King Saud University
Department of Mathematics

151
Second Midterm, December 2014

NAME:

Group Number:

ID:

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5	6	7	8
Answer								

I) Choose the correct answer (write it on the table above):

1) If $J = \{\{1\}, \{2, 3\}\}$ is a partition of the set $A = \{1, 2, 3\}$, then the equivalence relation associated with J is

(A) $\{(1, 2), (1, 3)\}$	(B) $\{((1, 1), (2, 2),$ $(2,\ 3),\ (3,\ 2),\ (3,\ 3)\}$	(C) $\{(1, 1), (1, 2), (1, 3),$ $(2,\ 1),\ (2,\ 2),\ (2,\ 3),$ $(3,\ 1),\ (3,\ 2),\ (3,\ 3)\}$
(D) None of the previous		

2) Let R be the relation defined on $\mathbb{Z}$ by

$$aRb \Longleftrightarrow a - b \geq 0.$$

The relation R is

(A) an equivalence relation	(B) a partial order relation	(C) symmetric	(D) None of the previous

3) Which pair is comparable for the relation R, on $\mathbb{Z}^+$, defined by

$$aRb \Longleftrightarrow a + b \quad \text{is a perfect square?}$$

(An integer number n is called *perfect square* if there exists an integer a, such that $n = a^2$).

(A) $(3, 5)$	(B) $(7, 2)$	(C) $(11, 3)$	(D) None of the previous

4) The partition of $\mathbb{Z}$ corresponding to the relation

$$R = \{(a, b) : a \equiv b \mod 5\}$$

is

(A) $\{\mathbb{N}, \{0\}, \mathbb{Z}^-\}$	(B) $\{[0], [1], [2], [3], [4]\}$	(C) $\{[1], [2], [3], [4]\}$	(D) None of the previous

For the following four questions, consider the relations

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a \geq b\}$$

and

$$R_2 = \{(a, b) \in \mathbb{R}^2, a \leq b\}.$$

5) $R_1 \cap R_2$ is

(A) $\{(a, b) \in \mathbb{R}^2 : a \neq b\}$	(B) $\{(a, b) \in \mathbb{R}^2 : a = b\}$	(C) $\emptyset$	(D) None of the previous

6) $R_1 \cup R_2$ is

(A) $\{(a, b) \in \mathbb{R}^2 : a \neq b\}$	(B) $\{(a, b) \in \mathbb{R}^2 : a = b\}$	(C) $\mathbb{R}^2$	(D) None of the previous

7) $R_1 - R_2$ is

(A) $\{(a, b) \in \mathbb{R}^2 : a > b\}$	(B) $\{(a, b) \in \mathbb{R}^2 : a < b\}$	(C) $\emptyset$	(D) None of the previous

8) R_2^2 is

(A) R_1	(B) R_2	(C) $R_1 \cup R_2$	(D) None of the previous

II) Prove that, for all positive integers $n \geq 4$, $3^n < (n+1)!$, using the first principle of mathematical induction.

III) Let

$$R = \{(1,1),(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$$

be a relation on the set $A = \{1,2,3,4\}$.

a) Represent R using a diagraph;

b) Is R reflexive? Justify the answer;

c) Is R symmetric? Justify the answer;

d) Is R transitive? Justify the answer;

e) Find the reflexive closure, the symmetric closure and the transitive closure of R.

IV) Let R be the relation on $\mathbb{Z}$, defined by

$$aRb \iff a - b \quad \text{is an even number.}$$

a) Prove that R is an equivalence relation;

b) Compute $[0]$ and $[1]$;

c) Find the partition of $\mathbb{Z}$ determined by R.