# Sinusoidal Alternating Waveforms

## 13.1 INTRODUCTION

***dc networks***: Voltages or Currents are fixed
***ac networks***: Voltages or Currents are varying with time in predefined manner

***ac*** ≡ **alternating current**

*ac voltage* and *ac current* ;
**alternating** ≡ changing between two levels in a set of time sequence

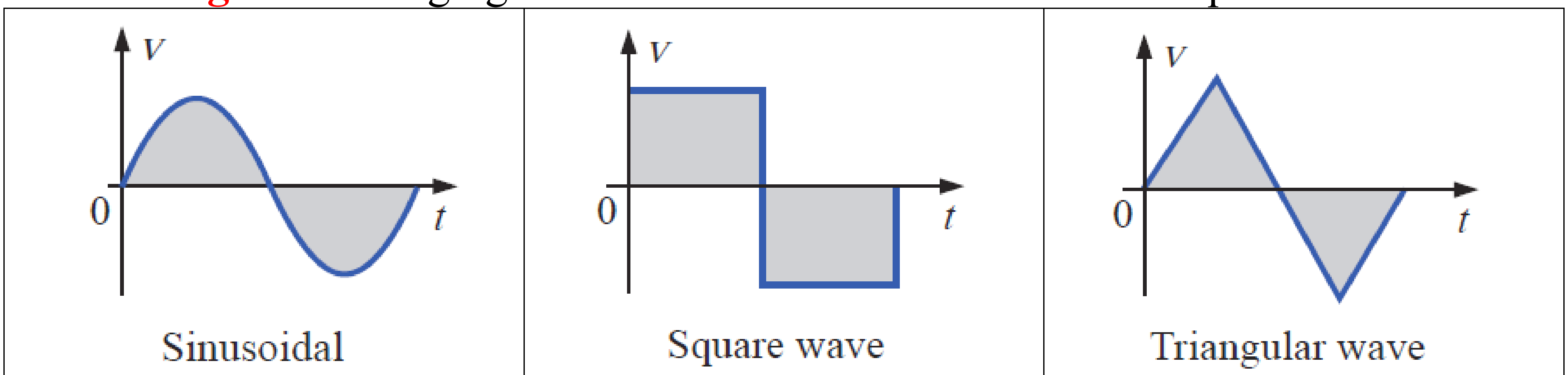



**FIG. 13.1**
*Alternating waveforms.*

# 13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

## Generation

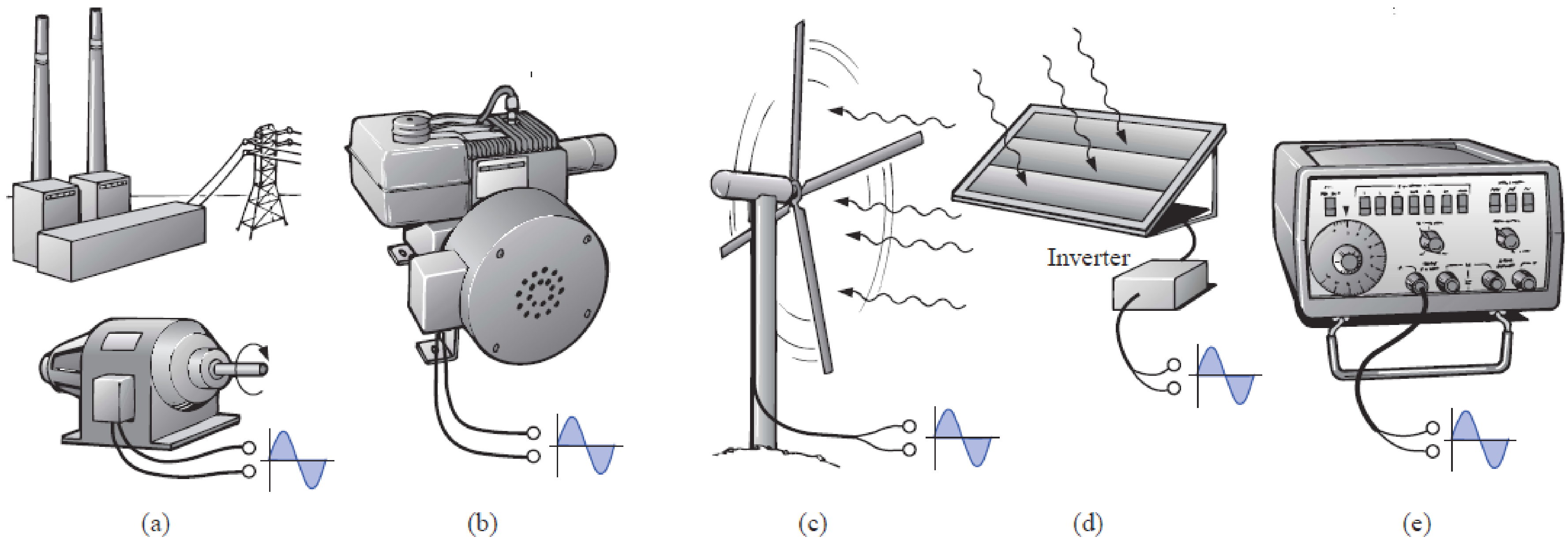



**FIG. 13.2**
*Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.*

# Definitions

**Vertical scale**: in Volts or Amperes

**Horizontal scale**: in units of times

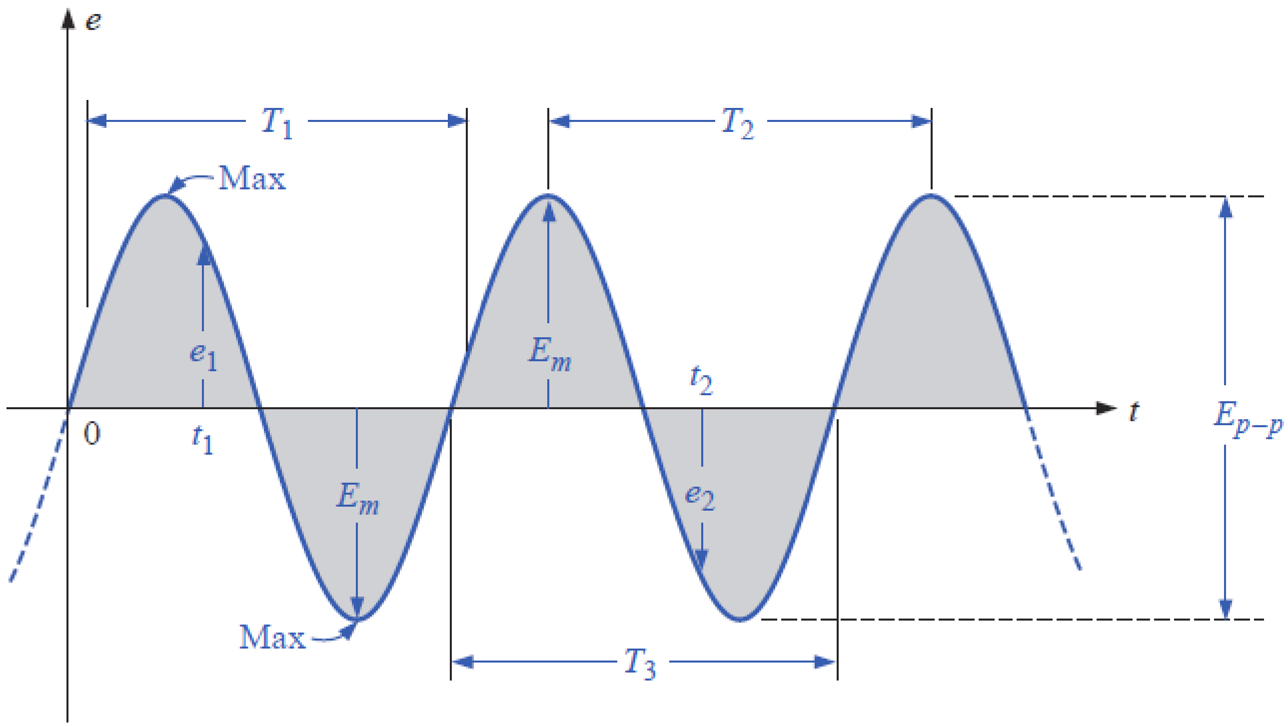



**FIG. 13.3**

*Important parameters for a sinusoidal voltage.*

**Waveform**: The path traced by a quantity plotted as a function of some variable such as time, position, degrees, radians, temperature, and so on.

**Instantaneous value**: The magnitude of a waveform **at any instant of time**; denoted by lowercase letters ($e1$, $e2$).

**Peak amplitude**: The maximum value of a waveform measured from its *average*, or *mean*, value, denoted by uppercase letters ($E_m$ and $V_m$).

**Peak value**: The maximum instantaneous value of a function as measured from the zero-volt level.

**Peak-to-peak value**: Denoted by $\boldsymbol{E_{p\text{-}p}}$ or $\boldsymbol{V_{p\text{-}p}}$, the full voltage between positive and negative peaks of the waveform.

**Periodic waveform**: A waveform that repeats itself after the same time interval.

**Period (*T*)**: The time between successive repetitions of a periodic waveform.

**Cycle**: The portion of a waveform contained in *one period* of time.

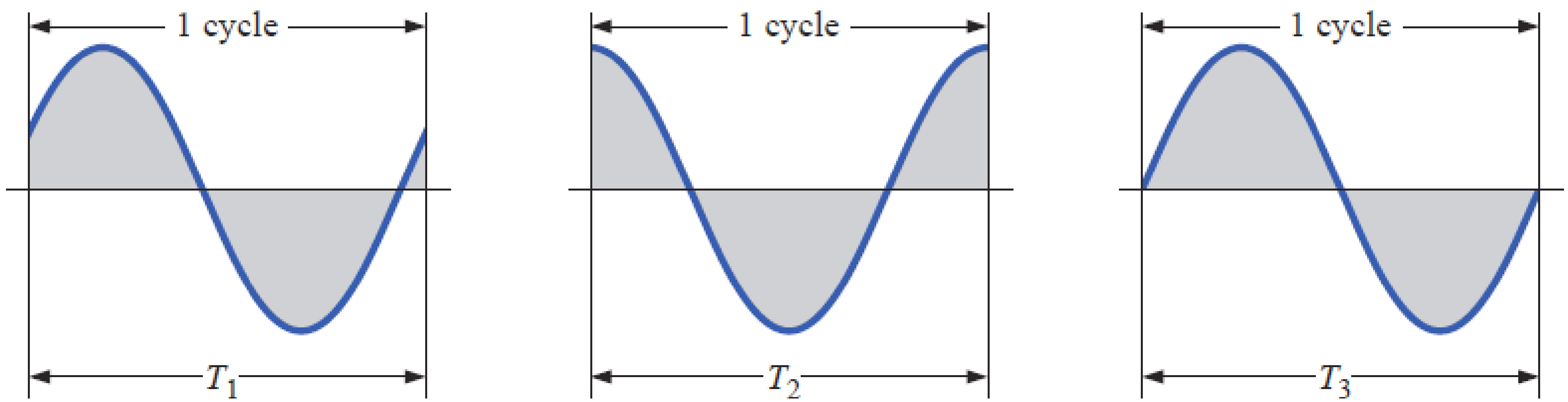



**FIG. 13.4**
*Defining the cycle and period of a sinusoidal waveform.*

**Frequency** ($f$): The number of cycles that occur in 1 s. The frequency of the waveform of Fig. 13.5(a) is 1 cycle per second, and for Fig. 13.5(b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13.5(c)], the frequency would be 2cycles per second.

