Descriptive Statistics

Measures of Symmetry and Peakedness

Descriptive Statistics

- Measures of Central Tendency
- Measures of Location
- Measures of Dispersion
- Measures of Symmetry
- Measures of Peakedness

Skewness

- The term *skewness* refers to the lack of symmetry. The lack of symmetry in a distribution is always determined with reference to a normal or Gaussian distribution. Note that a normal distribution is always symmetrical.

- The skewness may be either positive or negative. When the skewness of a distribution is positive (negative), the distribution is called a positively (negatively) skewed distribution. Absence of skewness makes a distribution symmetrical.

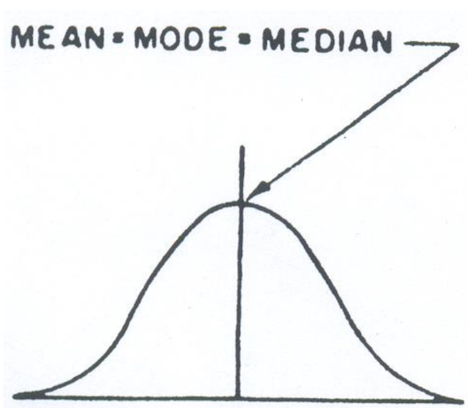
- It is important to emphasize that skewness of a distribution cannot be determined simply my inspection.

Measures of Symmetry

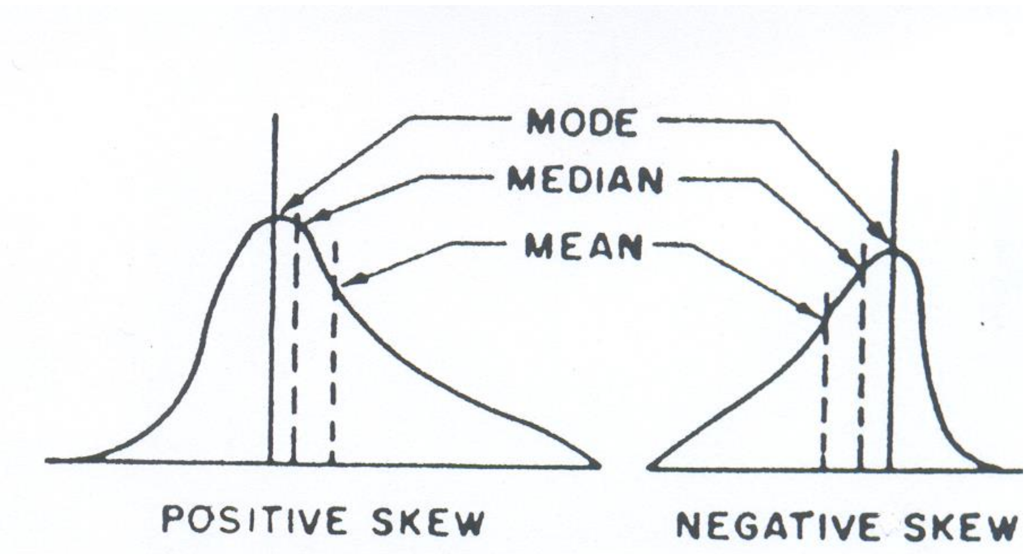
- Skewness
  - Symmetric distribution
  - Positively skewed distribution
  - Negatively skewed distribution

Measures of Skewness

- If Mean > Mode, the skewness is positive.
- If Mean < Mode, the skewness is negative.
- If Mean = Mode, the skewness is zero.



Symmetric



Measures of Symmetry

- Many distribution are not symmetrical.
- They may be tail off to right or to the left and as such said to be skewed.
- One measure of absolute skewness is difference between mean and mode. A measure of such would not be true meaningful because it depends of the units of measurement.
- The simplest measure of skewness is the Pearson’s coefficient of skewness:

$$\text{Pearson's coefficient of skewness} = \frac{\text{Mean - Mode}}{\text{Standard deviation}}$$

Measures of Skewness

$$\text{Pearson's coefficient of skewness} = \frac{\text{Mean - Mode}}{\text{Standard deviation}}$$

$$\text{Pearson's coefficient of skewness} = \frac{3(\text{Mean - Median})}{\text{Standard deviation}}$$

$$\text{Kelley's coefficient of skewness} = \frac{D_9 + D_1 - 2M_e}{D_9 - D_1}$$

$$\text{Bowley's coefficient of skewness} = \frac{(Q_3 + Q_1) - 2M_e}{Q_3 - Q_1}$$

