

Application of Integration (Solid of Revolution)

Bander Almutairi

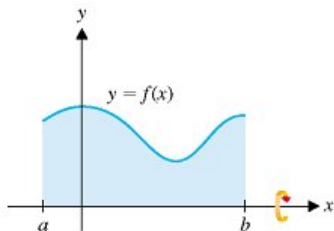
King Saud University

November 17, 2015

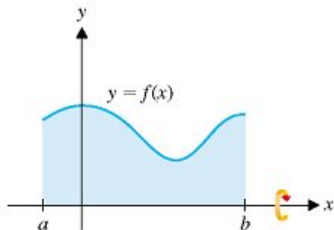
1 Solid of Revolution- Disk Method

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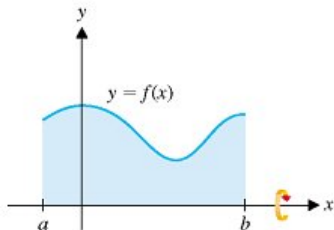


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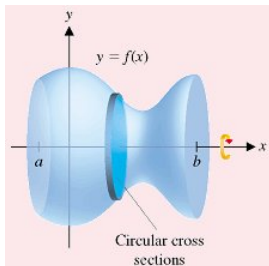


If we revolve the function $f(x)$ about the x -axis, it will produce the following solid:

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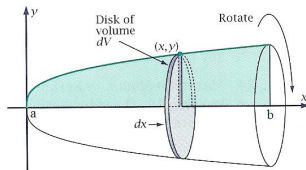


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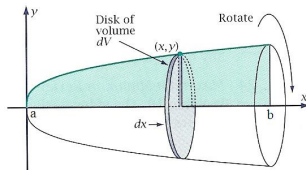


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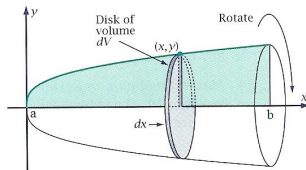


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$$V = \int_a^b \pi [f(x)]^2 dx$$

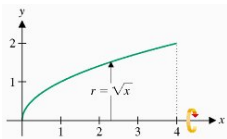
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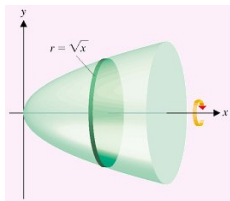
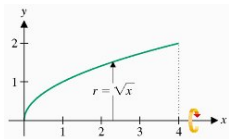
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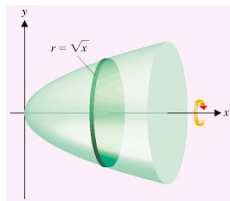
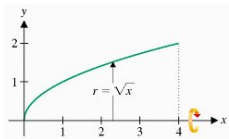
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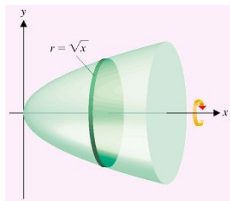
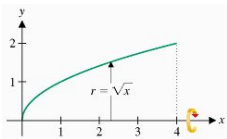
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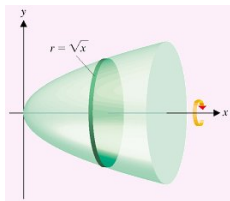
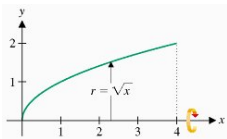


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$$V = \int_0^4 \pi(x^2 + 1)^2 dx$$

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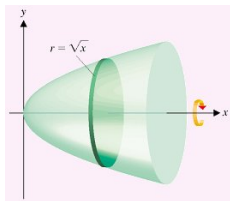
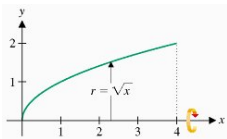


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$$V = \int_0^4 \pi(x^2 + 1)^2 dx = \frac{3772}{15} \pi \approx 790.$$

Example 1 (Swokowski, page 316): The region bounded by the x-axis, the graph of the equation $y = x^2 + 1$ and the lines $x = -1$ and $x = 1$ is revolved about the x-axis. *Find the volume of the resulting solid.*

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Example 2 (Swokowski, page 318): The region bounded by the graphs of the equations $x^2 = y - 2$ and $2y - x - 2 = 0$ and by the vertical lines $x = 0$ and $x = 1$ is revolved about the x-axis. Find the volume of the resulting solid.

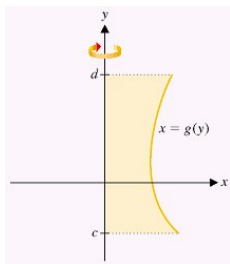
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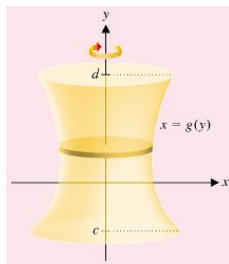
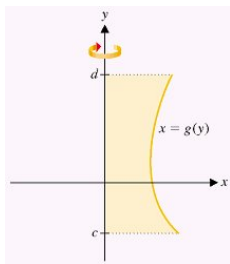
Example 2 (Swokowski, page 318): Find the volume of the solid generated by revolving the region described in (**Example 2**) about the line $y = 3$

Consider the following diagram of $g(y)$

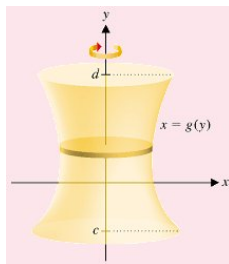
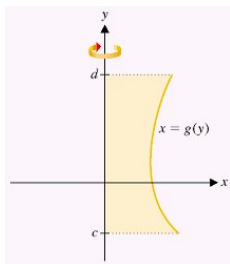
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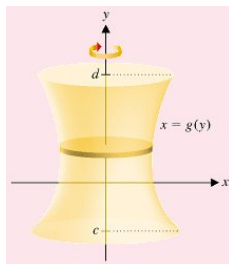
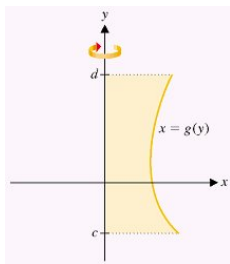


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Example 2 (Swokowski, page 319): The region in the first quadrant bounded by the graphs of $y = \frac{1}{8}x^3$ and $y = 2x$ is revolved about the y -axis. Find the volume of the resulting solid.