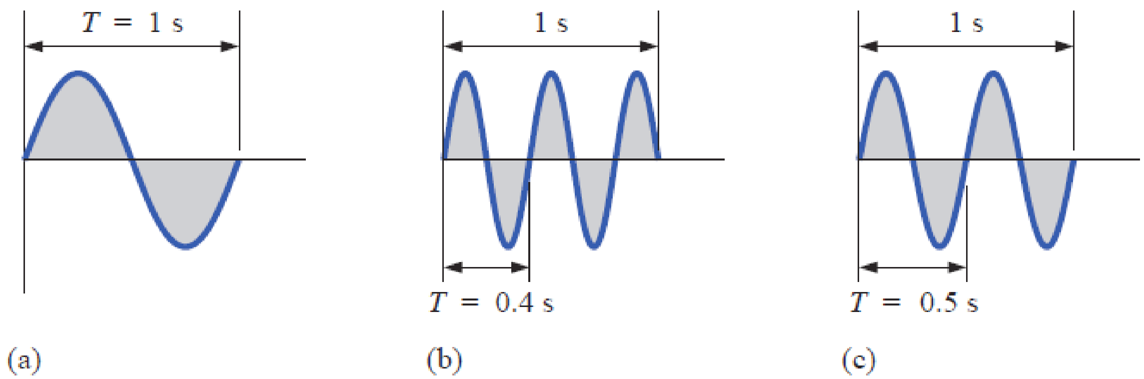



**FIG. 13.5**
*Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.*

The unit of measure for frequency is the *hertz* (Hz), where

$$\boxed{1 \text{ hertz (Hz)} = 1 \text{ cycle per second (c/s)}}$$

$$\boxed{f = \frac{1}{T}}$$

$f$ = Hz
$T$ = seconds (s)

$$\boxed{T = \frac{1}{f}}$$

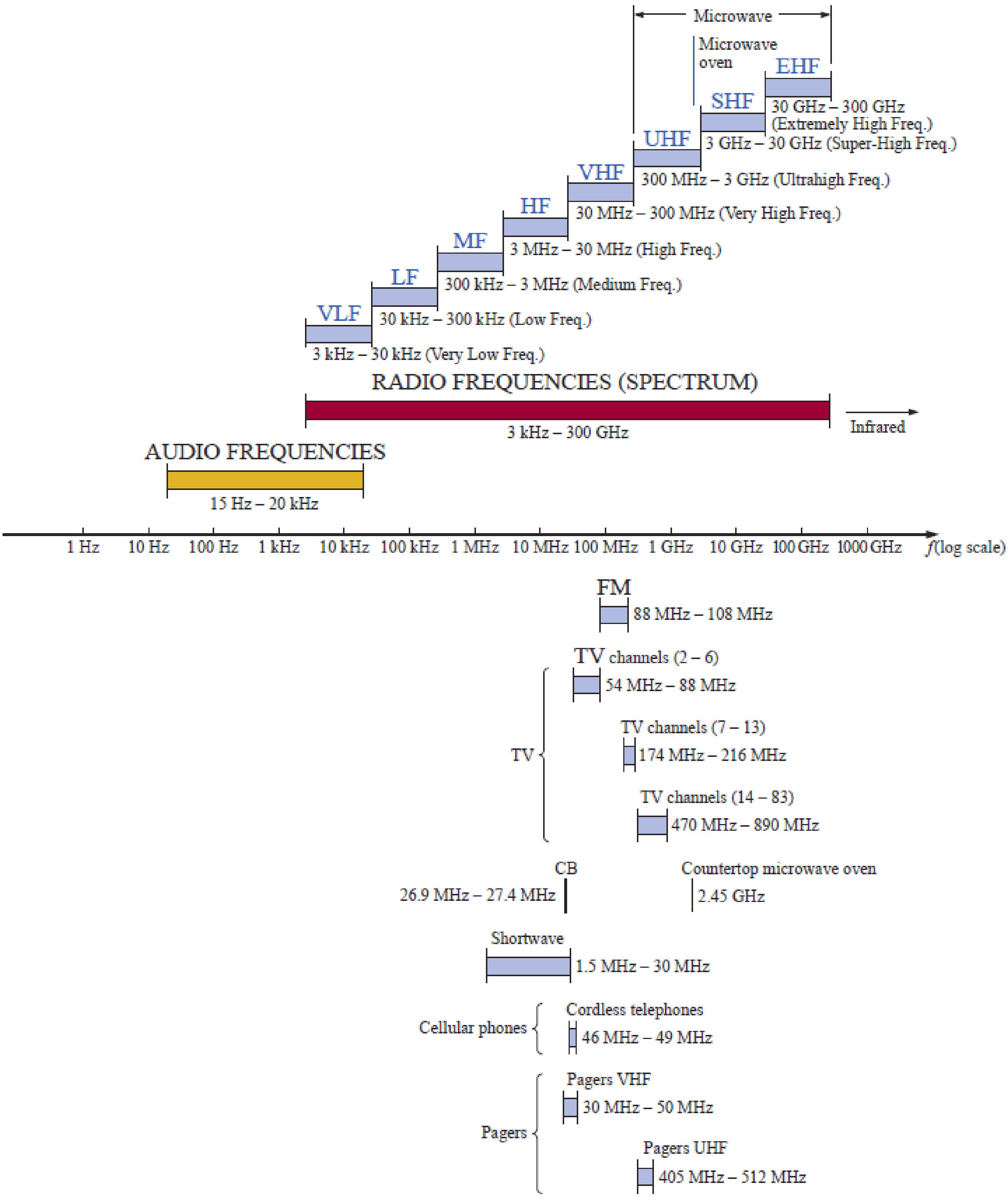



**FIG. 13.7**
*Areas of application for specific frequency bands.*

**EXAMPLE 13.1** Find the period of a periodic waveform with a frequency of

a. 60 Hz.
b. 1000 Hz.

***Solutions:***

a. $T = \dfrac{1}{f} = \dfrac{1}{60\text{ Hz}} \cong 0.01667\text{ s or } \mathbf{16.67\ ms}$

(a recurring value since 60 Hz is so prevalent)

b. $T = \dfrac{1}{f} = \dfrac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\ ms}$

**EXAMPLE 13.2** Determine the frequency of the waveform of Fig. 13.8.

***Solution:*** From the figure, $T = (25\text{ ms} - 5\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\ Hz}$$

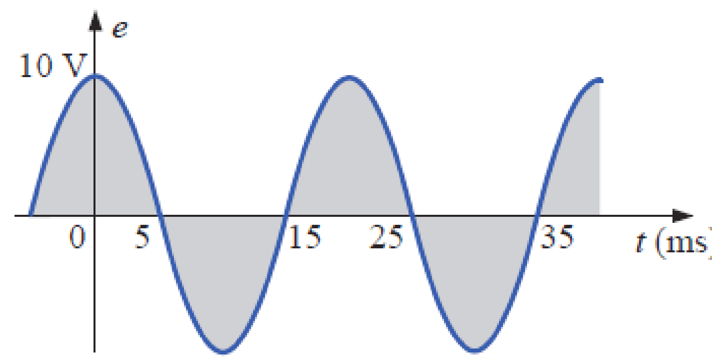
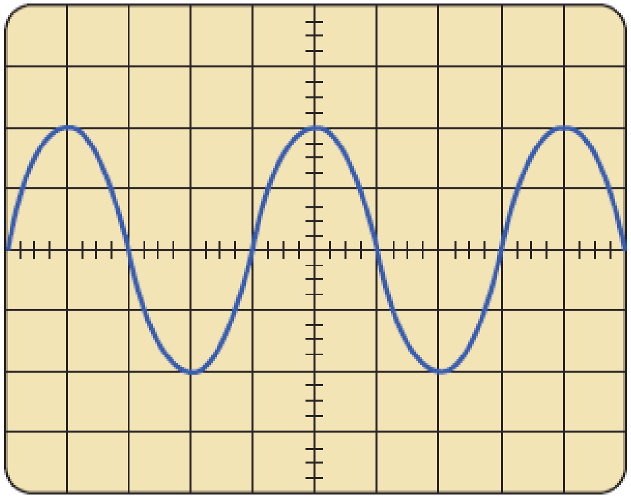



**FIG. 13.8**
*Example 13.2.*

**EXAMPLE 13.3** The **oscilloscope** is an instrument that will display alternating waveforms such as those described above. A sinusoidal pattern appears on the oscilloscope of Fig. 13.9 with the indicated vertical and horizontal sensitivities. The vertical sensitivity defines the voltage associated with each vertical division of the display. Virtually all oscilloscope screens are cut into a crosshatch pattern of lines separated by 1 cm in the vertical and horizontal directions. The horizontal sensitivity defines the time period associated with each horizontal division of the display.

For the pattern of Fig. 13.9 and the indicated sensitivities, determine the period, frequency, and peak value of the waveform.



Vertical sensitivity = 0.1 V/div.
Horizontal sensitivity = 50 $\mu$s/div.

**FIG. 13.9**
*Example 13.3.*

***Solution:*** One cycle spans 4 divisions. The period is therefore

$$T = 4\ \cancel{\text{div.}}\left(\frac{50\ \mu\text{s}}{\cancel{\text{div.}}}\right) = \mathbf{200\ \mu s}$$

and the frequency is

$$f = \frac{1}{T} = \frac{1}{200 \times 10^{-6}\ \text{s}} = \mathbf{5\ kHz}$$

The vertical height above the horizontal axis encompasses 2 divisions. Therefore,

$$V_m = 2\ \cancel{\text{div.}}\left(\frac{0.1\ \text{V}}{\cancel{\text{div.}}}\right) = \mathbf{0.2\ V}$$

## Defined Polarities and Direction

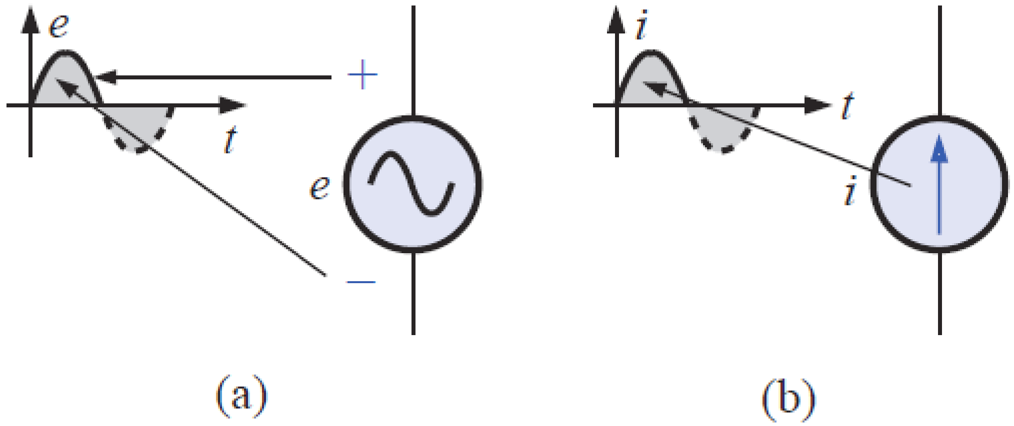
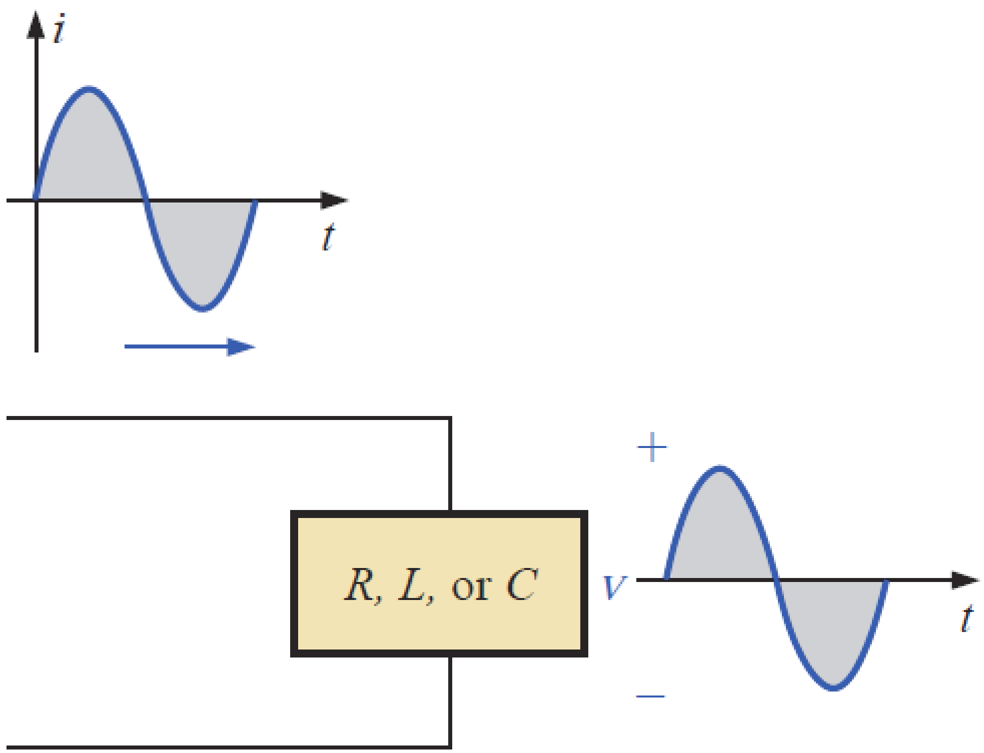



