## **Parallel Circuits**

### **6.2 PARALLEL ELEMENTS**





All elements are in parallel



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## **6.3 TOTAL CONDUCTANCE AND RESISTANCE**

For parallel elements, the total conductance is the sum of the individual conductances.



The total resistance of parallel resistors is always less than the value of the smallest resistor.

For N equal resistors in parallel
$$\frac{\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots + \frac{1}{R}}{N} = N\left(\frac{1}{R}\right)$$

$$\frac{R_T = \frac{R}{N}}{G_T = NG}$$

For two resistors in parallel:

For three resistors in parallel:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \qquad R_T = \frac{R_1 R_2}{R_1 + R_2} \qquad \qquad R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad \qquad R_T = \frac{R_1 R_2 R_3}{R_1 R_2 R_3 + R_2 R_3}$$

Parallel elements can be interchanged without changing the total resistance or input current.

**EXAMPLE 6.1** Determine the total conductance and resistance for the parallel network of Fig. 6.7.

#### Solution:

$$G_T = G_1 + G_2 = \frac{1}{3\Omega} + \frac{1}{6\Omega} = 0.333 \text{ S} + 0.167 \text{ S} = 0.5 \text{ S}$$

 $R_T = \frac{1}{G_T} = \frac{1}{0.5 \text{ S}} = 2 \Omega$ 

and

**EXAMPLE 6.2** Determine the effect on the total conductance and resistance of the network of Fig. 6.7 if another resistor of 10  $\Omega$  were added in parallel with the other elements.

#### Solution:

$$G_T = 0.5 \text{ S} + \frac{1}{10 \Omega} = 0.5 \text{ S} + 0.1 \text{ S} = 0.6 \text{ S}$$
  
 $R_T = \frac{1}{G_T} = \frac{1}{0.6 \text{ S}} \cong 1.667 \Omega$ 

Note, as mentioned above, that adding additional terms increases the conductance level and decreases the resistance level.



 $R_1 \lesssim 3 \Omega$ 

 $R_2 \leq 6 \Omega$ 

 $G_T$ 

 $R_{T}$ 

**EXAMPLE 6.3** Determine the total resistance for the network of Fig. 6.8.



**FIG. 6.8** *Example 6.3*.

Solution:

and

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2\Omega} + \frac{1}{4\Omega} + \frac{1}{5\Omega} = 0.5 \text{ S} + 0.25 \text{ S} + 0.2 \text{ S}$$
$$= 0.95 \text{ S}$$
$$R_T = \frac{1}{0.95 \text{ S}} = 1.053 \Omega$$





# For parallel resistors, the total resistance will always decrease as additional elements are added in parallel.

For series resistors, the total resistance will always increase as additional elements are added in series.

### **6.4 PARALLEL CIRCUITS**



$$I_{S} = \frac{E}{R_{T}} = E \cdot \left(\frac{1}{R_{T}}\right) = E \cdot \left(\frac{1}{R_{1}} + \frac{1}{R_{2}}\right) = \frac{E}{R_{1}} + \frac{E}{R_{2}} = I_{1} + I_{2}$$

$$I_s = I_1 + I_2$$

For single-source parallel networks, the source current  $(I_s)$  is equal to the sum of the individual branch currents.

$$P_{1} = V_{1}I_{1} = I_{1}^{2}R_{1} = \frac{V_{1}^{2}}{R_{1}}$$
$$P_{2} = V_{2}I_{2} = I_{2}^{2}R_{2} = \frac{V_{2}^{2}}{R_{2}}$$
$$P_{s} = EI_{s} = I_{s}^{2}R_{T} = \frac{E^{2}}{R_{T}}$$

The power dissipated by the resistors and delivered by the source are:

**EXAMPLE 6.11** For the parallel network of Fig. 6.22:

- a. Calculate  $R_T$ .
- b. Determine Is.
- c. Calculate  $I_1$  and  $I_2$ , and demonstrate that  $I_s = I_1 + I_2$ .
- d. Determine the power to each resistive load.
- e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

#### Solutions:

a. 
$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9 \ \Omega)(18 \ \Omega)}{9 \ \Omega + 18 \ \Omega} = \frac{162 \ \Omega}{27} = 6 \ \Omega$$
  
b.  $I_s = \frac{E}{R_T} = \frac{27 \ V}{6 \ \Omega} = 4.5 \ A$   
c.  $I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27 \ V}{9 \ \Omega} = 3 \ A$   
 $I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27 \ V}{18 \ \Omega} = 1.5 \ A$   
 $I_5 = I_1 + I_2$   
 $4.5 \ A = 3 \ A + 1.5 \ A$   
 $4.5 \ A = 4.5 \ A$  (checks)  
d.  $P_1 = V_1 I_1 = EI_1 = (27 \ V)(3 \ A) = 81 \ W$   
 $P_2 = V_2 I_2 = EI_2 = (27 \ V)(1.5 \ A) = 40.5 \ W$   
e.  $P_s = EI_s = (27 \ V)(4.5 \ A) = 121.5 \ W$ 



**EXAMPLE 6.12** Given the information provided in Fig. 6.23:

- a. Determine  $R_3$ .
- b. Calculate E.
- c. Find  $I_s$ .
- d. Find  $I_2$ .
- e. Determine  $P_2$ .

