HW (3)

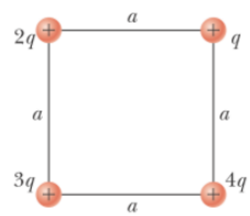
Electric Field

1. Two electrostatic point charges of 60 µC and 50 µC exert a repulsive force on each other of 175 N. What is the distance between the two charges?

$$\boldsymbol{F_e = \frac{k\, q_1 q_2}{r^2}}$$

$$\boldsymbol{r = \sqrt{\frac{k\, q_1 q_2}{F_e}} = \sqrt{\frac{(9\times10^9)(60\times10^{-6})(50\times10^{-6})}{175}} = 0.4\ m}$$

25. Four charged particles are at the corners of a square of side a as shown in Figure P23.25. Determine (a) the electric field at the location of charge q and (b) the total electric force exerted on q.



(a) $$\mathbf{E} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

$$\mathbf{E} = 3.06\frac{k_e q}{a^2}\hat{\mathbf{i}} + 5.06\frac{k_e q}{a^2}\hat{\mathbf{j}} = \boxed{5.91\frac{k_e q}{a^2}\ \text{at } 58.8°}$$

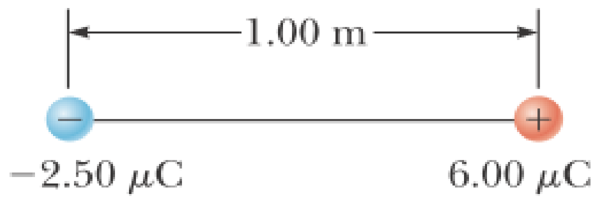
The electric field is calculated as $E = \sqrt{(E\ i)^2 + (E\ j)^2} = \frac{k_e q}{a^2}\sqrt{(3.06)^2 + (5.06)^2}$ $= 5.91\ \frac{k_e q}{a^2}\frac{N}{C}$

The direction of the electric field is calculated as $\theta = \tan^{-1}\frac{E_j}{E_i}$

$$\theta = \tan^{-1}\frac{5.06}{3.06} = 58.8°$$

(b) $$\mathbf{F} = q\mathbf{E} = \boxed{5.91\frac{k_e q^2}{a^2}\ \text{at } 58.8°}$$

29. **M** In Figure P23.29, determine the point (other than infinity) at which the electric field is zero.



The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are

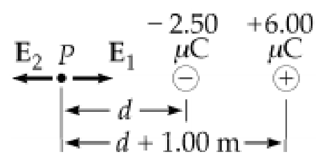


FIG. P23.15

$$E_1 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(2.50 \times 10^{-6} \text{ C}\right)}{d^2} \quad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(6.00 \times 10^{-6} \text{ C}\right)}{(d + 1.00 \text{ m})^2} \quad (2)$$

Equate the right sides of (1) and (2)

to get $(d + 1.00 \text{ m})^2 = 2.40 d^2$

or $d + 1.00 \text{ m} = \pm 1.55 d$

which yields $d = 1.82$ m

or $d = -0.392$ m .

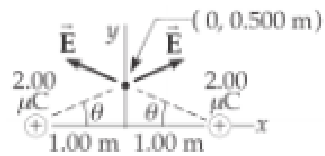
The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82 \text{ m to the left of the } -2.50\ \mu\text{C charge}}$.

34. Two 2.00-μC point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric field on the y axis at $y = 0.500$ m. (b) Calculate the electric force on a -3.00-μC charge placed on the y axis at $y = 0.500$ m.

(a) The distance from each charge to the point at $y = 0.500$ m is

$$d = \sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2} = 1.12 \text{ m}$$



ANS. FIG. P23.34

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.00 \times 10^{-6} \text{ C})}{(1.12 \text{ m})^2} = 14\,400 \text{ N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \quad E_y = 2(14\,400 \text{ N/C})\sin 26.6° = 1.29 \times 10^4 \text{ N/C}$$

so $\boxed{\vec{\mathbf{E}} = 1.29 \times 10^4 \hat{\mathbf{j}} \text{ N/C}}$.

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-3.00 \times 10^{-6} \text{ C})\left(1.29 \times 10^4 \text{ N/C}\hat{\mathbf{j}}\right)$$

$$= \boxed{-3.86 \times 10^{-2} \hat{\mathbf{j}} \text{ N}}$$

50. Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in Figure P23.50. Assume the three charges together create an electric field. (a) Sketch the field lines in the plane of the charges. (b) Find the location of one point (other than ∞) where the electric field is zero. What are (c) the magnitude and (d) the direction of the electric field at P due to the two charges at the base?

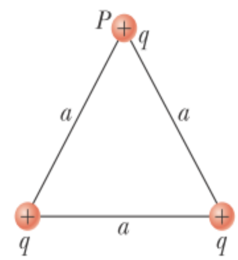
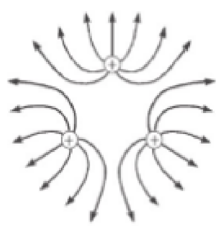


Figure P23.50



(a)

(b) It is zero **at the center**, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

(c)

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.$$

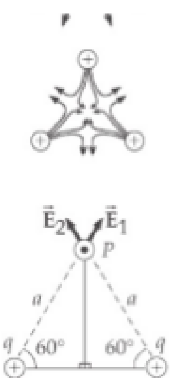
As shown in the bottom panel of ANS. FIG. P23.50,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

to the left and upward at 60°. So,



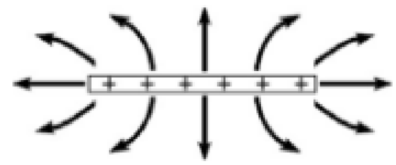
ANS. FIG. P23.50

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

47. A negatively charged rod of finite length carries charge with a uniform charge per unit length. Sketch the electric field lines in a plane containing the rod.

The field lines are shown in ANS. FIG. P23.47, where we've followed the rules for drawing field lines where field lines point toward negative charge, meeting the rod perpendicularly and ending there.



ANS. FIG. P23.47