# Methods of Analysis and Selected Topics (dc)

### 8.1 INTRODUCTION

- The steps shown in the previous chapters cannot be applied if the sources are not in series or parallel.
- Other methods have been developed to solve network with any arrangement of sources.
- These methods apply also for single source network,

#### These methods are:

- 1- Branch-current analysis
- 2- Mesh analysis
- 3- Nodal analysis

Any of these method can be applied, the best method cannot be defined. All the methods can be applied to <u>linear bilateral</u> networks.

- Linear = elements of the network are independent of the voltage applied across or the current through them,
- Bilateral  $\equiv$  no change in the behavior if current or voltage is reversed.

### **8.2 CURRENT SOURCES**

Current source is the dual of the voltage source:

Current source supplies fixed current to the branch in which it is located; the voltage across its terminal can vary.

The interest in current source is due to semiconductor devices (transistors),

# **Current-controlled devices**

- Transistor behaves like a current source.
- ➤ Symbol of current source = circle with an arrow.
- Arrow indicates the direction of current supplied.

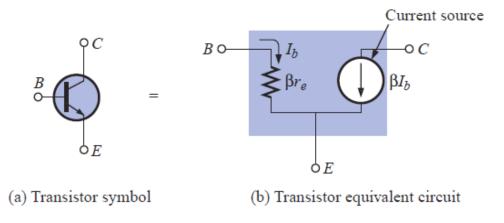


FIG. 8.1

Current source within the transistor equivalent circuit.

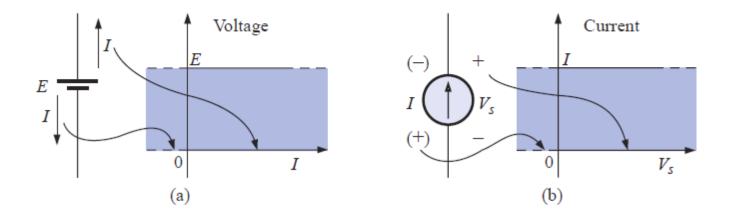


FIG. 8.2

Comparing the characteristics of an ideal voltage and current source.

- > A current source determines the current in the branch in which it is located
- The magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

**EXAMPLE 8.2** Find the voltage  $V_s$  and the currents  $I_1$  and  $I_2$  for the network of Fig. 8.4.

Solution:

$$V_s = E = \mathbf{12 V}$$

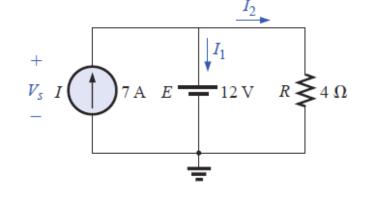
$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12 \text{ V}}{4 \Omega} = \mathbf{3 A}$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and

$$I_1 = I - I_2 = 7 A - 3 A = 4 A$$



**EXAMPLE 8.3** Determine the current  $I_1$  and the voltage  $V_s$  for the network of Fig. 8.5.

**Solution:** Using the current divider rule:

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1 \Omega)(6 A)}{1 \Omega + 2 \Omega} = 2 A$$

The voltage  $V_1$  is

$$V_1 = I_1 R_1 = (2 \text{ A})(2 \Omega) = 4 \text{ V}$$

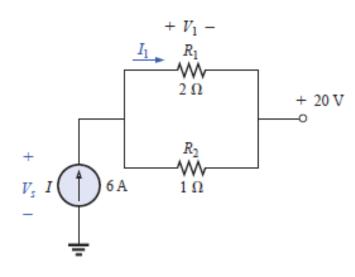
and, applying Kirchhoff's voltage law,

$$+V_s - V_1 - 20 V = 0$$

and

$$V_s = V_1 + 20 \text{ V} = 4 \text{ V} + 20 \text{ V}$$
  
= 24 V

Note the polarity of  $V_s$  as determined by the multisource network.