#### Solutions:

a. 
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{4\Omega} = \frac{1}{10\Omega} + \frac{1}{20\Omega} + \frac{1}{R_3}$$
$$0.25 \text{ S} = 0.1 \text{ S} + 0.05 \text{ S} + \frac{1}{R_3}$$
$$0.25 \text{ S} = 0.15 \text{ S} + \frac{1}{R_3}$$
$$\frac{1}{R_3} = 0.1 \text{ S}$$
$$R_3 = \frac{1}{0.1 \text{ S}} = 10 \Omega$$
$$b. E = V_1 = I_1 R_1 = (4 \text{ A})(10 \Omega) = 40 \text{ V}$$
$$c. I_5 = \frac{E}{R_T} = \frac{40 \text{ V}}{4\Omega} = 10 \text{ A}$$
$$d. I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40 \text{ V}}{20\Omega} = 2 \text{ A}$$
$$e. P_2 = I_2^2 R_2 = (2 \text{ A})^2 (20 \Omega) = 80 \text{ W}$$



FIG. 6.23 Example 6.12.

### **6.5 KIRCHHOFF'S CURRENT LAW**

Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.

The sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

 $\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$ 

$$I_1 + I_4 = I_2 + I_3$$
  
 $4 A + 8 A = 2 A + 10 A$   
 $12 A = 12 A$ 

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$

$$6 \text{ A} = 2 \text{ A} + 4 \text{ A}$$

$$6 \text{ A} = 6 \text{ A} \quad \text{(checks)}$$



**EXAMPLE 6.15** Determine the currents  $I_3$  and  $I_5$  of Fig. 6.29 through applications of Kirchhoff's current law.

**Solution:** Note that since node b has two unknown quantities and node a has only one, we must first apply Kirchhoff's current law to node a. The result can then be applied to node b.

For node a,

and

For node b,

and



 $I_3 = 7 \, \text{A}$ 

 $I_3 = I_4 + I_5$ 

 $7 A = 1 A + I_5$ 

 $I_5 = 7 A - 1 A = 6 A$ 





**EXAMPLE 6.16** Find the magnitude and direction of the currents  $I_3$ ,  $I_4$ ,  $I_6$ , and  $I_7$  for the network of Fig. 6.30. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.

**Solution:** Considering the overall system, we know that the current entering must equal that leaving. Therefore,

 $I_7 = I_1 = 10 \, \text{A}$ 

Since 10 A are entering node a and 12 A are leaving,  $I_3$  must be supplying current to the node.

Applying Kirchhoff's current law at node a,

$$I_1 + I_3 = I_2$$
  
10 A +  $I_3 = 12$  A

and

$$I_3 = 12 \text{ A} - 10 \text{ A} = 2 \text{ A}$$

At node b, since 12 A are entering and 8 A are leaving,  $I_4$  must be leaving. Therefore,

and

 $12 A = I_4 + 8 A$  $I_4 = 12 A - 8 A = 4 A$ 

 $I_2 = I_4 + I_5$ 

At node c,  $I_3$  is leaving at 2 A and  $I_4$  is entering at 4 A, requiring that  $I_6$  be leaving. Applying Kirchhoff's current law at node c,

 $I_4 = I_3 + I_6$  $4 A = 2 A + I_6$ 

and

 $I_6 = 4 \mathrm{A} - 2 \mathrm{A} = 2 \mathrm{A}$ 

As a check at node d,

$$I_5 + I_6 = I_7$$
  
8 A + 2 A = 10 A  
10 A = 10 A (checks)





### **6.6 CURRENT DIVIDER RULE**

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.



FIG. 6.32 Demonstrating how current will divide between unequal resistors.



The current through any parallel branch is equal to the product of the *total* resistance of the parallel branches and the input current divided by the resistance of the branch.

For the current  $I_1$ ,

and for  $I_2$ ,

$$I_1 = \frac{R_T}{R_1}$$

$$I_2 = \frac{R_T}{R_2}$$

and so on.