**FIG. 13.10**

*(a) Sinusoidal ac voltage sources;*
*(b) sinusoidal current sources.*

## 13.3 THE SINE WAVE

***The* sinusoidal waveform *is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.***



**Horizontal axis**: degree or **radian** (**rad**)

**Radian**: is defined by a quadrant of a circle where the distance subtended equals the radius

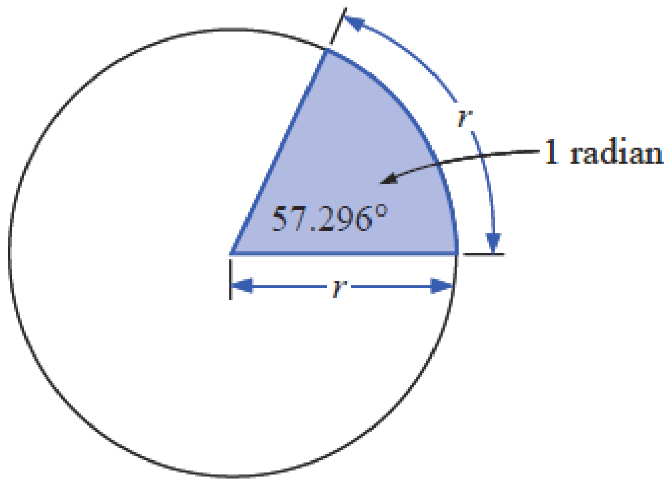
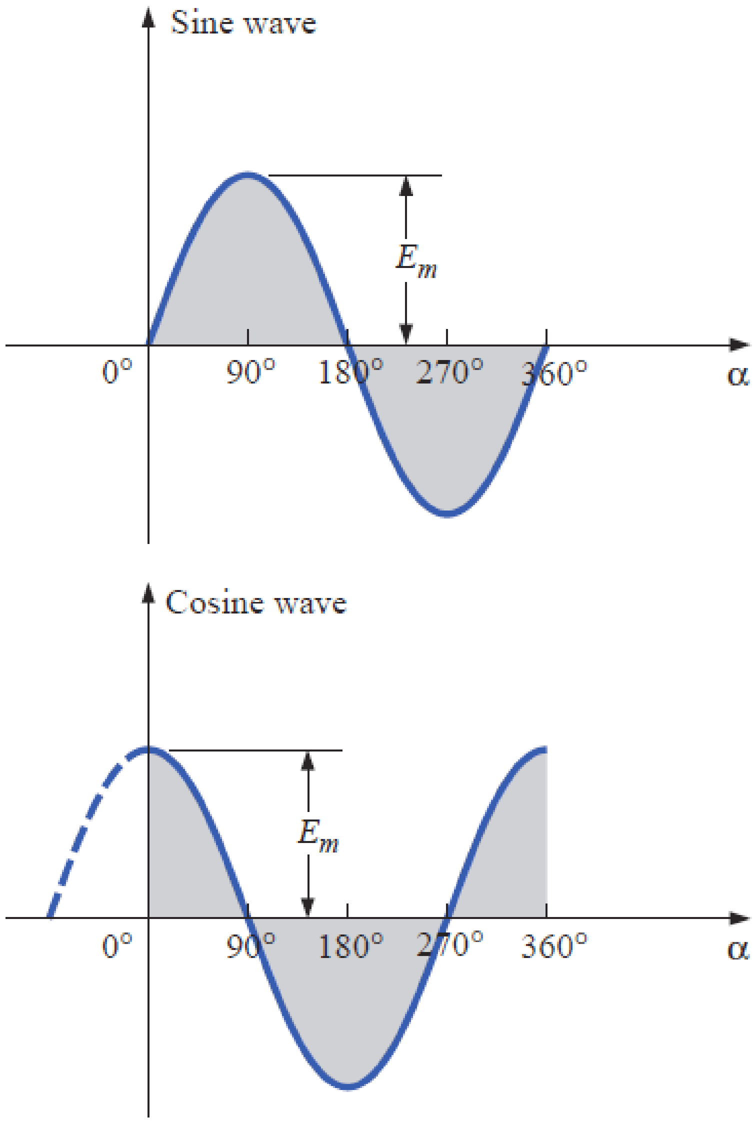



FIG. 13.13
*Defining the radian.*



$x \equiv$ Nº of intervals of $r$ (the radius) around the circumference of a circle:

$C = 2\pi r = x \cdot r \implies x = 2\pi$

There are $2\pi$ rad around a $360^{\circ}$ circle:

$\boxed{2\pi \text{ rad} = 360^{\circ}}$

$\boxed{1 \text{ rad} = 57.296^{\circ} \cong 57.3^{\circ}}$

***The quantity π is the ratio of the circumference of a circle to its diameter.***

π = 3.14159 26535 89793 23846 26433 . .

$\pi \cong 3.14$

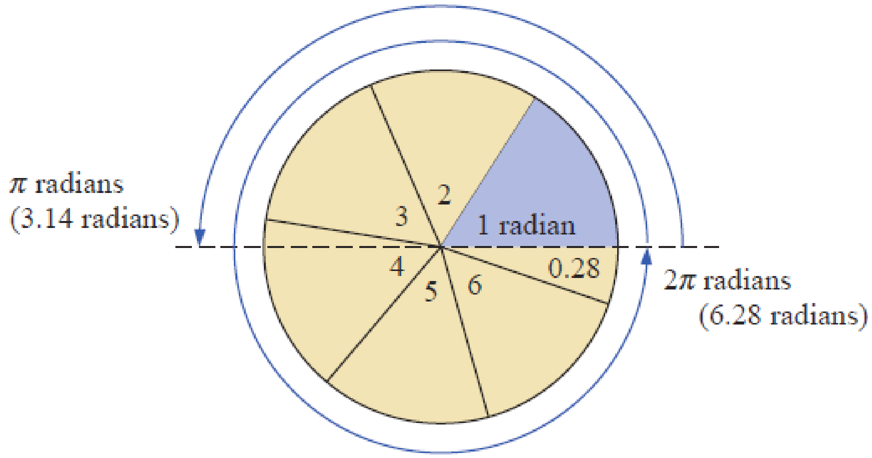



**FIG. 13.14**

*There are 2π radians in one full circle of 360°.*

***it is sometimes preferable to measure angles in radians rather than in degrees.***

$$\boxed{\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees})}$$

$$\boxed{\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians})}$$

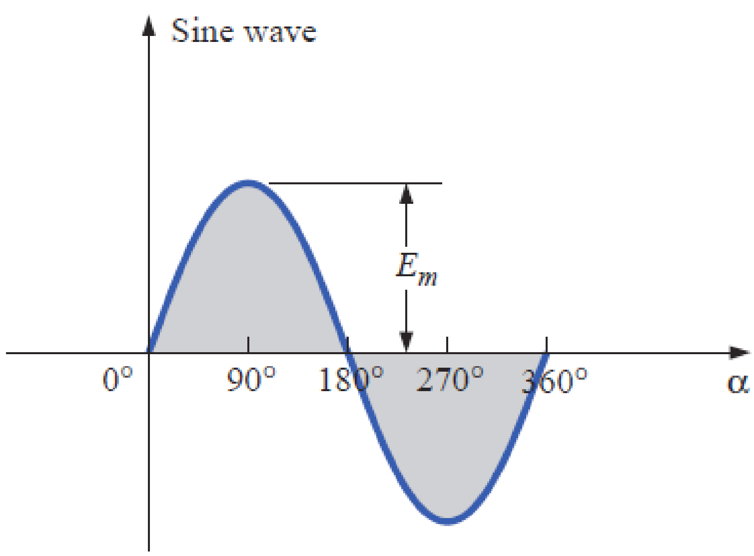
Applying these equations, we find

$$\mathbf{90°:}\quad \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\boldsymbol{\pi}}{\mathbf{2}}\ \textbf{rad}$$

$$\mathbf{30°:}\quad \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\boldsymbol{\pi}}{\mathbf{6}}\ \textbf{rad}$$

$$\frac{\boldsymbol{\pi}}{\mathbf{3}}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$



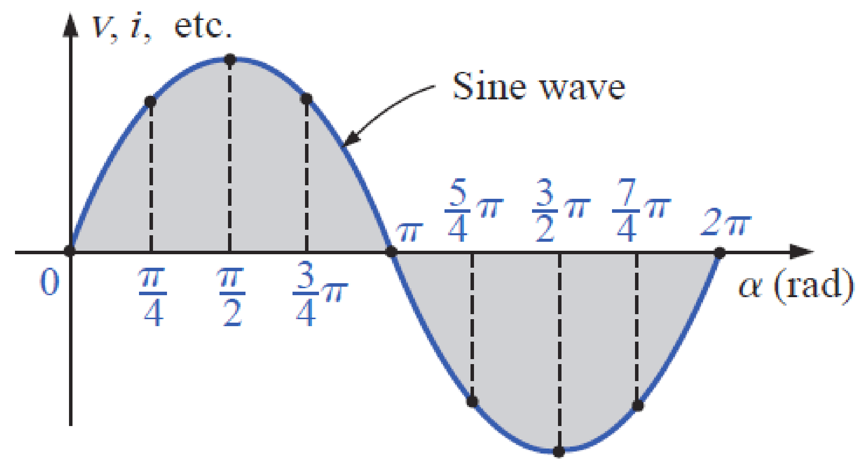




**FIG. 13.15**

*Plotting a sine wave versus radians.*

The sinusoidal waveform can be derived from the length of the vertical projection of radius vector rotating in a uniform circular motion about a fixed point:

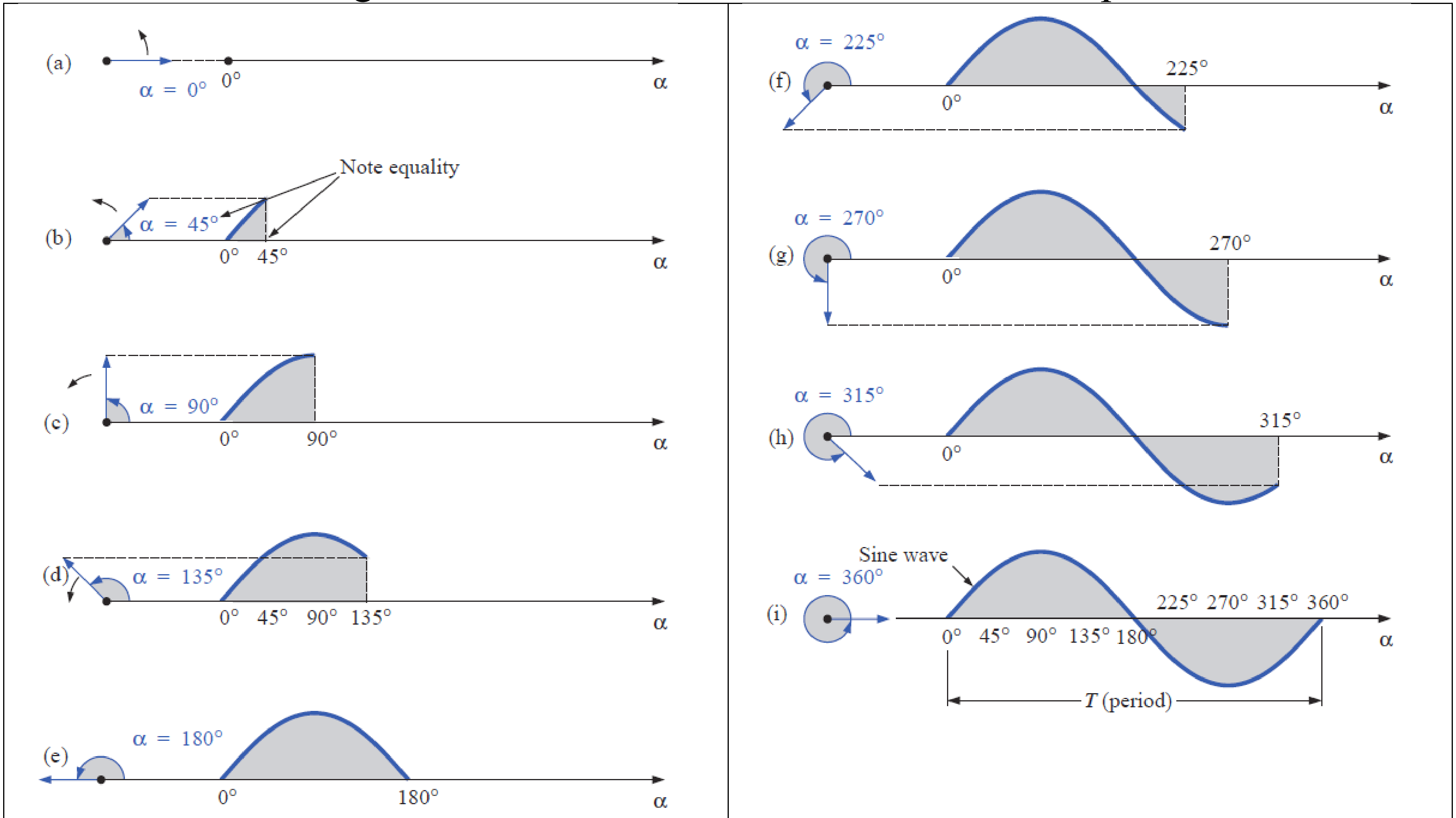



**FIG. 13.16**

*Generating a sinusoidal waveform through the vertical projection of a rotating vector.*

