General Physics II (111 phys)

HW-5
Capacitors and Dielectric

1- An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0 V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate?

(a) $\Delta V = Ed$

$$E = \frac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$$

(b) $E = \frac{\sigma}{\epsilon_0}$

$$\sigma = \left(1.11 \times 10^4\ \text{N/C}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{98.3\ \text{nC/m}^2}$$

(c) $$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\ \text{cm}^2\right)\left(1.00\ \text{m}/100\ \text{cm}\right)^2}{1.80 \times 10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$$

(d) $\Delta V = \frac{Q}{C}$

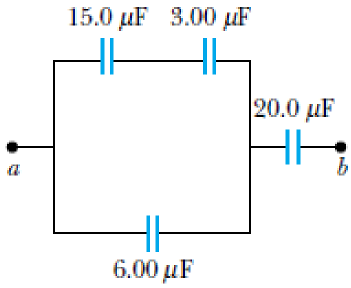
$$Q = (20.0\ \text{V})\left(3.74 \times 10^{-12}\ \text{F}\right) = \boxed{74.7\ \text{pC}}$$

2- When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

$$Q = \frac{\epsilon_0 A}{d}(\Delta V) \qquad \frac{Q}{A} = \sigma = \frac{\epsilon_0 (\Delta V)}{d}$$

$$d = \frac{\epsilon_0 (\Delta V)}{\sigma} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(30.0 \times 10^{-9}\ \text{C/cm}^2\right)\left(1.00 \times 10^4\ \text{cm}^2/\text{m}^2\right)} = \boxed{4.42\ \mu\text{m}}$$

3- Four capacitors are connected as shown in the figure. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor if ΔV_{ab} = 15.0 V.



(a) $$\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$$

$$C_s = 2.50\ \mu\text{F}$$

$$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $$Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}} \text{ on } 20.0\ \mu\text{F}$$

$$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$$

$$15.0 - 4.47 = 10.53\ \text{V}$$

$$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}} \text{ on } 6.00\ \mu\text{F}$$

$$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}} \text{ on } 15.0\ \mu\text{F and } 3.00\ \mu\text{F}$$

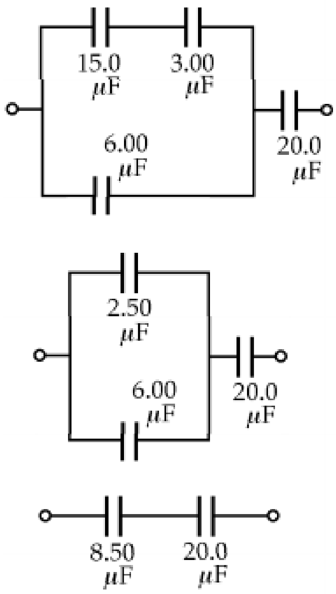
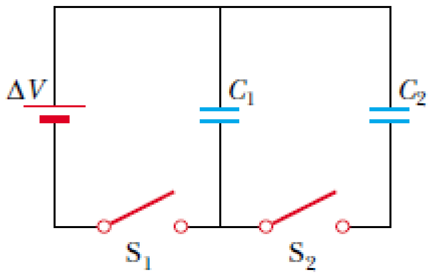


FIG. P26.21

4- Consider the circuit shown in the figure where C_1 = 6.00 μF, C_2 = 3.00 μF, and ΔV = 20.0 V. Capacitor $C1$ is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the initial charge acquired by C_1 and the final charge on each capacitor.



$C = \frac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \frac{Q}{20.0}$

and $Q = \boxed{120\ \mu C}$

$Q_1 = 120\ \mu C - Q_2$

and $\Delta V = \frac{Q}{C}$: $\frac{120 - Q_2}{C_1} = \frac{Q_2}{C_2}$

or $\frac{120 - Q_2}{6.00} = \frac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \frac{360}{9.00} = \boxed{40.0\ \mu C}$ $Q_1 = 120\ \mu C - 40.0\ \mu C = \boxed{80.0\ \mu C}$

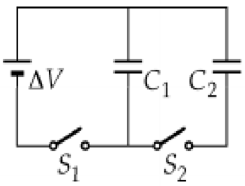


FIG. P26.23

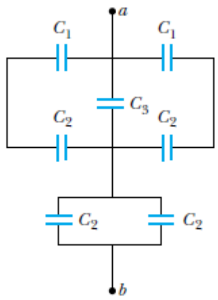
5- Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in the figure. Take C_1 = 5.00 μF, C_2 = 10.0 μF, and C_3 = 2.00 μF.

$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu F$$

$$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu F$$

$$C_{p2} = 2(10.0) = 20.0\ \mu F$$

$$C_{eq} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu F}$$



6- Two capacitors, C_1 = 25.0 μF and C_2 = 5.00 μF, are connected in parallel and charged with a 100 V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) **What If?** What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)? Draw a circuit diagram of this circuit.

$$U = \frac{1}{2}C(\Delta V)^2$$

The circuit diagram is shown at the right.

(a) $$C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$$

$$U = \frac{1}{2}\left(30.0\times10^{-6}\right)(100)^2 = \boxed{0.150\ \text{J}}$$

(b) $$C_s = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}}+\frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$$

$$U = \frac{1}{2}C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17\times10^{-6}}} = \boxed{268\ \text{V}}$$

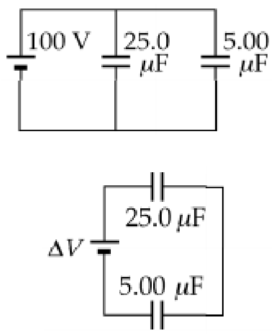


FIG. P26.33

7- A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor. Assume the liquid is an insulator.

Originally, $C = \frac{\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_i}$.

(a) The charge is the same before and after immersion, with value $Q = \frac{\epsilon_0 A(\Delta V)_i}{d}$.

$$Q = \frac{(8.85 \times 10^{-12}\ C^2/N \cdot m^2)(25.0 \times 10^{-4}\ m^2)(250\ V)}{(1.50 \times 10^{-2}\ m)} = \boxed{369\ pC}$$

(b) Finally,

$$C_f = \frac{\kappa \epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f} \qquad C_f = \frac{80.0(8.85 \times 10^{-12}\ C^2/N \cdot m^2)(25.0 \times 10^{-4}\ m^2)}{(1.50 \times 10^{-2}\ m)} = \boxed{118\ pF}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A(\Delta V)_i d}{\kappa \epsilon_0 Ad} = \frac{(\Delta V)_i}{\kappa} = \frac{250\ V}{80.0} = \boxed{3.12\ V}.$$

(c) Originally, $U_i = \frac{1}{2} C\ (\Delta V_i^2) = \frac{1}{2}\frac{A\varepsilon_0}{d}\ (\Delta V_i^2)$

Finally, $U_f = \frac{1}{2}\ C_f (\Delta V_f^2) = \frac{1}{2}\ \frac{k\ A\varepsilon_0}{d}\ (\Delta V_f^2)$

$$\Delta U = U_f - U_i$$

$$\Delta U = (5.74 \times 10^{-10}) - (4.6 \times 10^{-8})J = -45.4 \times 10^{-9} J = -45.4\ nJ$$