**EXAMPLE 6.18** Find the current  $I_1$  for the network of Fig. 6.36.

**Solution:** There are two options for solving this problem. The first is to use Eq. (6.9) as follows:

$$\frac{1}{R_T} = \frac{1}{6\Omega} + \frac{1}{24\Omega} + \frac{1}{48\Omega} = 0.1667 \text{ S} + 0.0417 \text{ S} + 0.0208 \text{ S}$$
$$= 0.2292 \text{ S}$$

and

$$R_T = \frac{1}{0.2292 \text{ S}} = 4.363 \Omega$$

with

$$I_1 = \frac{R_T}{R_1} I = \frac{4.363 \ \Omega}{6 \ \Omega} (42 \text{ mA}) = 30.54 \text{ mA}$$

The second option is to apply Eq. (6.10) once after combining  $R_2$  and  $R_3$  as follows:

$$24 \Omega \parallel 48 \Omega = \frac{(24 \Omega)(48 \Omega)}{24 \Omega + 48 \Omega} = 16 \Omega$$

and

Both options generated the same answer, leaving you with a choice for future calculations involving more than two parallel resistors.

 $I_1 = \frac{16 \Omega (42 \text{ mA})}{16 \Omega + 6 \Omega} = 30.54 \text{ mA}$ 



**EXAMPLE 6.20** Determine the resistance  $R_1$  to effect the division of current in Fig. 6.38.

Solution: Applying the current divider rule,

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$

and

$$(R_1 + R_2)I_1 = R_2I$$
  

$$R_1I_1 + R_2I_1 = R_2I$$
  

$$R_1I_1 = R_2I - R_2I_1$$
  

$$R_1 = \frac{R_2(I - I_1)}{I_1}$$

Substituting values:

$$R_1 = \frac{7 \Omega(27 \text{ mA} - 21 \text{ mA})}{21 \text{ mA}}$$
$$= 7 \Omega\left(\frac{6}{21}\right) = \frac{42 \Omega}{21} = 2 \Omega$$

An alternative approach is

and  

$$I_2 = I - I_1 \quad \text{(Kirchhoff's current law)}$$

$$= 27 \text{ mA} - 21 \text{ mA} = 6 \text{ mA}$$

$$V_2 = I_2 R_2 = (6 \text{ mA})(7 \Omega) = 42 \text{ mV}$$

$$V_1 = I_1 R_1 = V_2 = 42 \text{ mV}$$

$$R_1 = \frac{V_1}{I_1} = \frac{42 \text{ mV}}{21 \text{ mA}} = 2 \Omega$$



### Current seeks the path of least resistance.

- **1.** More current passes through the smaller of two parallel resistors.
- 2. The current entering any number of parallel resistors divides into these resistors as the inverse ratio of their ohmic values.



FIG. 6.39 Current division through parallel branches.

### **6.7 VOLTAGE SOURCES IN PARALLEL**

Voltage source are placed in parallel only if they have the **same voltage rating**.

Primary reason to increase the current rating,  $\Rightarrow$  increase power.



Parallel voltage sources.



### **6.8 OPEN AND SHORT CIRCUITS**



A short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.





**EXAMPLE 6.21** Determine the voltage  $V_{ab}$  for the network of Fig. 6.45.

**Solution:** The open circuit requires that *I* be zero amperes. The voltage drop across both resistors is therefore zero volts since V = IR = (0)R = 0 V. Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = 20 \text{ V}$$



**FIG. 6.45** *Example 6.21.* 



FIG. 6.46

Example 6.22.

**EXAMPLE 6.22** Determine the voltages  $V_{ab}$  and  $V_{cd}$  for the network of Fig. 6.46.

**Solution:** The current through the system is zero amperes due to the open circuit, resulting in a 0-V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 6.47. The voltage  $V_{ab}$  is then directly across the 10-V battery, and

$$V_{ab} = E_1 = 10 \, \text{V}$$

The voltage  $V_{cd}$  requires an application of Kirchhoff's voltage law:



or



The negative sign in the solution simply indicates that the actual voltage  $V_{cd}$  has the opposite polarity of that appearing in Fig. 6.46.

FIG. 6.47 Circuit of Fig. 6.46 redrawn.



**EXAMPLE 6.24** Calculate the current *I* and the voltage *V* for the network of Fig. 6.50.

**Solution:** The 10-k $\Omega$  resistor has been effectively shorted out by the jumper, resulting in the equivalent network of Fig. 6.51. Using Ohm's law,

$$I = \frac{E}{R_1} = \frac{18 \text{ V}}{5 \text{ k}\Omega} = 3.6 \text{ mA}$$

and





**EXAMPLE 6.25** Determine V and I for the network of Fig. 6.52 if the resistor  $R_2$  is shorted out.

**Solution:** The redrawn network appears in Fig. 6.53. The current through the 3- $\Omega$  resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since  $V_{3\Omega} = IR = (0)R = 0$  V, the voltage V is directly across the short, and

$$V = 0 \mathbf{V}$$

 $I = \frac{E}{R_1} = \frac{6 \mathrm{V}}{2 \Omega} = \mathbf{3} \mathrm{A}$ 

with



 $R_3$