Solution-10

1. A green line of wavelength 4.86x10 ^{7} m is observed in the emission spectrum of hydrogen. Calculate the energy of one photon of this green light.

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(4.86 \times 10^{-7}\ \text{m})} = 4.09 \times 10^{-19}\ \text{J/photon}$$

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2. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV.

- Determine the work function of the metal.

$$K_1 = \frac{hc}{\lambda_1} - \phi$$
$$K_2 = \frac{hc}{\lambda_2} - \phi$$

where $\lambda_2 = 0.5\,\lambda_1$, so

$$K_2 - 2K_1 = \phi$$

and therefore $\phi = 2\,\text{eV}$ when $K_1 = 1\,\text{eV}$ and $K_2 = 4\,\text{eV}$.

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3. What is the energy of a quantum of light with a frequency of 3.87 x 10^{19} Hz?

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4. The temperature of an electric heating element is 150°C. At what wavelength does the radiation emitted from the heating element reach its peak?

The absolute temperature of the heating element is

$$T = 150°\text{C} + 273 = 423\ \text{K}$$

The peak wavelength is, from Equation 40.2,

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{423\ \text{K}} = 6.85 \times 10^{-6}\ \text{m}$$

or 6.85 μm, which is in the infrared region of the spectrum.

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5. Molybdenum has a work function of 4.20 eV. (a) Find the cutoff wavelength and cutoff frequency for the photoelectric effect. (b) What is the stopping potential if the incident light has a wavelength of 180 nm?

$$\lambda_c = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(4.20\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = \boxed{295\ \text{nm}}$$

which corresponds to a frequency of

$$f_c = \frac{c}{\lambda_c} = \frac{2.998\times10^{8}\ \text{m/s}}{295\times10^{-9}\ \text{m}} = \boxed{1.02\times10^{15}\ \text{Hz}}$$

(b) We find the stopping potential from $\frac{hc}{\lambda} = \phi + e\Delta V_S$:

$$\frac{(6.626\times10^{-34})(2.998\times10^{8})}{180\times10^{-9}} = (4.20\ \text{eV})(1.602\times10^{-19}\ \text{J/eV}) + (1.602\times10^{-19})\Delta V_S$$

Therefore, $\boxed{\Delta V_S = 2.69\ \text{V}}$.

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6. The work function for zinc is 4.31 eV. (a) Find the cutoff wavelength for zinc. (b) What is the lowest frequency of light incident on zinc that releases photoelectrons from its surface? (c) If photons of energy 5.50 eV are incident on zinc, what is the maximum kinetic energy of the ejected photoelectrons?

(a) At the cutoff wavelength, the energy of the photons is equal to the work function ($K_{max} = 0$):

$$\frac{hc}{\lambda} = \phi \quad \rightarrow \quad \lambda = \frac{hc}{\phi} = \frac{1\,240\ \text{nm}\cdot\text{eV}}{4.31\ \text{eV}} = \boxed{288\ \text{nm}}$$

(b) This is the cutoff frequency:

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{288\times10^{-9}\ \text{m}} = \boxed{1.04\times10^{15}\ \text{Hz}}$$

(c) The maximum kinetic energy is the difference between the energy of the photons and the work function:

$$K_{max} = E - \phi = 5.50\ \text{eV} - 4.31\ \text{eV} = \boxed{1.19\ \text{eV}}$$

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7. Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ=546.1 nm) is used, a stopping potential of 0.376 V reduces the photocurrent to zero.

(a) Based on this measurement, what is the work function for this metal?

(b) What stopping potential would be observed when using the yellow light from a helium discharge tube (λ= 587.5 nm)?

(a) Einstein's photoelectric effect equation is $K_{max} = hf - \phi$ and the energy required to raise an electron through a 1-V potential is 1 eV, so that

$$K_{max} = e\Delta V_s = 0.376 \text{ eV}$$

The energy of a photon from the mercury lamp is:

$$hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^{8} \text{ m/s})}{546.1\times10^{-9} \text{ m}}\left(\frac{1 \text{ eV}}{1.602\times10^{-19} \text{ J}}\right)$$

$$= \frac{1\ 240 \text{ eV}\cdot\text{nm}}{546.1 \text{ nm}} = 2.27 \text{ eV}$$

Therefore, the work function for this metal is:

$$\phi = hf - K_{max} = 2.27 \text{ eV} - 0.376 \text{ eV} = \boxed{1.89 \text{ eV}}$$

(b) For the yellow light, λ = 587.5 nm and the photon energy is

$$hf = \frac{hc}{\lambda} = \frac{1\ 240 \text{ eV}\cdot\text{nm}}{587.5 \text{ nm}} = 2.11 \text{ eV}$$

Therefore the maximum energy that can be given to an ejected electron is

$$K_{max} = hf - \phi = 2.11 \text{ eV} - 1.89 \text{ eV} = 0.216 \text{ eV}$$

so the stopping voltage is

$$\Delta V_s = \boxed{0.216 \text{ V}}$$

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8. Electrons are ejected from a metallic surface with speeds of up to 4.60×10^5 m/s when light with a wavelength of 625 nm is used. (a) What is the work function of the surface? (b) What is the cutoff frequency for this surface?

The maximum kinetic energy of the electrons is

$$K_{max} = \frac{1}{2}mu_{max}^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{kg}\right)\left(4.60\times10^{5}\text{m/s}\right)^2$$

$$= 9.64\times10^{-20} \text{ J} = 0.602 \text{ eV}$$

(a) The work function is

$$\phi = E - K_{max} = \frac{1\ 240 \text{ eV}\cdot\text{nm}}{625 \text{ nm}} - 0.602 \text{ eV} = \boxed{1.38 \text{ eV}}$$

(b) At the cutoff frequency, the energy of the photons equals the work function:

$$E = hf = \phi \quad \rightarrow \quad f = \frac{\phi}{h} = \frac{1.38 \text{ eV}}{6.626\times10^{-34} \text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right)$$

$$= \boxed{3.34\times10^{14} \text{ Hz}}$$