The velocity with which the radius vector rotates about the center, called the **angular velocity,** can be determined from the following equation:

$$\boxed{\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}} \qquad \textbf{(13.8)}$$

| | |
|---|---|
| $\boldsymbol{\omega}$ typically given in **radians per second**<br>$\boldsymbol{\alpha}$ the angle typically in **radians** | $\boxed{\omega = \frac{\alpha}{t}}$<br>$\boxed{\alpha = \omega t}$ |
| • the time required to complete one revolution is equal to the period ($T$) of the sinusoidal waveform<br>• The radians subtended in this time interval are $2\pi$ | $\boxed{\omega = \frac{2\pi}{T}}$ (rad/s)<br>$\boxed{\omega = 2\pi f}$ (rad/s) |

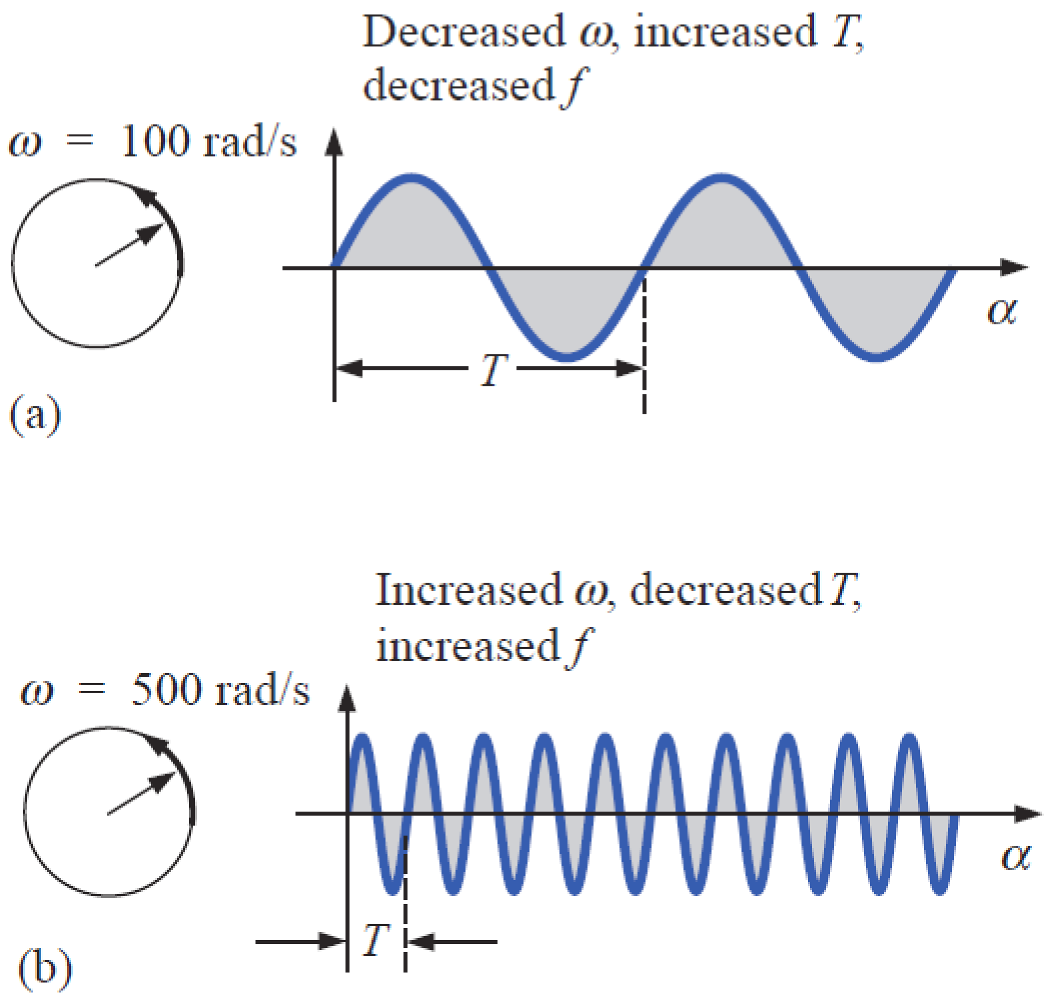



**FIG. 13.17**

*Demonstrating the effect of ω on the frequency and period.*

**EXAMPLE 13.4** Determine the angular velocity of a sine wave having a frequency of 60 Hz.

***Solution:***

$$\omega = 2\pi f = (2\pi)(60\text{ Hz}) \cong \mathbf{377\ rad/s}$$

(a recurring value due to 60-Hz predominance)

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**EXAMPLE 13.5** Determine the frequency and period of the sine wave of Fig. 13.17(b).

***Solution:*** Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \mathbf{12.57\ ms}$$

and $$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\ Hz}$$

**EXAMPLE 13.6** Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

***Solution:*** Eq. (13.10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, $\alpha$ must be substituted as $\pi/2$ (= 90°) since $\omega$ is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2\ \text{rad}}{200\ \text{rad/s}} = \frac{\pi}{400}\ \text{s} = \mathbf{7.85\ ms}$$

**EXAMPLE 13.7** Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

***Solution:*** Eq. (13.11): $\alpha = \omega t$, or

$$\alpha = 2\pi f t = (2\pi)(60\ \text{Hz})(5 \times 10^{-3}\ \text{s}) = \mathbf{1.885\ rad}$$

If not careful, one might be tempted to interpret the answer as 1.885°. However,

$$\alpha\ (°) = \frac{180°}{\pi\ \text{rad}}(1.885\ \text{rad}) = \mathbf{108°}$$

## 13.4 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is:

$$A_m \sin \alpha$$

$A_m$ ≡ peak amplitude (value) of the waveform

$\alpha$ ≡ unit of measure for the horizontal axis

$\alpha = \omega t$ ≡ the angle of the rotating vector

$$A_m \sin \omega t$$

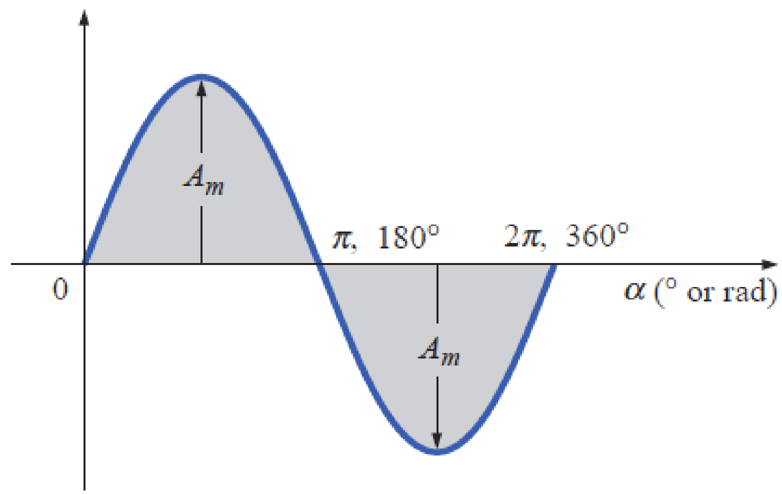



**FIG. 13.18**
*Basic sinusoidal function.*

$I_m$ and $E_m$ ≡ amplitude

$i$ and $e$ ≡ instantaneous value

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

The angle at which a particular voltage level is attained can be determined by rearranging the equation:

$$e = E_m \sin\alpha \quad \Longrightarrow \quad \sin\alpha = \frac{e}{E_m} \quad \Longrightarrow \quad \boldsymbol{\alpha = \sin^{-1}\left(\frac{e}{E_m}\right)}$$

For a voltage level: $\boldsymbol{\alpha = \sin^{-1}\left(\frac{e}{E_m}\right)}$

For a current level: $\boldsymbol{\alpha = \sin^{-1}\left(\frac{i}{I_m}\right)}$

The function ***sin***$^{-1}$ is available in calculators (the inverse of the ***sin*** function)

## EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
b. Determine the time at which the magnitude is attained.

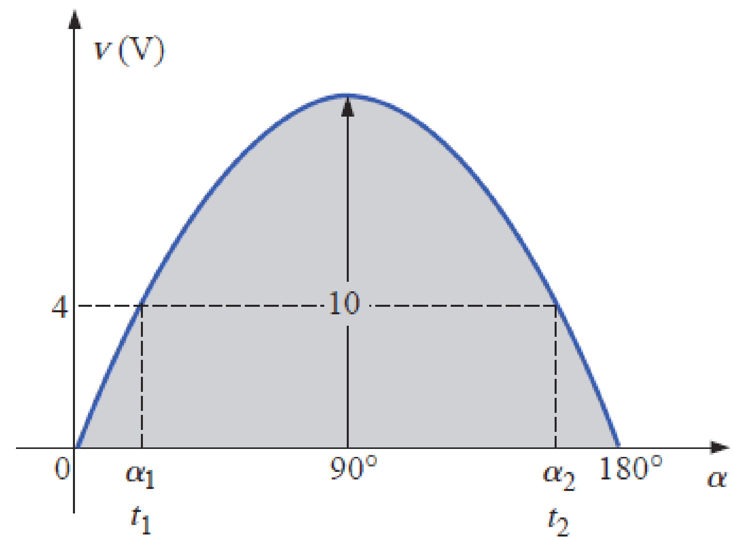
***Solutions:***

a. Eq. (13.15):

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.578^\circ}$$

However, Figure 13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.422^\circ}$$




In general, therefore, keep in mind that Equations (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90°.

b. Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, $\alpha$ must be in radians. Thus,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(23.578^\circ) = 0.411\text{ rad}$$

and
$$t_1 = \frac{\alpha}{\omega} = \frac{0.411\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\text{ ms}}$$

For the second intersection,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\text{ ms}}$$

**EXAMPLE 13.10** Sketch $e = 10 \sin 314t$ with the abscissa

a. angle ($\alpha$) in degrees.
b. angle ($\alpha$) in radians.
c. time ($t$) in seconds.

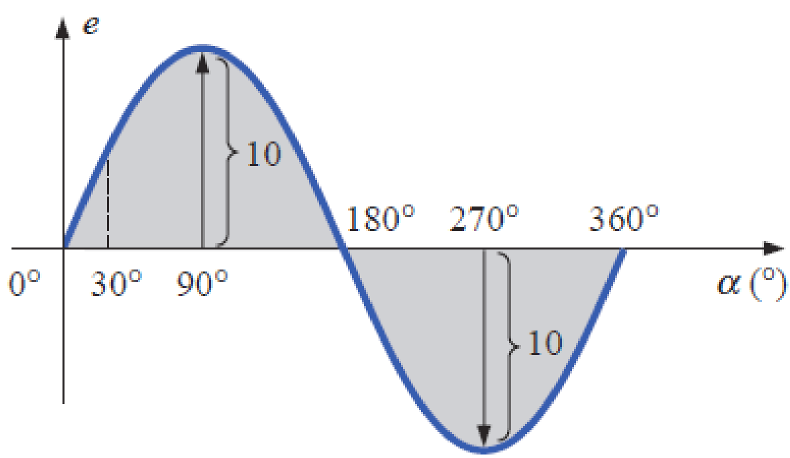



**FIG. 13.20**
*Example 13.10, horizontal axis in degrees.*

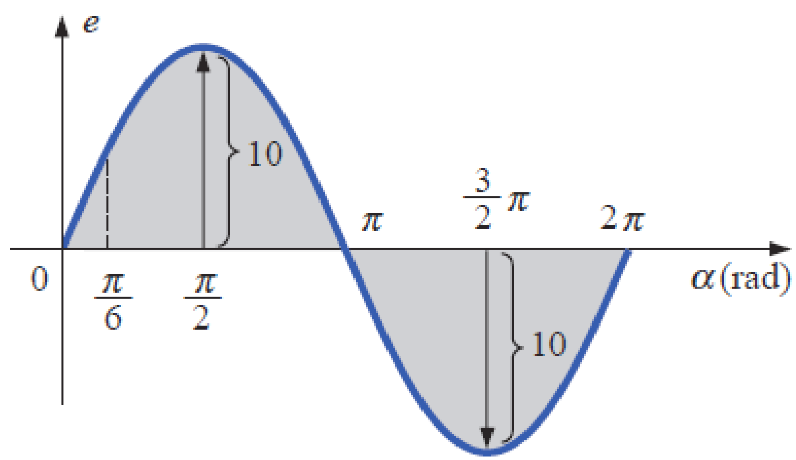



**FIG. 13.21**
*Example 13.10, horizontal axis in radians.*

Sketch $e = 10 \sin 314t$ ·

c. 360°: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{314} = 20 \text{ ms}$