### **8.3 SOURCE CONVERSIONS**

The current (voltage) source described are *ideal sources* ⇒ No internal resistance Real sources (voltage or current) have internal resistance

Real Voltage source:

- $\triangleright$   $E \equiv$  voltage source rating
- $ightharpoonup R_s \equiv \text{internal resistance (series)}$

 $R_s = 0$  (much smaller than any series resistor)  $\Rightarrow$  **Ideal voltage source** 

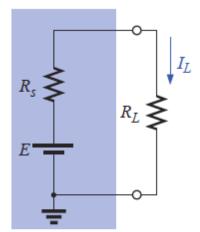


FIG. 8.6

Practical voltage source.

$$I_L = \frac{E}{R_S + R_L}$$

Real Current source:

- $ightharpoonup I \equiv$  current source rating
- $ightharpoonup R_s \equiv \text{internal resistance (parallel)}$

 $R_s = \infty$  (much larger than any parallel resistor)

**⇒ Ideal Current source** 

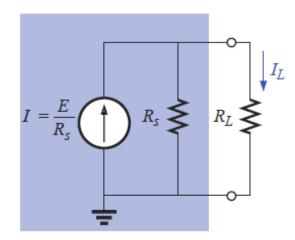


FIG. 8.7

Practical current source.

$$I_L = \frac{R_S \cdot I}{R_S + R_L}$$

Sources with their internal resistance included can be converted from one type to the other type.

### Source conversions are equivalent only at their external terminals.

- $\triangleright$   $R_s$  is the same (position changed)
- $\triangleright E = I \cdot R_S$

As far as terminal point (a) and (b) and the circuit in between there is no difference:

- Same current
- Same voltage
- Same power

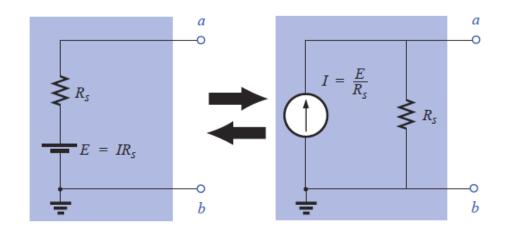


FIG. 8.8
Source conversion.

#### **EXAMPLE 8.4**

- a. Convert the voltage source of Fig. 8.9(a) to a current source, and calculate the current through the 4- $\Omega$  load for each source.
- b. Replace the 4- $\Omega$  load with a 1-k $\Omega$  load, and calculate the current  $I_L$  for the voltage source.
- c. Repeat the calculation of part (b) assuming that the voltage source is ideal ( $R_s = 0 \Omega$ ) because  $R_L$  is so much larger than  $R_s$ . Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

#### Solutions:

a. See Fig. 8.9.

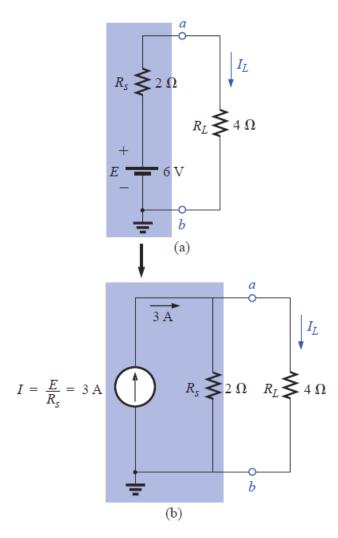
Fig. 8.9(a): 
$$I_L = \frac{E}{R_s + R_L} = \frac{6 \text{ V}}{2 \Omega + 4 \Omega} = 1 \text{ A}$$

Fig. 8.9(b): 
$$I_L = \frac{R_s I}{R_s + R_L} = \frac{(2 \Omega)(3 \text{ A})}{2 \Omega + 4 \Omega} = \mathbf{1} \text{ A}$$

b. 
$$I_L = \frac{E}{R_s + R_L} = \frac{6 \text{ V}}{2 \Omega + 1 \text{ k}\Omega} \approx 5.99 \text{ mA}$$

c. 
$$I_L = \frac{E}{R_L} = \frac{6 \text{ V}}{1 \text{ k}\Omega} = 6 \text{ mA} \approx 5.99 \text{ mA}$$

Yes,  $R_L \gg R_s$  (voltage source).



