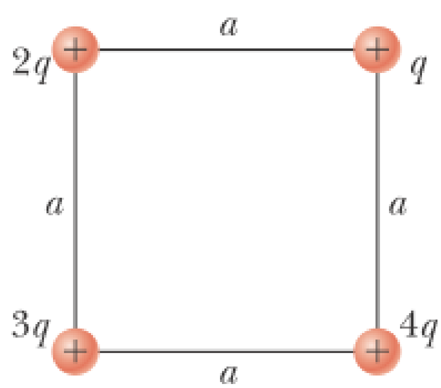
HW (3)

Electric Field

25. Four charged particles are at the corners of a square of side a as shown in Figure P23.25. Determine (a) the electric field at the location of charge q and (b) the total electric force exerted on q.



P23.25 We sum the electric fields from each of the other charges using Equation 23.7 for the definition of the electric field.

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

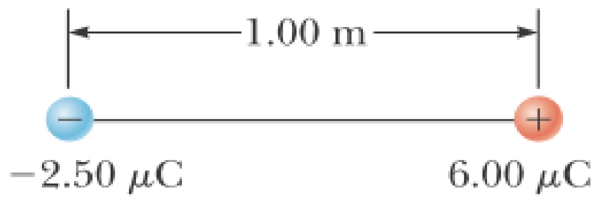
$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

29. **M** In Figure P23.29, determine the point (other than infinity) at which the electric field is zero.



The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are

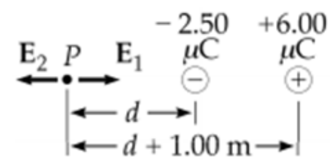


FIG. P23.15

$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2.50 \times 10^{-6} \text{ C})}{d^2} \quad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(6.00 \times 10^{-6} \text{ C})}{(d + 1.00 \text{ m})^2} \quad (2)$$

Equate the right sides of (1) and (2)

to get $(d + 1.00 \text{ m})^2 = 2.40 d^2$

or $d + 1.00 \text{ m} = \pm 1.55 d$

which yields $d = 1.82$ m

or $d = -0.392$ m.

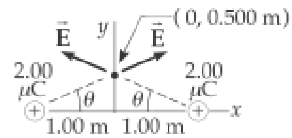
The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d =$ 1.82 m to the left of the -2.50 μC charge.

34. Two 2.00-μC point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric field on the y axis at $y = 0.500$ m. (b) Calculate the electric force on a -3.00-μC charge placed on the y axis at $y = 0.500$ m.

***P23.34** (a) The distance from each charge to the point at $y = 0.500$ m is

$$d = \sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2} = 1.12 \text{ m}$$



ANS. FIG. P23.34

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^{-6} \text{ C})}{(1.12 \text{ m})^2} = 14\ 400 \text{ N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \quad E_y = 2(14\ 400 \text{ N/C})\sin 26.6° = 1.29 \times 10^4 \text{ N/C}$$

so $\boxed{\vec{E} = 1.29 \times 10^4 \hat{\mathbf{j}} \text{ N/C}}$.

(b) The electric force at this point is given by

$$\vec{F} = q\vec{E} = (-3.00 \times 10^{-6} \text{ C})(1.29 \times 10^4 \text{ N/C}\hat{\mathbf{j}})$$

$$= \boxed{-3.86 \times 10^{-2} \hat{\mathbf{j}} \text{ N}}$$

50. Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in Figure P23.50. Assume the three charges together create an electric field. (a) Sketch the field lines in the plane of the charges. (b) Find the location of one point (other than ∞) where the electric field is zero. What are (c) the magnitude and (d) the direction of the electric field at P due to the two charges at the base?

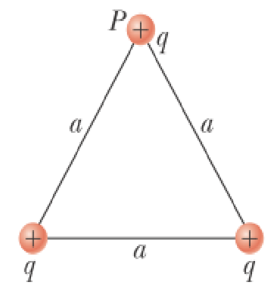


Figure P23.50

(a) The electric field has the general appearance shown in ANS. FIG. P23.50 below.

(b) It is zero **at the center**, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second panel of ANS. FIG. P23.50 indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(c) You may need to review vector addition in Chapter 1. The electric field at point *P* can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.$$

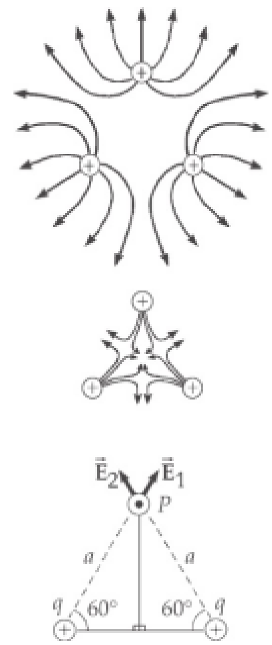
As shown in the bottom panel of ANS. FIG. P23.50,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

to the left and upward at 60°. So,



ANS. FIG. P23.50

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

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51. AMT M A proton accelerates from rest in a uniform electric field of 640 N/C. At one later moment, its speed is 1.20 Mm/s (nonrelativistic because *v* is much less than the speed of light). (a) Find the acceleration of the proton. (b) Over what time interval does the proton reach this speed? (c) How far does it move in this time interval? (d) What is its kinetic energy at the end of this interval?

Uniform Electric Field

P23.51 (a) We obtain the acceleration of the proton from the particle under a net force model, with $F = qE$ representing the electric force:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(640\ \text{N/C}\right)}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$$

(b) The particle under constant acceleration model gives us $v_f = v_i + at$, from which we obtain

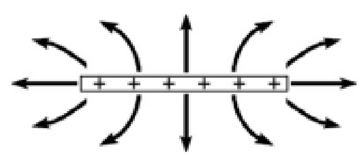
$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^6\ \text{m/s}}{6.14 \times 10^{10}\ \text{m/s}^2} = \boxed{19.5\ \mu\text{s}}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(6.14 \times 10^{10}\ \text{m/s}^2)(19.5 \times 10^{-6}\ \text{s})^2$$
$$= \boxed{11.7\ \text{m}}$$

47. A negatively charged rod of finite length carries charge with a uniform charge per unit length. Sketch the electric field lines in a plane containing the rod.

P23.47 The field lines are shown in ANS. FIG. P23.47, where we've followed the rules for drawing field lines where field lines point toward negative charge, meeting the rod perpendicularly and ending there.



ANS. FIG. P23.47

57. **M** A proton moves at 4.50×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find (a) the time interval required for the proton to travel 5.00 cm horizontally, (b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

P23.57 $\vec{\mathbf{E}}$ is directed along the y direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $$t = \frac{x}{v_{xi}} = \frac{0.050\,0\text{ m}}{4.50 \times 10^5\text{ s}} = 1.11 \times 10^{-7}\text{ s} = \boxed{111\text{ ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{(1.602 \times 10^{-19}\text{ C})(9.60 \times 10^3\text{ N/C})}{1.67 \times 10^{-27}\text{ kg}} = 9.21 \times 10^{11}\text{ m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2:$$

$$y_f = \frac{1}{2}(9.21 \times 10^{11}\text{ m/s}^2)(1.11 \times 10^{-7}\text{ s})^2$$

$$= 5.68 \times 10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$$

(c) $$v_x = 4.50 \times 10^5\text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = (9.21 \times 10^{11}\text{ m/s}^2)(1.11 \times 10^{-7}\text{ s}) = 1.02 \times 10^5\text{ m/s}$$

$$\vec{\mathbf{v}} = \boxed{(450\hat{\mathbf{i}} + 102\hat{\mathbf{j}})\text{ km/s}}$$