180°: $\dfrac{T}{2} = \dfrac{20 \text{ ms}}{2} = 10 \text{ ms}$

90°: $\dfrac{T}{4} = \dfrac{20 \text{ ms}}{4} = 5 \text{ ms}$

30°: $\dfrac{T}{12} = \dfrac{20 \text{ ms}}{12} = 1.67 \text{ ms}$

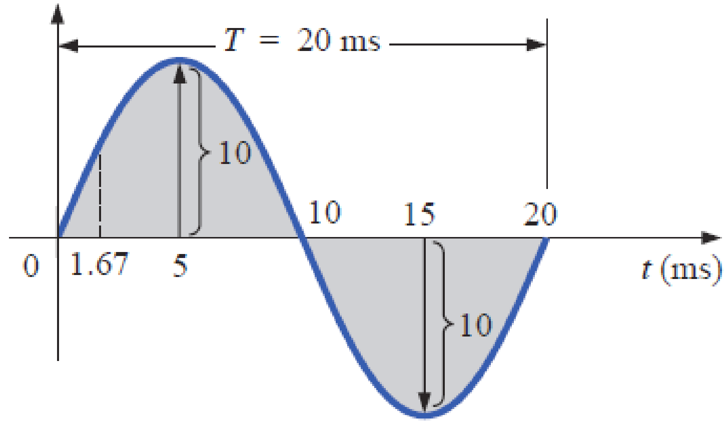



**FIG. 13.22**
*Example 13.10, horizontal axis in milliseconds.*

**EXAMPLE 13.11** Given $i = 6 \times 10^{-3} \sin 1000t$, determine $i$ at $t = 2$ ms

***Solution:***

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

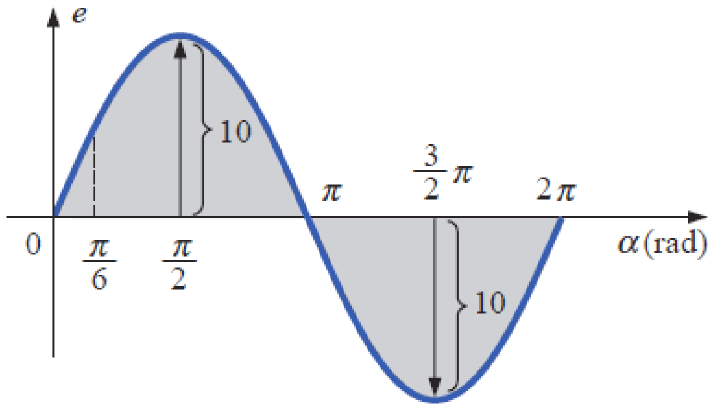
$$\alpha\,(°) = \frac{180°}{\pi \text{ rad}}(2 \text{ rad}) = 114.59°$$

$$\begin{aligned} i &= (6 \times 10^{-3})(\sin 114.59°) \\ &= (6 \text{ mA})(0.9093) = \mathbf{5.46 \text{ mA}} \end{aligned}$$

# 13.5 PHASE RELATIONS

The sine wave we considered $A_m \sin \omega t$ has:

- maxima at: $\frac{\pi}{2}$ and $\frac{3\pi}{2}$
- zero value at: $0, \ \pi,$ and $2\pi$




If the waveform is shifted to the right or left of 0°, the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

$\theta$ is the angle in degree or radians that the waveform has been shifted

If the waveform passes the horizontal axis with a positive going slope before $0^{o}$ as shown the expression is:

$$A_m \sin(\omega t + \theta)$$

At $\omega t = \alpha = 0°$ the magnitude is

$$A_m \sin(\theta)$$

This wave and the sine wave are **out of phase by θ**.

This wave **leads** the sine wave by **θ**.

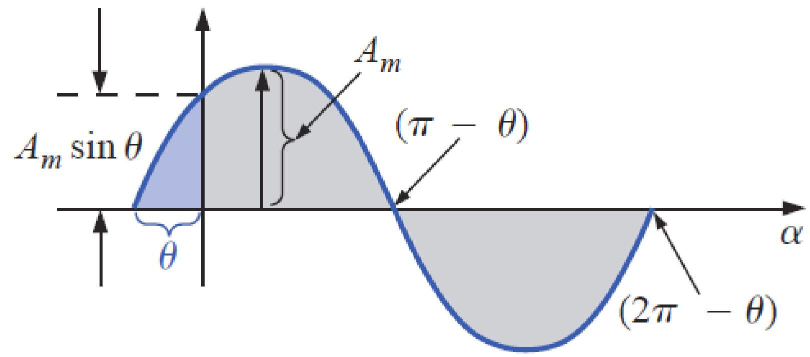



**FIG. 13.23**

*Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.*

If the waveform passes the horizontal axis with a positive going slope after $0^{o}$ as shown the expression is:

$$A_m \sin(\omega t - \theta)$$

At $\omega t = \alpha = 0°$ the magnitude is

$$A_m \sin(-\theta) = -A_m \sin(\theta)$$

This wave and the sine wave are **out of phase by θ**.

This wave **lags** the sine wave by **θ**.

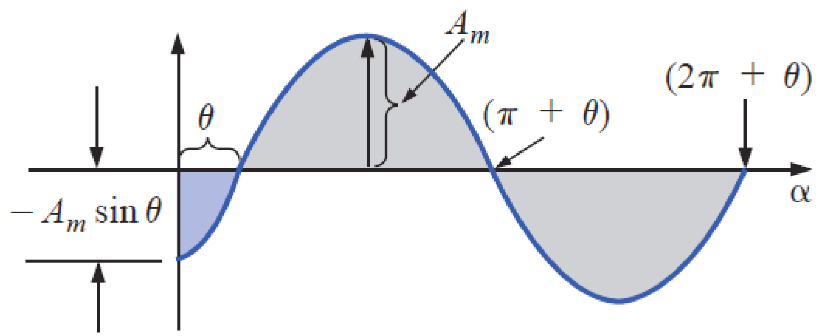



**FIG. 13.24**

*Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.*

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2) sooner, it is called a ***cosine wave***;

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

or

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

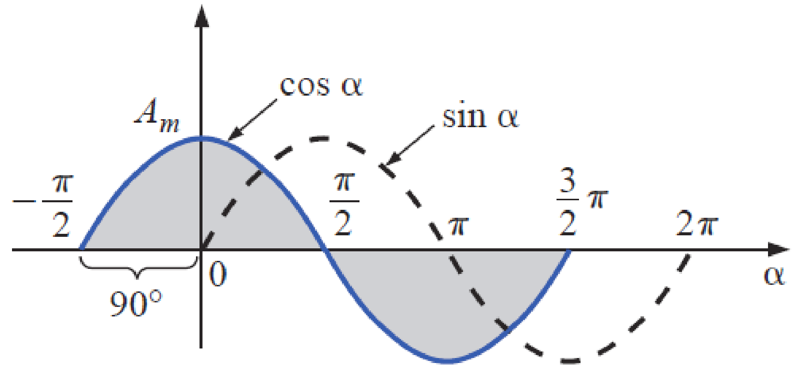



**FIG. 13.25**
*Phase relationship between a sine wave and a cosine wave.*

The terms **lead** and **lag** are used to indicate the relationship between two sinusoidal waveforms of the **same frequency** plotted on the same set of axes:

- The *cosine* curve ***leads*** the *sine* curve by 90°.
- The *sine* curve ***lags*** the *cosine* curve by 90°.
- 90° is the ***phase angle*** between the two waveform,
- The waveforms are ***out of phase*** by 90°.
- The phase angle is measured between those two points on **the horizontal axis** through which each passes with the ***same slope***.

$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

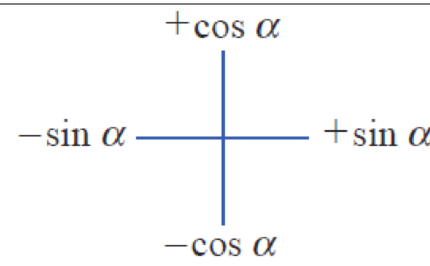
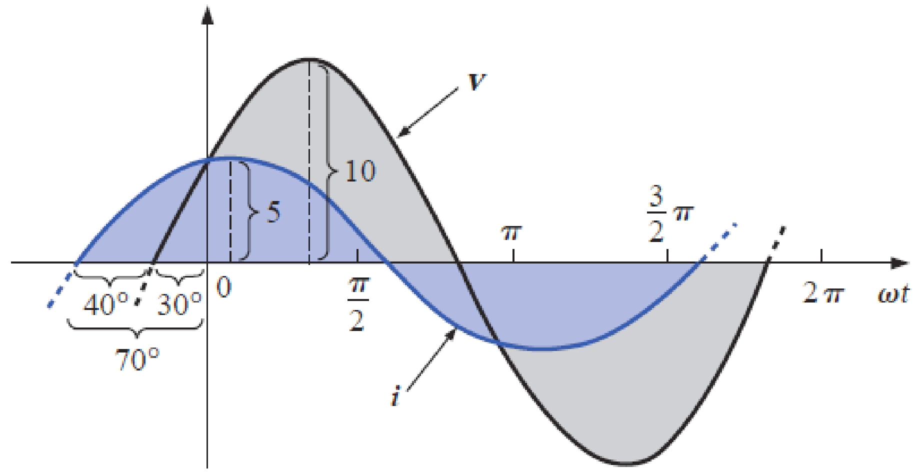



**FIG. 13.26**
*Graphic tool for finding the relationship between specific sine and cosine functions.*

$$e = -E_m \sin\omega t \Longrightarrow e = E_m(-\sin\omega t) \quad \Longrightarrow \boldsymbol{e = E_m \sin(\omega t \pm 180^\circ)}$$

The **phase relationship** between two waveform indicates which **leads** or **lags** and by how many degrees or radians.

**EXAMPLE 13.12** What is the phase relationship between the sinusoidal waveforms of each of the following sets?

| | |
|---|---|
| a. $v = 10 \sin(\omega t + 30°)$<br>$i = 5 \sin(\omega t + 70°)$<br><br>a. See Fig. 13.27.<br>***i* leads *v* by 40°, or *v* lags *i* by 40°.** | <br><br>**FIG. 13.27**<br>*Example 13.12; i leads v by 40°.* |

b. $i = 15 \sin(\omega t + 60°)$
$v = 10 \sin(\omega t - 20°)$

b. See Fig. 13.28.
***i* leads *v* by 80°, or *v* lags *i* by 80°.**

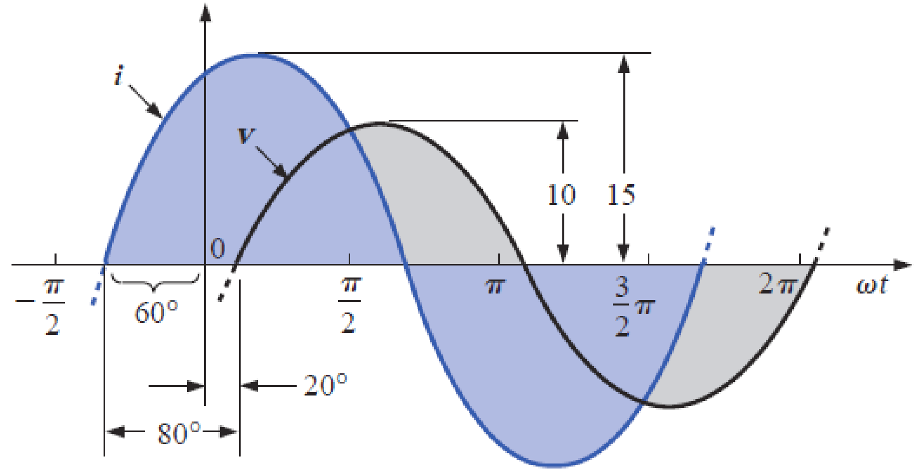



**FIG. 13.28**
*Example 13.12; i leads v by 80°.*

c. $i = 2 \cos(\omega t + 10°)$
$v = 3 \sin(\omega t - 10°)$

c. See Fig. 13.29.

$$i = 2 \cos(\omega t + 10°) = 2 \sin(\omega t + 10° + 90°)$$
$$= 2 \sin(\omega t + 100°)$$

***i* leads *v* by 110°, or *v* lags *i* by 110°.**

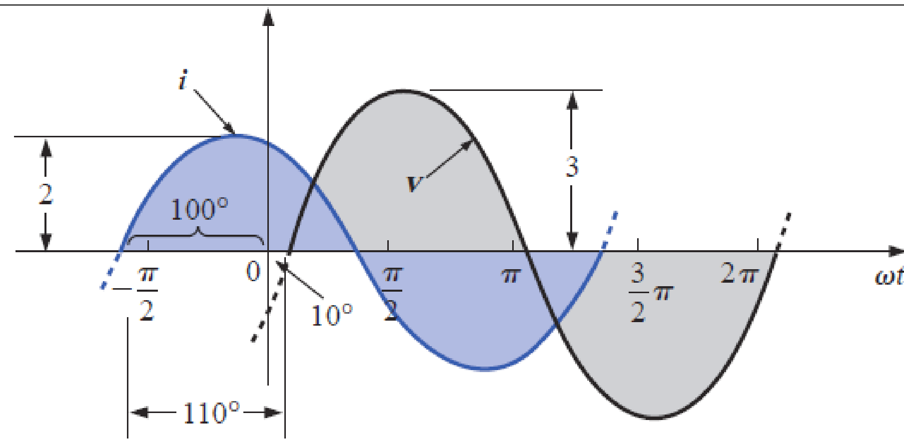



**FIG. 13.29**
*Example 13.12; i leads v by 110°.*

d. $i = -\sin(\omega t + 30°)$
$v = 2\sin(\omega t + 10°)$

d. See Fig. 13.30.