#### **EXAMPLE 8.5**

- a. Convert the current source of Fig. 8.10(a) to a voltage source, and find the load current for each source.
- b. Replace the 6-k $\Omega$  load with a 10- $\Omega$  load, and calculate the current  $I_L$  for the current source.
- c. Repeat the calculation of part (b) assuming that the current source is ideal  $(R_s = \infty \Omega)$  because  $R_L$  is so much smaller than  $R_s$ . Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

#### **Solutions:**

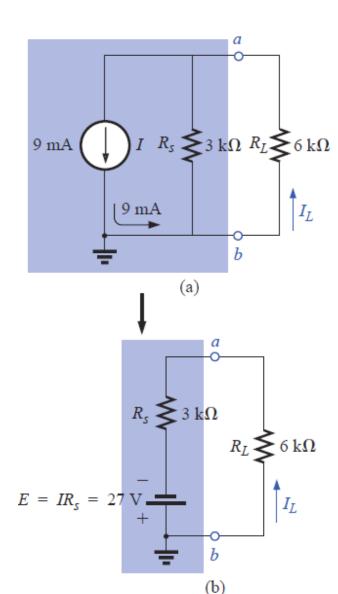
a. See Fig. 8.10.

Fig. 8.10(a): 
$$I_L = \frac{R_s I}{R_s + R_L} = \frac{(3 \text{ k}\Omega)(9 \text{ mA})}{3 \text{ k}\Omega + 6 \text{ k}\Omega} = 3 \text{ mA}$$
  
Fig. 8.10(b):  $I_L = \frac{E}{R_s + R_L} = \frac{27 \text{ V}}{3 \text{ k}\Omega + 6 \text{ k}\Omega} = \frac{27 \text{ V}}{9 \text{ k}\Omega} = 3 \text{ mA}$ 

b. 
$$I_L = \frac{R_s I}{R_s + R_L} = \frac{(3 \text{ k}\Omega)(9 \text{ mA})}{3 \text{ k}\Omega + 10 \Omega} = 8.97 \text{ mA}$$

c. 
$$I_L = I = 9 \text{ mA} \cong 8.97 \text{ mA}$$

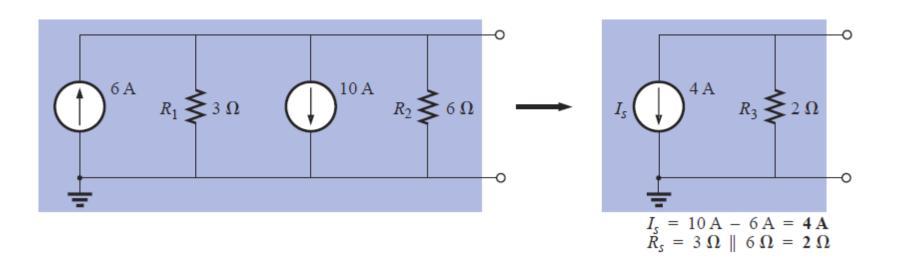
Yes,  $R_s \gg R_L$  (current source).

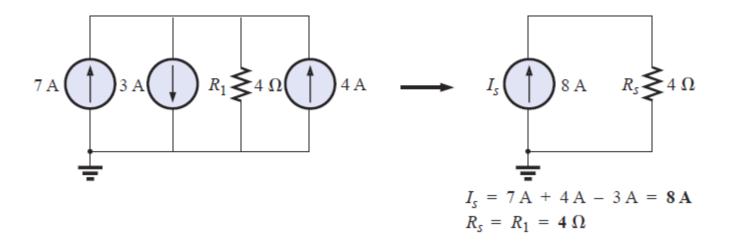


### 8.4 CURRENT SOURCES IN PARALLEL

Current sources in parallel: they may all be replaced by one current source having the magnitude and direction of the resultant:

