KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1432/1433)

Solution First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	d	a	c	c	d	c	a	a	c	c

Q. No: 1 The sum $\sum\limits_{k=1}^{n}(3+k)^2$ is equal to:

(a) $\frac{1}{6}(2n^3+21n^2+54n)$ (b) $\frac{1}{6}(2n^3+19n^2+73n)$

(c) $\frac{1}{6}(n^3+21n^2+73n)$ (d) $\frac{1}{6}(2n^3+21n^2+73n)$

Q. No: 2 The value of the integral $\int \sin(1+3x)\,dx$ is equal to:

(a) $\frac{-1}{3}\cos(1+3x)+c$ (b) $3\cos(1+3x)+c$

(c) $\frac{1}{3}\cos(1+3x)+c$ (d) $-\cos(1+3x)+c$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x)=x^2$ on $[-2,0]$ is:

(a) $-\sqrt{\frac{8}{3}}$ (b) $\sqrt{\frac{8}{3}}$ (c) $\frac{-2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

Q. No: 4 The average value of $f(x)=\sqrt{x+1}$ on $[-1,0]$ is equal to:

(a) $\frac{-3}{2}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Q. No: 5 If $F(x)=\int_x^{2x} f'(t)\,dt$, then $F'(x)$ is equal to:

(a) $f(2x)-f(x)$ (b) $2f(2x)-f(x)$ (c) $2f'(2x)$ (d) $2f'(2x)-f'(x)$

Q. No: 6 The value of the integral $\int \frac{5^{\cosh(x)}}{csch(x)}dx$ is equal to:

(a) $5^{\cosh(x)}+c$ (b) $(\ln 5)\,5^{\sinh(x)}+c$ (c) $\frac{5^{\cosh(x)}}{\ln 5}+c$ (d) $\frac{5^{\sinh(x)}}{\ln 5}+c$

Q. No: 7 The derivative of the function $f(x)=\cosh^{-1}(\sqrt{x})$ is equal to:

(a) $\frac{1}{2\sqrt{x^2-x}}$ (b) $\frac{1}{\sqrt{2x^2-x}}$ (c) $\frac{1}{2x\sqrt{x+1}}$ (d) $\frac{1}{2x\sqrt{x^2-1}}$

Q. No: 8 The value of the integral $\int (\sin x)(\sec x)^2\,dx$ is equal to:

(a) $\frac{1}{\cos x}+c$ (b) $\frac{1}{\sin x}+c$ (c) $\frac{1}{\sec x}+c$ (d) $\frac{1}{3}(\sec x)^3+c$

Q. No: 9 If $\int \frac{e^{\cos^{-1}(x)}}{\sqrt{1-x^2}}dx = f(x) + c$, then $f(x)$ is equal to:

$(a)\ e^{\cos^{-1}(x)}$ $(b)\ e^{-\cos^{-1}(x)}$ $(c)\ -e^{\cos^{-1}(x)}$ $(d)\ e^{\sin^{-1}(x)}$

Q. No: 10 The value of the integral $\int \frac{e^{2x}}{\sqrt{e^{4x}-1}}dx$ is equal to:

$(a)\ \frac{1}{2}\sin^{-1}(e^{2x}) + c$ $(b)\ \frac{1}{2}\sinh^{-1}(e^{2x}) + c$ $(c)\ \frac{1}{2}\cosh^{-1}(e^{2x}) + c$ $(d)\ \cosh^{-1}(e^{2x}) + c$

Full Questions

Question No: 11 Approximate the integral $\int_0^1 e^{4x}dx$ using the **Simpson's rule** for $n = 4$. [**3**]

Solution:

Let $f(x) = e^{4x}$.

$\Delta x = \frac{1}{4} = 0.25$

$x_0 = 0,\ x_1 = 0.25,\ x_2 = 0.5,\ x_3 = 0.75$ and $x_4 = 1.$ (1)

$$\begin{aligned}\int_0^1 e^{4x}dx &\approx \frac{1}{3\times 4}\{f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)\} && (1)\\ &= \frac{1}{12}\{1 + 4(2.7183) + 2(7.3891) + 4(20.086) + 54.598\}\\ &= \frac{1}{12}\{1 + 10.873 + 14.778 + 80.344 + 54.598\}\\ &= \frac{1}{12}\{161.59\} \approx 13.466 && (1)\end{aligned}$$

Question No: 12 If $y = (\cosh x)^{2x+1}$, then find y'. [**2**]

Solution:

$$\begin{aligned}\ln y &= (2x+1)\ln(\cosh x) && (0.5)\\ \frac{y'}{y} &= 2\ln(\cosh x) + (2x+1)\tanh x && (1)\\ y' &= [2\ln(\cosh x) + (2x+1)\tanh x](\cosh x)^{2x+1} && (0.5)\end{aligned}$$

Question No: 13 Evaluate the integral $\int \frac{x-2}{\sqrt{8-2x^2}}dx.$ [**3**]

Solution:

$$\begin{aligned}\int \frac{x-2}{\sqrt{8-2x^2}}dx &= \int \frac{x}{\sqrt{8-2x^2}}dx - 2\int \frac{1}{\sqrt{8-2x^2}}dx \\ &= -\frac{1}{2}\sqrt{8-2x^2} - \frac{2}{\sqrt{2}}\sin^{-1}\left(\frac{x}{2}\right). \qquad (1)+(2)\end{aligned}$$

Question No: 14 Evaluate the integral $\int \frac{1}{x\sqrt{4+(\ln x)^2}}dx.$ [**2**]

Solution:

$$\begin{aligned}&\text{Let } u = \ln x && \Rightarrow du = \tfrac{1}{x}dx && (0.5)\\ &\int \frac{1}{x\sqrt{4+(\ln x)^2}}dx && = \int \frac{1}{\sqrt{4+u^2}}du && (0.5)\\ & && = \sinh^{-1}\left(\tfrac{u}{2}\right) + c && (0.5)\\ & && = \sinh^{-1}\left(\tfrac{\ln x}{2}\right) + c && (0.5)\end{aligned}$$