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - \overset{\text{Note}}{180°})$$
$$= \sin(\omega t - 150°)$$

**$v$ leads $i$ by 160°, or $i$ lags $v$ by 160°.**

**$i$ leads $v$ by 200°, or $v$ lags $i$ by 200°.**

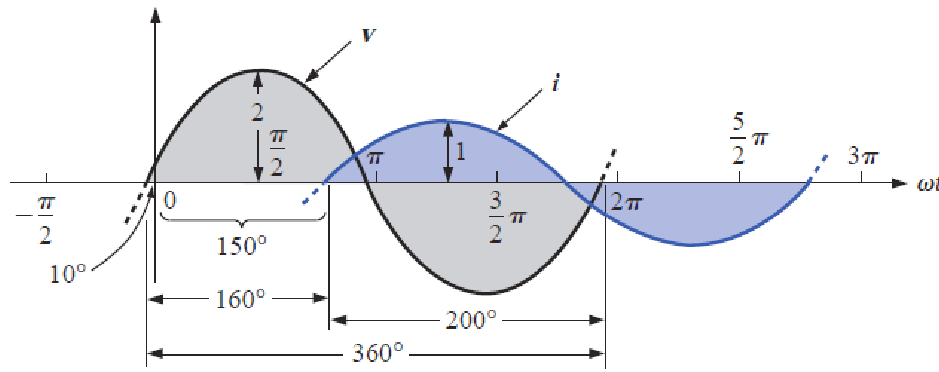



**FIG. 13.30**
*Example 13.12; v leads i by 160°.*

e. $i = -2\cos(\omega t - 60°)$
$v = 3\sin(\omega t - 150°)$

e. See Fig. 13.31.

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° - \overset{\text{By choice}}{180°})$$
$$= 2\cos(\omega t - 240°)$$

However, $\cos\alpha = \sin(\alpha + 90°)$

so that $2\cos(\omega t - 240°) = 2\sin(\omega t - 240° + 90°)$
$= 2\sin(\omega t - 150°)$

**$v$ and $i$ are in phase.**

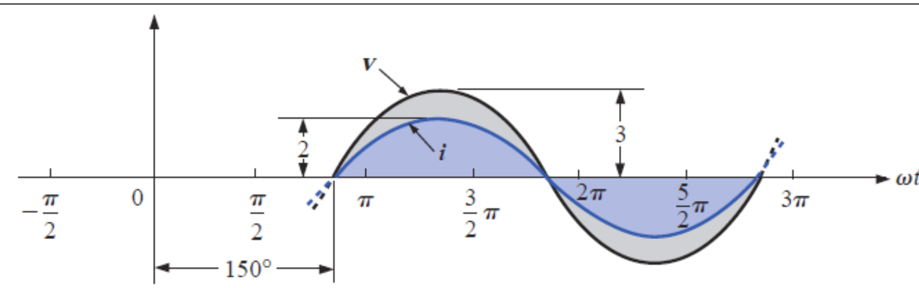



**FIG. 13.31**
*Example 13.12; v and i are in phase.*

$$\frac{360°}{T \text{ (no. of div.)}} = \frac{\theta}{\text{phase shift (no. of div.)}}$$

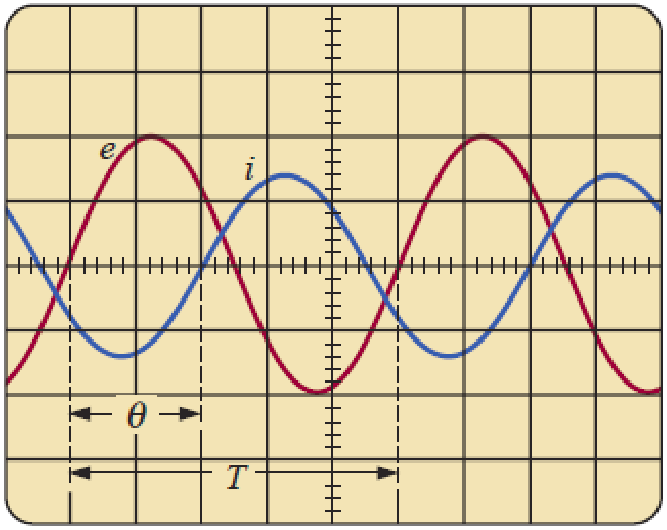
and

$$\boxed{\theta = \frac{\text{phase shift (no. of div.)}}{T \text{ (no. of div.)}} \times 360°}$$

Substituting into Eq. (13.24) will result in

$$\theta = \frac{(2 \text{ div.})}{(5 \text{ div.})} \times 360° = 144°$$

and $e$ leads $i$ by 144°.




Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

**FIG. 13.32**
*Finding the phase angle between waveforms using a dual-trace oscilloscope.*

# 13.6 AVERAGE VALUE

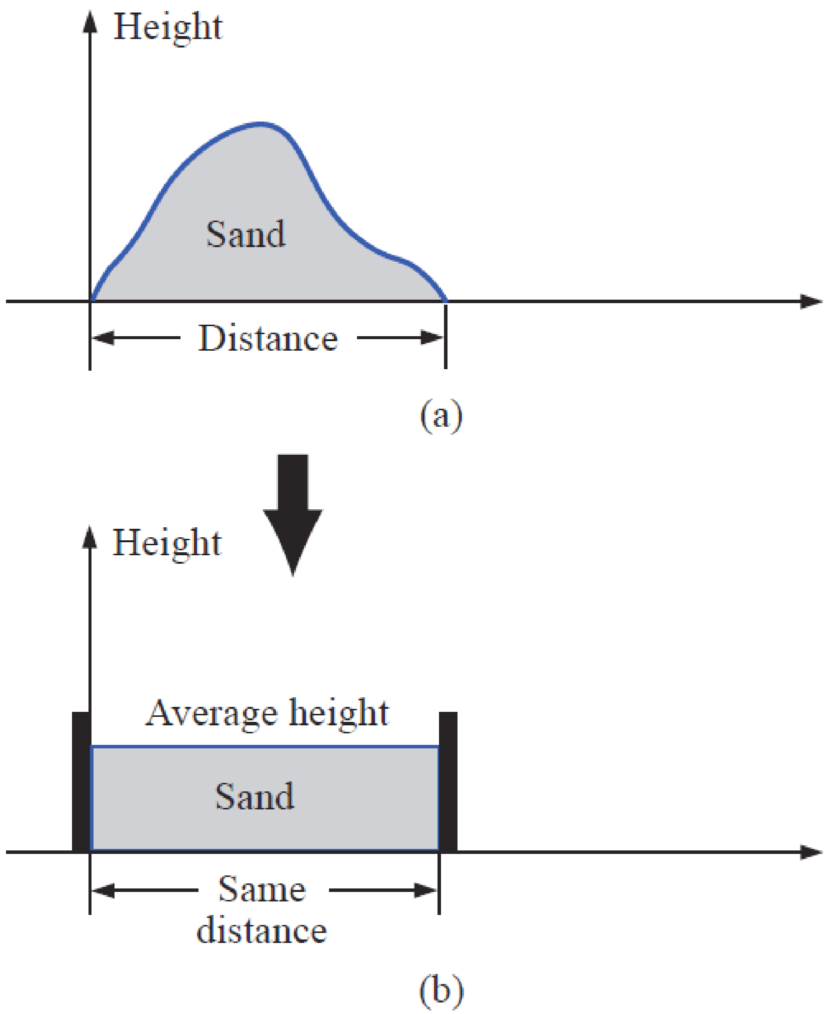



**FIG. 13.33**
*Defining average value.*

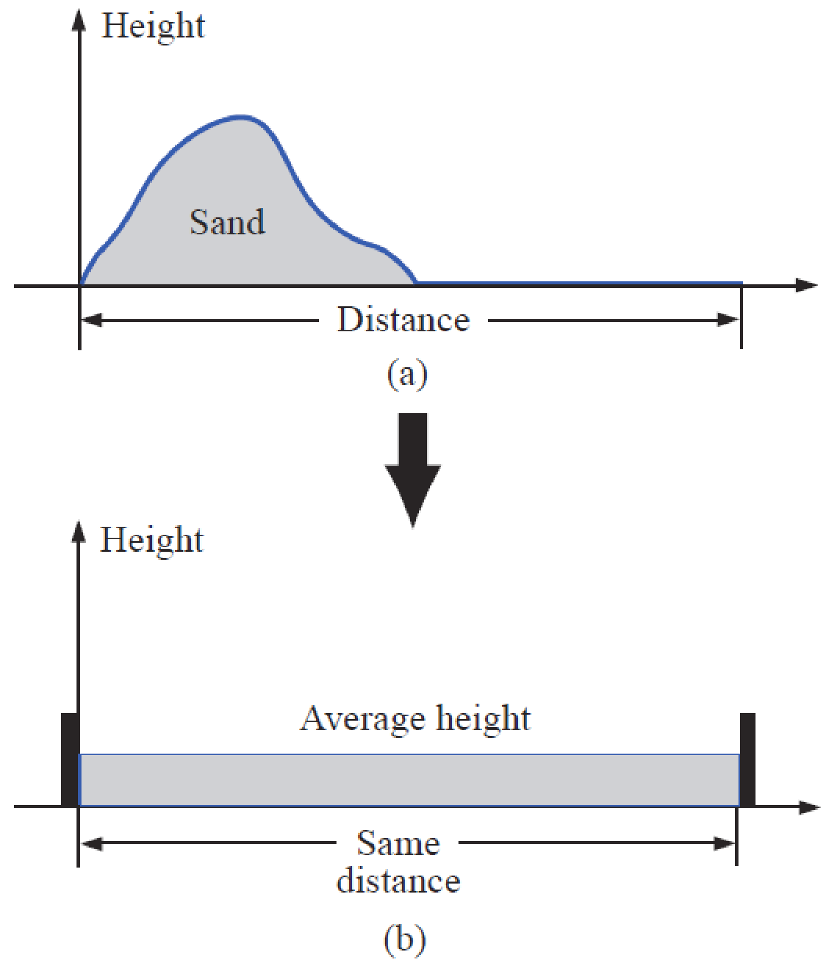



**FIG. 13.34**
*Effect of distance (length) on average value.*

$$Average\ height = \frac{total\ amount}{total\ distance}$$

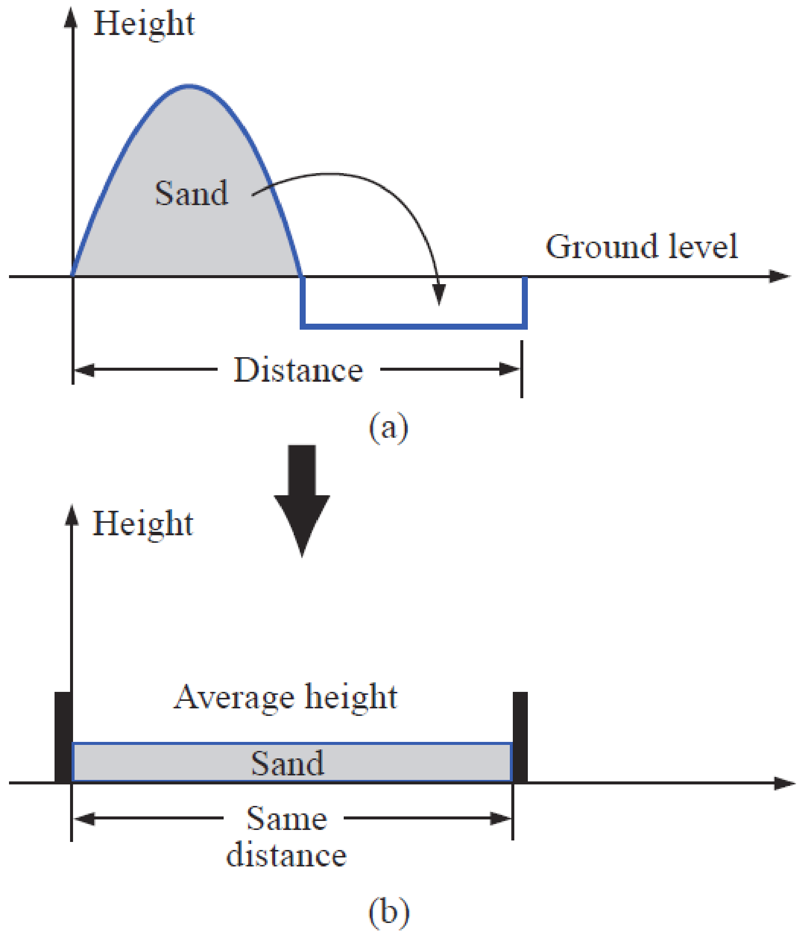



**FIG. 13.35**
*Effect of depressions (negative excursions) on average value.*

$A_1$ = 60 mi/h for 2 hours

A break of 1/2 hour

$A_2$ = 50 mi/h for 2.5 hours

$$\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}}$$

$$\begin{aligned}\text{Average speed} &= \frac{A_1 + A_2}{5\text{ h}} \\ &= \frac{(60\text{ mi/h})(2\text{ h}) + (50\text{ mi/h})(2.5\text{ h})}{5\text{ h}} \\ &= \frac{225}{5}\text{ mi/h} \\ &= \mathbf{45\ mi/h}\end{aligned}$$