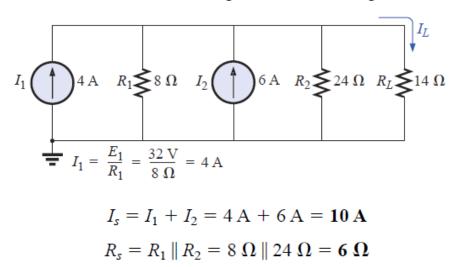
- Summing the currents in one direction
- Subtracting the sum of the currents in the opposite direction
- New parallel resistance is found like any parallel resistors in parallel





**EXAMPLE 8.7** Reduce the network of Fig. 8.13 to a single current source, and calculate the current through  $R_L$ .

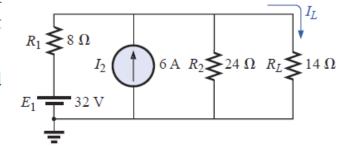
**Solution:** In this example, the voltage source will first be converted to a current source as shown in Fig. 8.14. Combining current sources,



Applying the current divider rule to the resulting network of Fig. 8.15,

and

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6 \Omega)(10 \text{ A})}{6 \Omega + 14 \Omega} = \frac{60 \text{ A}}{20} = 3 \text{ A}$$



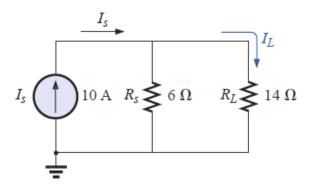
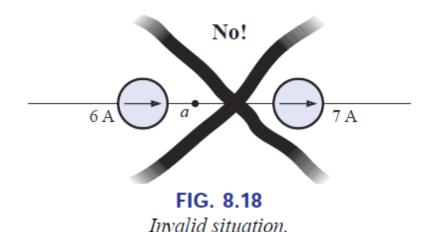


FIG. 8.15
Network of Fig. 8.14 reduced to its simplest form.

## 8.5 CURRENT SOURCES IN SERIES

current sources of different current ratings are not connected in series,

Voltage sources of different voltage ratings are not connected in parallel.



# 8.7 MESH ANALYSIS (GENERAL APPROACH)

 $Mesh \equiv closed loop (like a fence)$ 

- 1. Assign a current in clockwise direction to each independent, closed loop of the network.
  - *Not necessarily the clockwise direction.*
  - Any direction can be chosen ---
  - Remaining steps follow the choice properly.

However, by choosing the clockwise direction as a standard, we can develop a shorthand method for writing the required equations that will save time and possibly prevent some common errors.

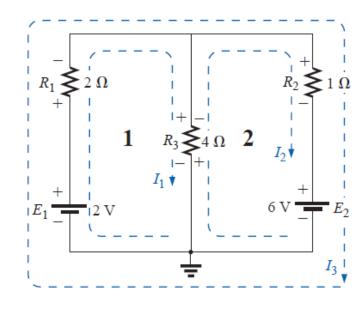


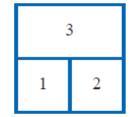
FIG. 8.26

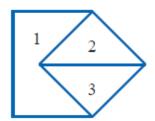
Defining the mesh currents for a "two-window" network.

Independent closed loop









2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.

Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.26, that the  $4\Omega$  resistor have two sets of polarities across it.

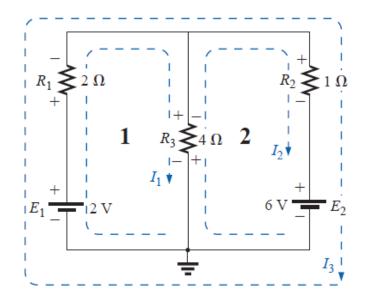


FIG. 8.26

Defining the mesh currents for a "two-window" network.

- 3. Apply Kirchhoff's voltage law around each closed loop.

