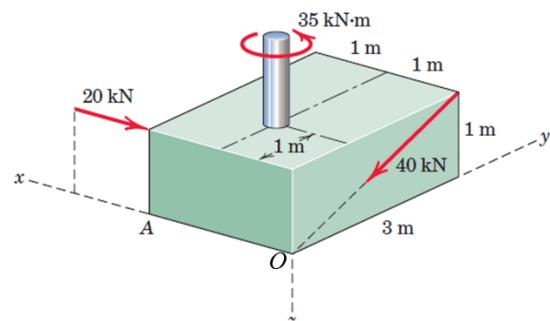
The given forces form various couples. The total moment of these couples are

$$P \cos 45^\circ \times 1 - P \sin 45^\circ \times 3 - 10 \times 1 = -50 \\ \Rightarrow P = 28.3 \text{ N} \quad \text{Ans.}$$

5 marks

Question # 2 (a) (5 points)

Replace the system of forces and couple shown in the figure by a single force-couple at O .

**Solution**

$$\mathbf{F}_1 = -20\mathbf{i} \quad \mathbf{F}_2 = 40 \times \left[\frac{-3\mathbf{j} + \mathbf{k}}{\sqrt{9+1}} \right] = -37.95\mathbf{j} + 12.65\mathbf{k}$$

2 marks

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = -20\mathbf{i} - 37.95\mathbf{j} + 12.65\mathbf{k} \quad \text{Ans.}$$

1 mark

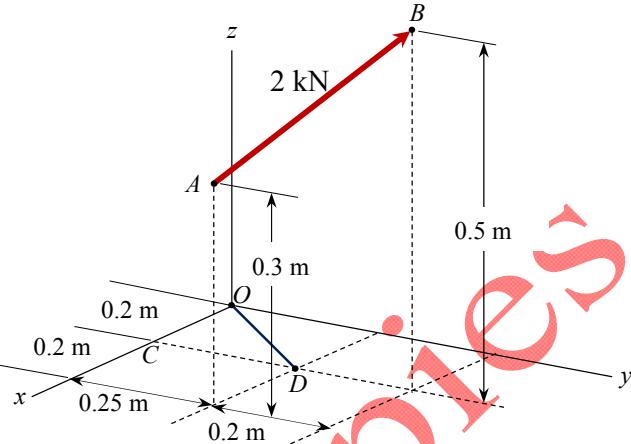
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 - 35\mathbf{k} = (-\mathbf{k}) \times (-20\mathbf{i}) - 35\mathbf{k} = 20\mathbf{j} - 35\mathbf{k} \quad \text{kN.m} \quad \text{Ans.}$$

2 marks

Student name	Marks obtained for Q.2	page 3/4
Student number		
Question # 2(b) (15 points)		

For the given force of 2 kN, determine in a vector form

- (i) Moment about the point O (\mathbf{M}_O)
- (ii) Moment about the point A (\mathbf{M}_A)
- (iii) Moment about the line OD (\mathbf{M}_{OD})



Solution

Coordinates: $O(0, 0, 0)$; $A(0.4, 0.25, 0.3)$; $B(0.2, 0.45, 0.5)$; $C(0.2, 0, 0)$; $D(0.2, 0.25, 0)$

(i) Moment about the point O

$$\mathbf{F} = 2\mathbf{n}_{AB} = 2 \times \left[\frac{(0.2 - 0.4)\mathbf{i} + (0.45 - 0.25)\mathbf{j} + (0.5 - 0.3)\mathbf{k}}{\sqrt{(0.2 - 0.4)^2 + (0.45 - 0.25)^2 + (0.5 - 0.3)^2}} \right] = -1.16\mathbf{i} + 1.16\mathbf{j} + 1.16\mathbf{k} \text{ kN}$$
4 marks

$$\mathbf{r}_{OA} = 0.4\mathbf{i} + 0.25\mathbf{j} + 0.3\mathbf{k} \text{ m};$$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & 0.25 & 0.3 \\ -1.16 & 1.16 & 1.16 \end{vmatrix} = -0.058\mathbf{i} - 0.812\mathbf{j} + 0.754\mathbf{k} \text{ kN.m} \quad \text{Ans.}$$
1 mark
3 marks

(ii) Moment about the point A

$\mathbf{M}_A = 0$ (Line of action of the given force is passing through point A)

1 mark

(iii) Moment about the line OD

$$\mathbf{n}_{OD} = \left[\frac{(0.2 - 0)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k}}{\sqrt{(0.2 - 0)^2 + (0.25 - 0)^2 + (0 - 0)^2}} \right] = 0.625\mathbf{i} + 0.781\mathbf{j} + 0\mathbf{k}$$
2 marks

In scalar form:

$$M_{OD} = \mathbf{M}_O \cdot \mathbf{n}_{OD} = (-0.058\mathbf{i} - 0.812\mathbf{j} + 0.754\mathbf{k}) \cdot (0.625\mathbf{i} + 0.781\mathbf{j}) = -0.67 \text{ kN.m}$$
2 marks

In vector form:

$$\mathbf{M}_{OD} = (\mathbf{M}_O \cdot \mathbf{n}_{OD})\mathbf{n}_{OD} = -0.67(0.625\mathbf{i} + 0.781\mathbf{j}) = -0.42\mathbf{i} - 0.52\mathbf{j} \text{ kN.m} \quad \text{Ans.}$$
2 marks