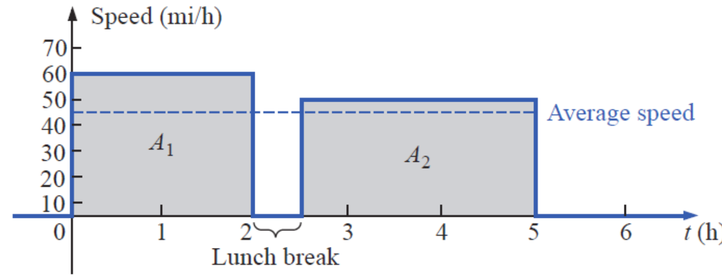



**FIG. 13.36**
*Plotting speed versus time for an automobile excursion.*

For any other quantity such as voltage or current:

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

- Algebraic sum of areas:
    - Area above horizontal axis is positive
    - Area below horizontal axis is negative

- Positive average value = above axis
- Negative average value = below axis

The average value of any current or voltage is the value indicated on a dc meter

**EXAMPLE 13.13** Determine the average value of the waveforms of Fig. 13.37.

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

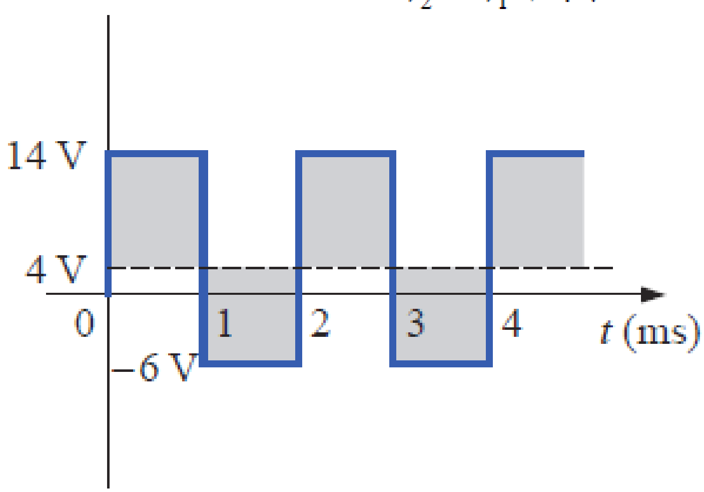
$$= \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

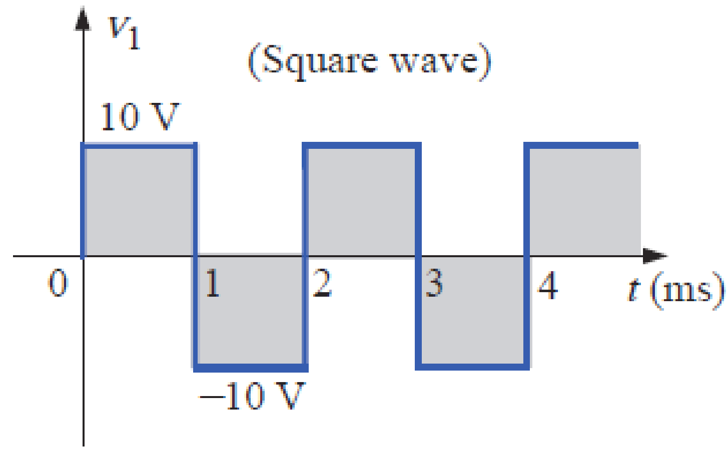
$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

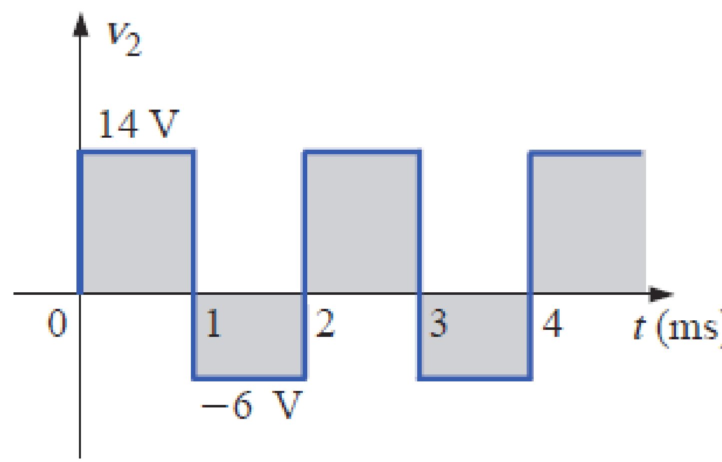
In reality, the waveform of Fig. 13.37(b) is simply the square wave of Fig. 13.37(a) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4\text{ V}$$




(a)

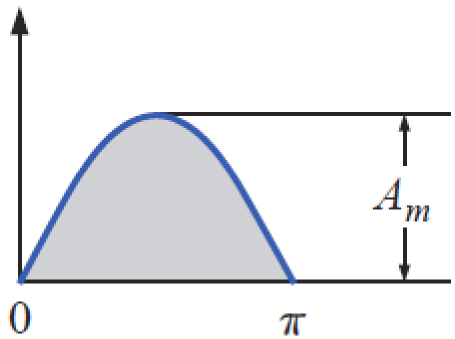



(b)

For a sine wave:

Area = $\int_0^{\pi} A_m \sin\alpha \, d\alpha = A_m[-\cos\alpha]_0^{\pi} = 2A_m$

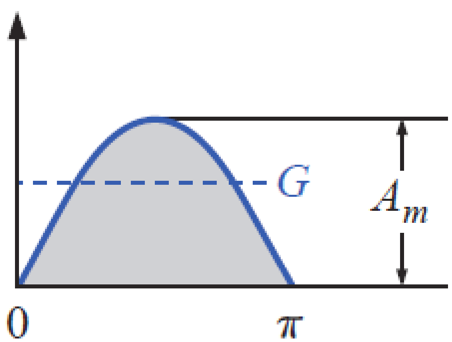
$$\boxed{\text{Area} = 2A_m}$$




$$\boxed{G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}}$$

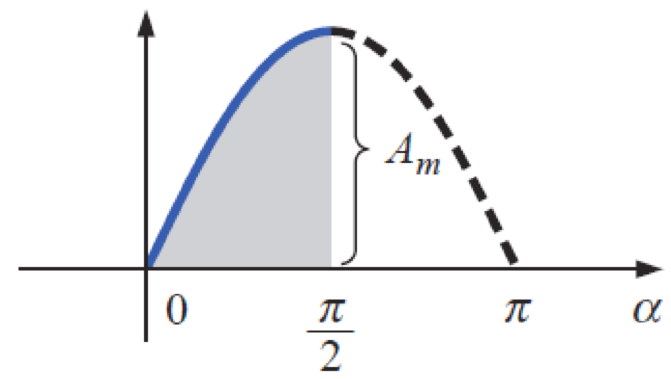
$$G = \frac{2A_m}{\pi}$$

$$\boxed{G = 0.637A_m}$$



For the waveform of Fig. 13.45,

$$G = \frac{(2A_m/2)}{\pi/2} = \frac{2A_m}{\pi} \qquad \text{(average the same as for a full pulse)}$$



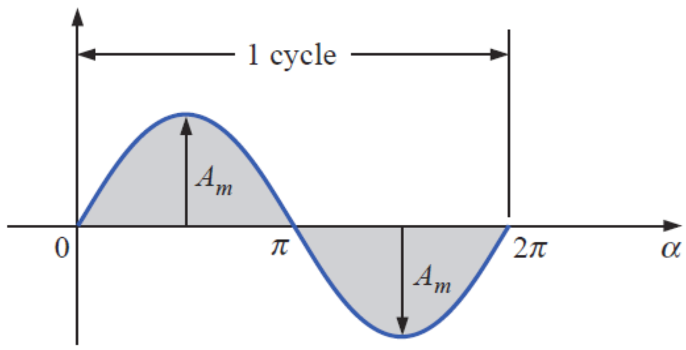

**EXAMPLE 13.15** Determine the average value of the sinusoidal waveform of Fig. 13.46.

***Solution:*** By inspection it is fairly obvious that

***the average value of a pure sinusoidal waveform over one full cycle is zero.***

Eq. (13.26):

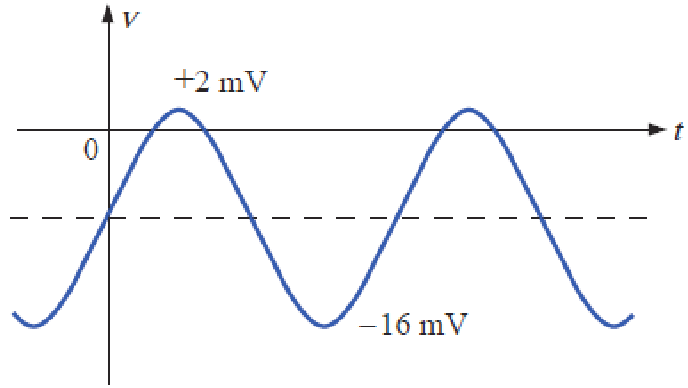
$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$




***The average of a pure sinusoidal waveform over a full cycle is zero***

**EXAMPLE 13.16** Determine the average value of the waveform of Fig. 13.47.

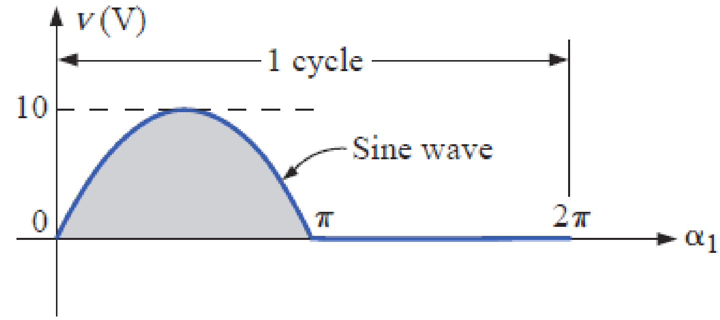
***Solution:*** The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from −16 mV) results in an average or dc level of −7 mV, as noted by the dashed line of Fig. 13.47.




**EXAMPLE 13.17** Determine the average value of the waveform of Fig. 13.48.

***Solution:***

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\text{ V})}{2\pi} \cong \mathbf{3.18\ V}$$



## 13.6 EFFECTIVE (rms) VALUE

It gives a relation between dc and ac with respect to power delivered:

- A sinusoidal quantity (average zero) can it deliver a net power?