  Again, the clockwise direction is chosen to establish uniformity.
  - a. If a resistor has two or more assumed currents through it,  $I_T = \sum I_T$
  - b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.
  - 4. Solve the resulting simultaneous linear equations for the assumed loop currents.

**EXAMPLE 8.11** Consider the basic network as in Fig. 8.26.

### **Solution:**

**Step 1:** Two loop currents ( $I_1$  and  $I_2$ ) are assigned in the clockwise direction in the windows of the network. A third loop ( $I_3$ ) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4  $\Omega$  resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each

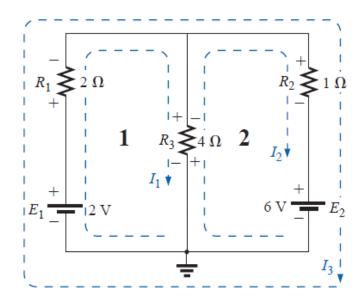


FIG. 8.26

Defining the mesh currents for a "two-window" network.

resistor is determined by V = IR, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions.

If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents will always be subtracted from the loop current of the loop being analyzed.

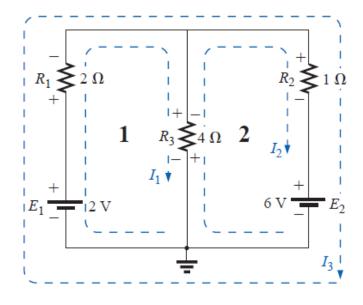


FIG. 8.26

Defining the mesh currents for a "two-window" network.

loop 1: 
$$+E_1 - V_1 - V_3 = 0$$
 (clockwise starting at point a)

$$+2~\text{V}-(2~\Omega)~I_1-\overbrace{(4~\Omega)(I_1-I_2)=0}^{\text{Voltage drop across}} \\ \text{Subtracted since}~I_2~\text{is opposite in direction to}~I_1.$$

loop 2: 
$$-V_3 - V_2 - E_2 = 0$$
 (clockwise starting at point b)  
- $(4 \Omega)(I_2 - I_1) - (1 \Omega)I_2 - 6 V = 0$ 

**Step 4:** The equations are then rewritten as follows (without units for clarity):

loop 1: 
$$+2 - 2I_1 - 4I_1 + 4I_2 = 0$$

loop 2: 
$$-4I_2 + 4I_1 - 1I_2 - 6 = 0$$

loop 1: 
$$+2 - 6I_1 + 4I_2 = 0$$

loop 2: 
$$-5I_2 + 4I_1 - 6 = 0$$

loop 1: 
$$-6I_1 + 4I_2 = -2$$

loop 2: 
$$+4I_1 - 5I_2 = +6$$

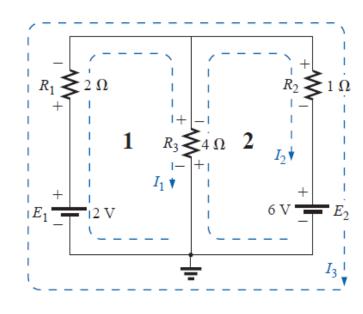


FIG. 8.26

Defining the mesh currents for a "two-window" network.

### **Solution is:**

$$I_1 = -1 A$$
 and  $I_2 = -2 A$ 

The minus signs  $\implies$  the currents have a direction opposite to the assumed loop current.

The actual current through the 2-V source and 2- $\Omega$  resistor is therefore 1 A in the other direction, The current through the 4- $\Omega$  resistor is determined by the following equation from the original network:

loop 1: 
$$I_{4\Omega} = I_1 - I_2 = -1 \text{ A} - (-2 \text{ A}) = -1 \text{ A} + 2 \text{ A}$$
  
= **1 A** (in the direction of  $I_1$ )

**EXAMPLE 8.12** Find the current through each branch of the network of Fig. 8.27.

#### Solution:

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the  $6-\Omega$  resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: 
$$+E_1 - V_1 - V_2 - E_2 = 0$$
 (clockwise starting at point a)  $+5 \text{ V} - (1 \Omega)I_1 - (6 \Omega)(I_1 - I_2) - 10 \text{ V} = 0$ 