$$\text{Moment based coefficient of skewness} = \frac{\mu_3^2}{\mu_2^3}$$

Central Moments

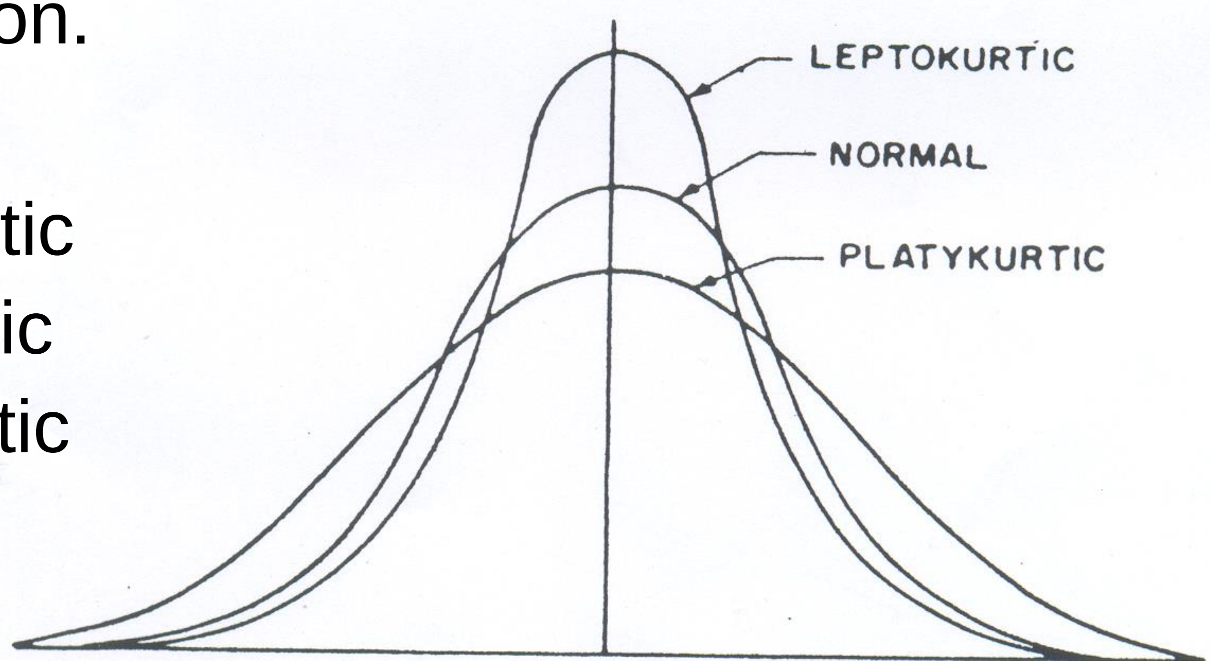
- First Moment $$\mu_1 = \frac{\sum f(x-\bar{x})}{N}$$
- Second Moment $$\mu_2 = \frac{\sum f(x-\bar{x})^2}{N}$$
- Third Moment $$\mu_3 = \frac{\sum f(x-\bar{x})^3}{N}$$
- Fourth Moment $$\mu_4 = \frac{\sum f(x-\bar{x})^4}{N}$$

Exercise-1: Find coefficient of Skewness using Pearson's, Kelly's and Bowley's formula

Income of daily Labour	Frequency (f)	(x)
40-50	3	45
50-60	5	55
60-70	10	65
70-80	8	75
80-90	4	85
90-100	4	95
100-110	1	105
Sum	N=35	

Measures of Peakedness

- **Kurtosis**:
  - is the degree of peakedness of a distribution, usually taken in relation to a normal distribution.
  - Leptokurtic
  - Platykurtic
  - Mesokurtic



Kurtosis

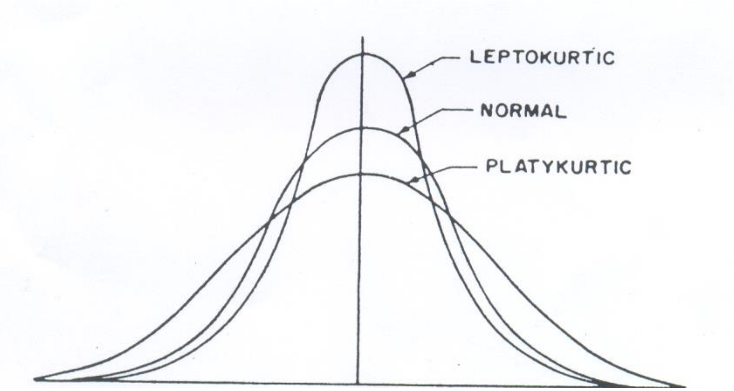
- A curve having relatively higher peak than the normal curve, is known as Leptokurtic.
- On the other hand, if the curve is more flat-topped than the normal curve, it is called Platykurtic.
- A normal curve itself is called Mesokurtic, which is neither too peaked nor too flat-topped.

Measures of Kurtosis

- If $\beta_2 - 3 > 0,$ the distribution is leptokurtic.
- If , $\beta_2 - 3 < 0$ the distribution is platykurtic.
- If , $\beta_2 - 3 = 0$ the distribution is mesokurtic.

The most important measure of kurtosis based on the second and fourth moments is

$$\beta_2 = \frac{\mu_4}{\mu_2{}^2}$$



Co-efficient of Skewness and Kurtosis using Moments

- Co-efficient of Skewness $$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

- Kurtosis $$\beta_2 = \frac{\mu_4}{\mu_2^{\ 2}}$$

Exercise-2: Find coefficient of skewness and Kurtosis using moments

Class	Frequency (f)	x	fx	f(x-μ)	$f(x-\mu)^2$	$f(x-\mu)^3$	$f(x-\mu)^4$
25-30	2						
30-35	8						
35-40	18						
40-45	27						
45-50	25						
50-55	16						
55-60	7						
60-65	2						
Sum	N=35						