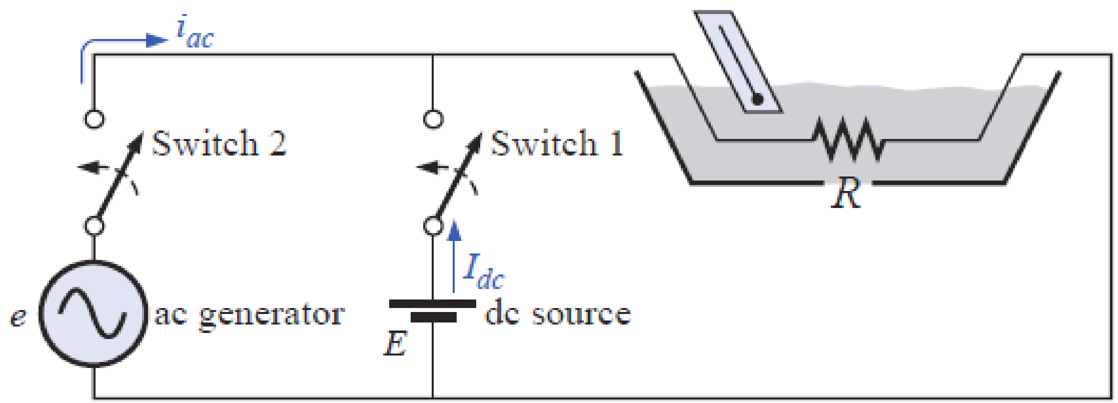



**FIG. 13.52**

*An experimental setup to establish a relationship between dc and ac quantities.*

The instantaneous power delivered by the ac supply is:

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2(\omega t))R$$

Using the trigonometric identity: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$P_{ac} = I_m^2 \left[\frac{1}{2}(1 - \cos 2\omega t)\right] R \implies P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

$$\boxed{P_{\text{ac}} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t}$$

The *average power* delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{\text{av(ac)}} = P_{\text{dc}}$$

$$\frac{I_m^2 R}{2} = I_{\text{dc}}^2 R \quad \text{and} \quad I_m = \sqrt{2} I_{\text{dc}}$$

or

$$I_{\text{dc}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

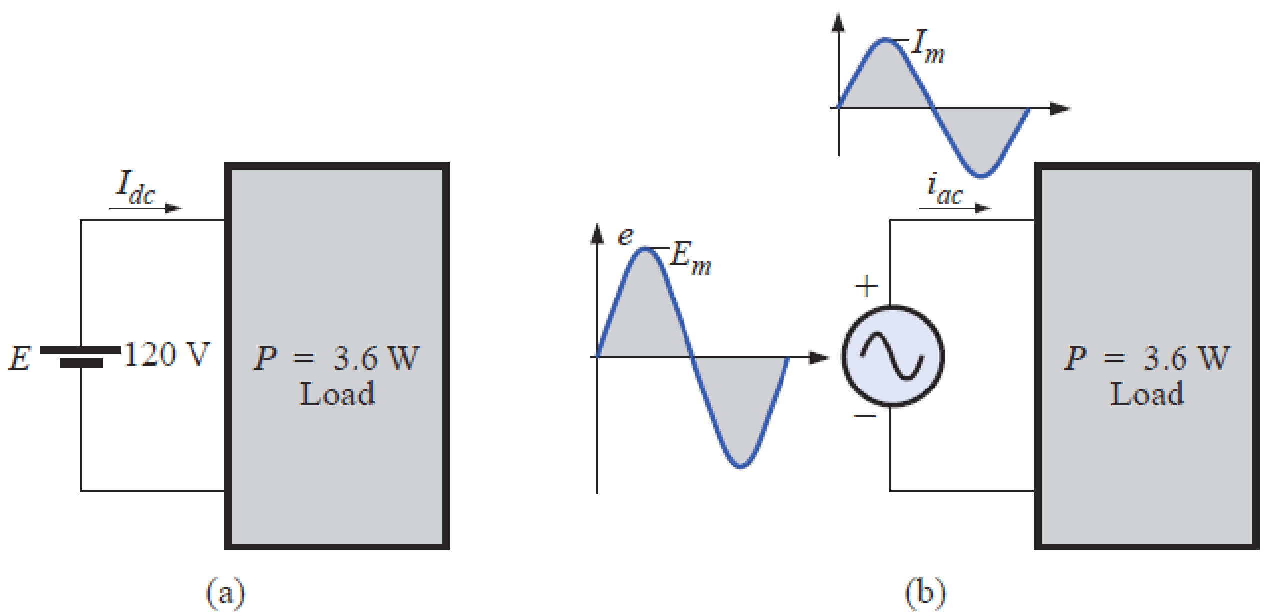
***the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its maximum value.***

The equivalent dc value is called the **effective value** of the sinusoidal quantity.

| | |
|---|---|
| $$I_{\text{eff}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$ | $$I_{\text{eq(dc)}} = I_{\text{eff}} = 0.707 I_m$$ |
| $$I_{\text{eff}} = \sqrt{\frac{\text{area } (i^2(t))}{T}}$$ | $$I_m = \sqrt{2} I_{\text{eff}} = 1.414 I_{\text{eff}}$$ |
| | $$E_{\text{eff}} = 0.707 E_m$$ |
| | $$E_m = \sqrt{2} E_{\text{eff}} = 1.414 E_{\text{eff}}$$ |

$\boldsymbol{I_{eff}}$ $\equiv$ root mean square value (rms value)

**EXAMPLE 13.20** The 120-V dc source of Fig. 13.54(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage ($E_m$) and the current ($I_m$) if the ac source [Fig. 13.54(b)] is to deliver the same power to the load.




***Solution:***

$$P_{dc} = V_{dc}I_{dc}$$

and
$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2}I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2}E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

**EXAMPLE 13.21** Find the effective or rms value of the waveform of Fig. 13.55.

***Solution:***

$v^2$ (Fig. 13.56):

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.236\ V}$$

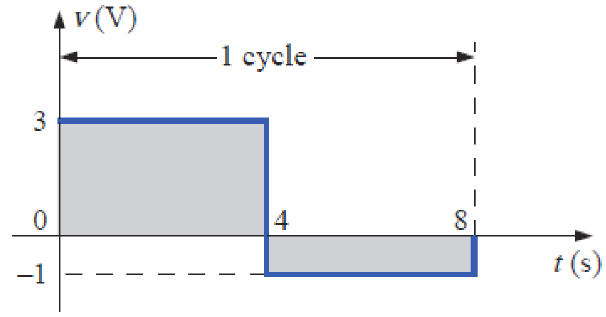



**FIG. 13.55**
*Example 13.21.*

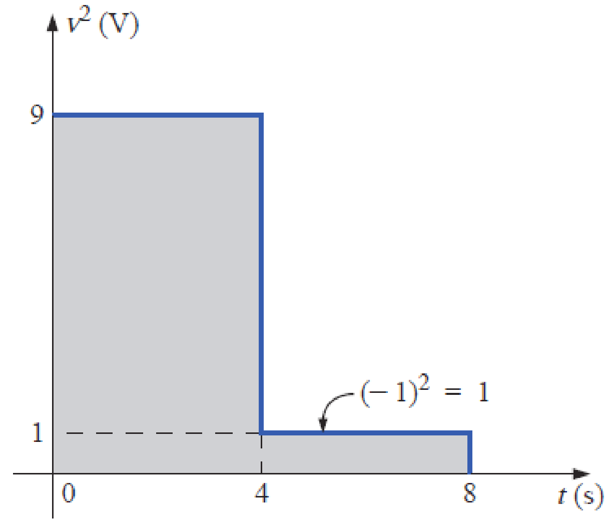
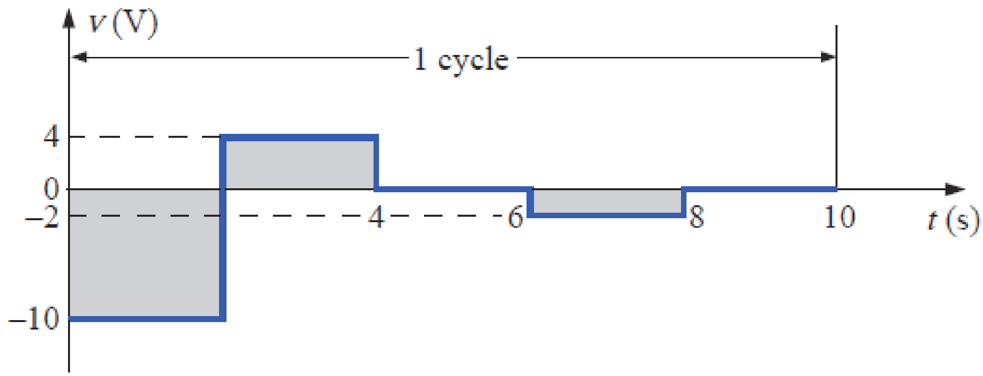



**FIG. 13.56**
*The squared waveform of Fig. 13.55.*

**EXAMPLE 13.22** Calculate the rms value of the voltage of Fig. 13.57.




***Solution:***

$v^2$ (Fig. 13.58):

$$V_{\text{rms}} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$

$$= \mathbf{4.899\ V}$$

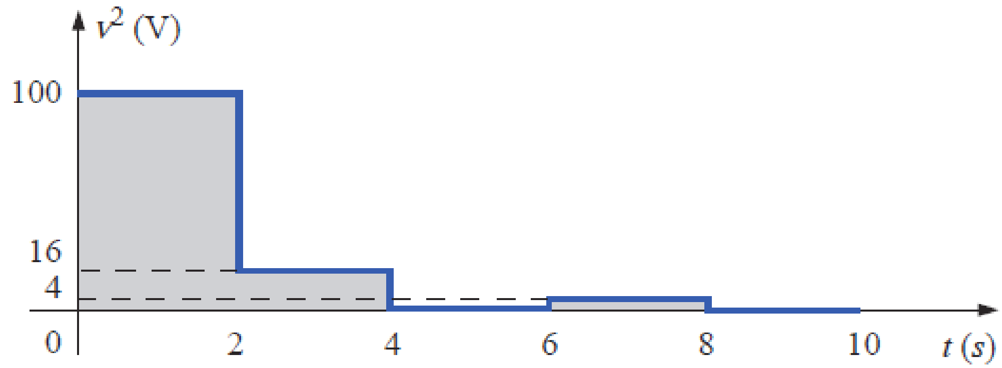



**FIG. 13.58**

*The squared waveform of Fig. 13.57.*

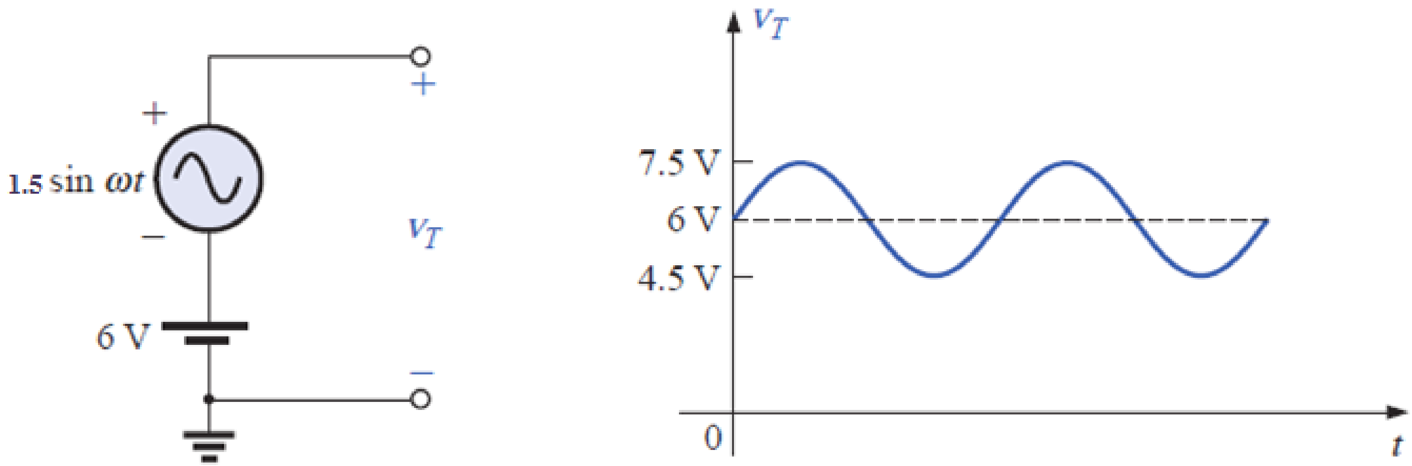



**FIG. 13.61**
*Generation and display of a waveform having a dc and an ac component.*

$$V_T = 6 + 1.5\sin\omega t$$

$$V_{eff} \neq V_{ac\ rms} + V_{dc} = 0.707 \times (1.5) + 6$$

$$V_{eff} = \sqrt{V_{dc}^2 + V_{ac\ rms}^2}$$

$$\begin{aligned} V_{\text{rms}} &= \sqrt{(6\text{ V})^2 + (1.06\text{ V})^2} \\ &= \sqrt{37.124}\text{ V} \\ &\cong \mathbf{6.1\ V} \end{aligned}$$