 $I_2$  flows through the 6- $\Omega$  resistor in the direction opposite to  $I_1$ .

loop 2: 
$$E_2 - V_2 - V_3 = 0$$
 (clockwise starting at point b)  
+10 V -  $(6 \Omega)(I_2 - I_1) - (2 \Omega)I_2 = 0$ 

The equations are rewritten as

$$5 - I_1 - 6I_1 + 6I_2 - 10 = 0 - 7I_1 + 6I_2 = 5$$
$$10 - 6I_2 + 6I_1 - 2I_2 = 0 + 6I_1 - 8I_2 = -10$$

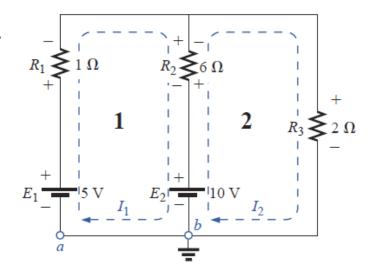


FIG. 8.27 Example 8.12.

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1} \mathbf{A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2} \mathbf{A}$$

Since  $I_1$  and  $I_2$  are positive and flow in opposite directions through the 6- $\Omega$  resistor and 10-V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1$$
 (2 A > 1 A)

Therefore,

$$I_{R_2} = I_2 - I_1 = 2 A - 1 A = 1 A$$
 in the direction of  $I_2$ 

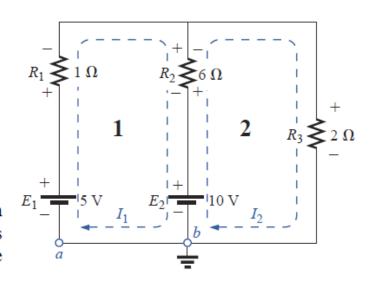


FIG. 8.27 Example 8.12.

#### **EXAMPLE 8.13** Find the branch currents of the network of Fig. 8.28.

#### Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: 
$$-E_1 - I_1 R_1 - E_2 - V_2 = 0$$
 (clockwise from point *a*)  
 $-6 \text{ V} - (2 \Omega) I_1 - 4 \text{ V} - (4 \Omega) (I_1 - I_2) = 0$   
loop 2:  $-V_2 + E_2 - V_3 - E_3 = 0$  (clockwise from point *b*)  
 $-(4 \Omega) (I_2 - I_1) + 4 \text{ V} - (6 \Omega) (I_2) - 3 \text{ V} = 0$ 

which are rewritten as

$$-10 - 4I_1 - 2I_1 + 4I_2 = 0 -6I_1 + 4I_2 = +10 + 1 + 4I_1 - 4I_2 - 6I_2 = 0 +4I_1 - 10I_2 = -1$$

$$I_1 = -2.182 \text{ A};$$
  
 $I_2 = -0.773 \text{ A};$ 

The current in central branch ( $R_2$  and  $E_2$ ) is:

$$I_1 - I_2 = -2.182 \,\mathrm{A} - (-0.773 \,\mathrm{A})$$
  
= -2.182 A + 0.773 A  
= -1.409 A

Revealing that it is 1.409 A in a direction opposite (due to the minus sign) to  $I_I$  in loop 1.

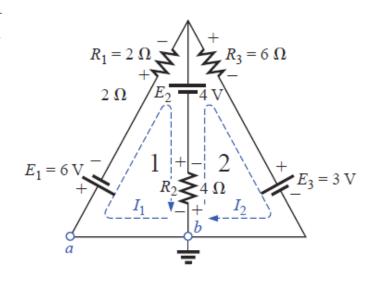


FIG. 8.28 Example 8.13.

## **Supermesh Currents**

When there is a current source in the network:

- Start as before step 1 and 2, (current source is treated as resistor)
- Mentally remove the current source (open circuit) and apply KVL to the remaining loops
- Any resulting open window, including two or more mesh current current is said supermesh.
- Relate the currents to the independent current sources, and solve.

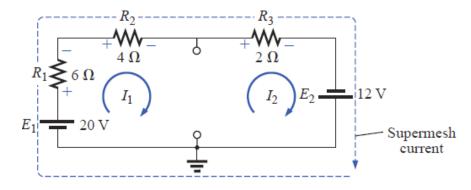


FIG. 8.31

Defining the supermesh current.

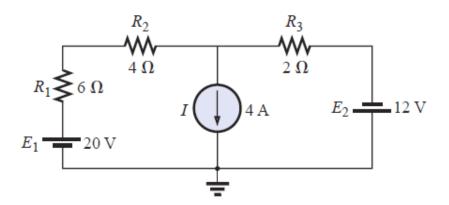


FIG. 8.29 Example 8.14.

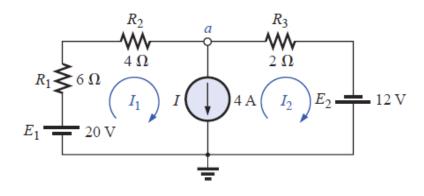


FIG. 8.30

Defining the mesh currents for the network of Fig. 8.29.

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

Applying determinants:

$$I_{1} = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = 3.33 \text{ A}$$

and

$$I_2 = I_1 - I = 3.33 \text{ A} - 4 \text{ A} = -0.67 \text{ A}$$

In the above analysis, it might appear that when the current source was removed,  $I_1 = I_2$ . However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

**EXAMPLE 8.15** Using mesh analysis, determine the currents for the network of Fig. 8.32.

**Solution:** The mesh currents are defined in Fig. 8.33. The current sources are removed, and the single supermesh path is defined in Fig. 8.34.

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$

$$-(I_2 - I_1)2 \Omega - I_2(6 \Omega) - (I_2 - I_3)8 \Omega = 0$$

$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$

$$2I_1 - 16I_2 + 8I_3 = 0$$

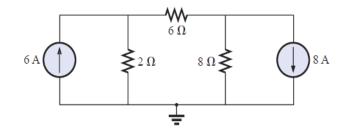
Introducing the relationship between the mesh currents and the current sources:

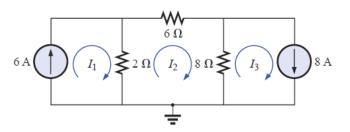
$$I_1 = 6 A$$
$$I_3 = 8 A$$

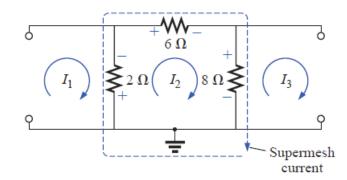
results in the following solutions:

and 
$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6 \text{ A}) - 16I_2 + 8(8 \text{ A}) = 0$$
$$I_2 = \frac{76 \text{ A}}{16} = \textbf{4.75 A}$$
Then 
$$I_{2\Omega} \downarrow = I_1 - I_2 = 6 \text{ A} - 4.75 \text{ A} = \textbf{1.25 A}$$
and 
$$I_{8\Omega} \uparrow = I_3 - I_2 = 8 \text{ A} - 4.75 \text{ A} = \textbf{3.25 A}$$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.







# 8.8 MESH ANALYSIS (FORMAT APPROACH)

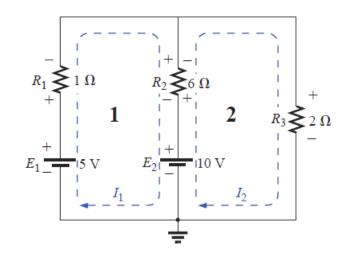
The equations obtained are

$$-7I_1 + 6I_2 = 5$$
  
 $6I_1 - 8I_2 = -10$ 
 $7I_1 - 6I_2 = -5$   
 $8I_2 - 6I_1 = 10$ 

and expanded as

Col. 1 Col. 2 Col. 3  

$$(1+6)I_1 - 6I_2 = (5-10)$$
  
 $(2+6)I_2 - 6I_1 = 10$ 



|         | Col. 1               | Col. 2           | Col. 3             |
|---------|----------------------|------------------|--------------------|
| Loop 1: | $(R_1+R_2)\cdot I_1$ | $-R_2 \cdot I_2$ | $(E_1-E_2)$        |
| Loop 2: | $(R_2+R_3)\cdot I_2$ | $-R_2 \cdot I_1$ | $\boldsymbol{E_2}$ |

**Col. 1:** Loop current × sum Resistor in loop

Col. 2: Resistor common to another loop  $\times$  that Loop current

Col. 3: Algebraic sum of voltage sources in the loop

- 1. Assign a loop current to each independent, closed loop in a clockwise direction.
- 2. The  $N^o$  of required equations =  $N^o$  of independent, closed loops.
  - Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.
- 3. the mutual terms are always subtracted from the first column. A mutual term is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each term is the product of the mutual resistor and the other loop current passing through the same element.
- 4. The column to the right of the equality sign is the algebraic sum of the voltage sources in the loop considered.

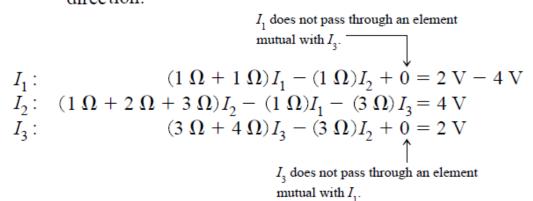


5. Solve the resulting simultaneous equations for the desired loop currents.

#### **EXAMPLE 8.17** Write the mesh equations for the network

#### Solution:

Step 1: Each window is assigned a loop current in the clockwise direction:



Summing terms yields

$$2I_1 - I_2 + 0 = -2$$
  
 $6I_2 - I_1 - 3I_3 = 4$   
 $7I_3 - 3I_2 + 0 = 2$ 

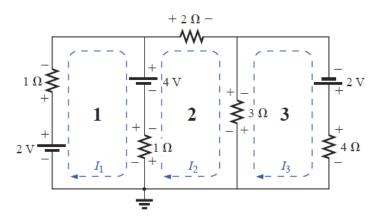
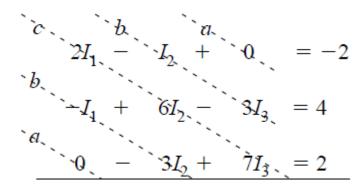


FIG. 8.37 Example 8.17.

which are rewritten for determinants as



Note that the coefficients of the a and b diagonals are equal. This symmetry about the c-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

$$I_{1}: \qquad (8 \Omega + 3 \Omega)I_{1} - (8 \Omega)I_{3} - (3 \Omega)I_{2} = 15 \text{ V}$$

$$I_{2}: \qquad (3 \Omega + 5 \Omega + 2 \Omega)I_{2} - (3 \Omega)I_{1} - (5 \Omega)I_{3} = 0$$

$$I_{3}: \qquad (8 \Omega + 10 \Omega + 5 \Omega)I_{3} - (8 \Omega)I_{1} - (5 \Omega)I_{2} = 0$$

$$11I_{1} - 8I_{3} - 3I_{2} = 15$$

$$10I_{2} - 3I_{1} - 5I_{3} = 0$$

$$23I_{3} - 8I_{1} - 5I_{2} = 0$$

$$11I_{1} - 3I_{2} - 8I_{3} = 15$$

$$-3I_{1} + 10I_{2} - 5I_{3} = 0$$

$$-8I_{1} - 5I_{2} + 23I_{3} = 0$$

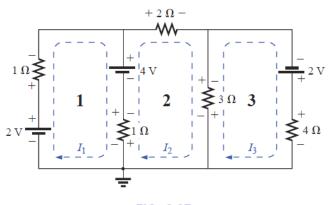


FIG. 8.37
Example 8.17.

