Revision

Bander Almutairi

King Saud University

22 Oct 2013

Derivatives	Integrals
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(\ln(f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln\lvert f(x)\rvert + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$	$\int f'(x)e^{f(x)} dx = e^{f'(x)}$

Derivatives	Integrals
$\frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{\ln(a)} \cdot \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\|x\| + C$
$\frac{d}{dx}\left(\log_a(f(x))\right) = \frac{1}{\ln(a)} \cdot \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln\|f(x)\| + C$
$\frac{d}{dx}\left(a^x\right) = \ln(a)a^x$	$\int a^x dx = \frac{1}{\ln(a)} a^x + C$
$\frac{d}{dx}\left(a^{f(x)}\right) = \ln(a)f'(x)a^{f(x)}$	$\int f'(x)a^{f(x)} dx = \frac{1}{\ln(a)} a^{f'(x)}$

Derivatives	Integrals
$\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{u'}{\sqrt{1-u^2}}$	$\int \frac{1}{\sqrt{1-u^2}}du = \sin^{-1}(u) + C$
$\frac{d}{dx}\left(\cos^{-1}(u)\right) = \frac{-u'}{\sqrt{1-u^2}}$	$\int \frac{-1}{\sqrt{a^2-u^2}}du = -\sin^{-1}(\frac{u}{a}) + C$
$\frac{d}{dx}\left(\tan^{-1}(u)\right) = \frac{u'}{1+u^2}$	$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$
$\frac{d}{dx}\left(\cot^{-1}(u)\right) = \frac{-u'}{1+u^2}$	$\int \frac{-1}{a^2+u^2}du = -\frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$
$\frac{d}{dx}\left(\sec^{-1}(u)\right) = \frac{u'}{u\sqrt{u^2-1}}$	$\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$
$\frac{d}{dx}\left(\csc^{-1}(u)\right) = \frac{-u'}{u\sqrt{u^2-1}}$	$\int \frac{-1}{u\sqrt{u^2-a^2}}du = -\frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$

$\sinh(x)$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &
\end{aligned}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\coth(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}
\end{aligned}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{sech}(x)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{sech}(x) = \frac{1}{\cosh(x)}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}
\end{aligned}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &
\end{aligned}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &= \frac{1}{\sinh(x)}
\end{aligned}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &= \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}.
\end{aligned}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &= \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}.
\end{aligned}$$

Hyperbolic Trigonometric Formulas:

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &= \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}.
\end{aligned}$$

Hyperbolic Trigonometric Formulas:

$$\begin{aligned}
\cosh^2(x) - \sinh^2(x) &= 1 \\
1 - \tanh^2(x) &= \operatorname{sech}^2(x)
\end{aligned}$$

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\coth(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
\operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\
\operatorname{csch}(x) &= \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}.
\end{aligned}$$

Hyperbolic Trigonometric Formulas:

$$\begin{aligned}
\cosh^2(x) - \sinh^2(x) &= 1 \\
1 - \tanh^2(x) &= \operatorname{sech}^2(x) \\
\coth^2(x) - 1 &= \operatorname{csch}^2(x)
\end{aligned}$$

Derivatives	Integrals
$\frac{d}{dx}(\sinh(u)) = \cosh(u)u'$	$\int \sinh(x)dx = \cosh(x) + c$
$\frac{d}{dx}(\cosh(u)) = \sinh(u)u'$	$\int \cosh(x)dx = \sinh(x) + c$
$\frac{d}{dx}(\tanh(u)) = \operatorname{sech}^2(u)u'$	$\int \operatorname{sech}^2(x)dx = \tanh(x) + c$
$\frac{d}{dx}(\coth(u)) = -\operatorname{csch}^2(u)u'$	$\int \operatorname{csch}^2(x)dx = -\coth(x) + c$
$\frac{d}{dx}(\operatorname{sech}(u)) = -\operatorname{sech}(u)\tanh(u)u'$	$\int \operatorname{sech}(x)\tanh(x)dx = -\operatorname{sech}(x) + c$
$\frac{d}{dx}(\operatorname{csch}(u)) = -\operatorname{csch}(u)\coth(u)u'$	$\int \operatorname{csch}(x)\coth(x)dx = -\operatorname{csch}(x) + c$

$$y = \sinh^{-1}(x) \iff x = \sinh(y)$$

$$y = \sinh^{-1}(x) \iff x = \sinh(y)$$
$$y = \cosh^{-1}(x) \iff x = \cosh(y)$$

$$
\begin{aligned}
y = \sinh^{-1}(x) \quad &\Longleftrightarrow \quad x = \sinh(y) \\
y = \cosh^{-1}(x) \quad &\Longleftrightarrow \quad x = \cosh(y) \\
y = \tanh^{-1}(x) \quad &\Longleftrightarrow \quad x = \tanh(y)
\end{aligned}
$$

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y) \\
y = \cosh^{-1}(x) &\iff x = \cosh(y) \\
y = \tanh^{-1}(x) &\iff x = \tanh(y) \\
y = \coth^{-1}(x) &\iff x = \coth(y)
\end{aligned}
$$

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y) \\
y = \cosh^{-1}(x) &\iff x = \cosh(y) \\
y = \tanh^{-1}(x) &\iff x = \tanh(y) \\
y = \coth^{-1}(x) &\iff x = \coth(y) \\
y = \operatorname{sech}^{-1}(x) &\iff x = \operatorname{sech}(y)
\end{aligned}
$$

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y)\\
y = \cosh^{-1}(x) &\iff x = \cosh(y)\\
y = \tanh^{-1}(x) &\iff x = \tanh(y)\\
y = \coth^{-1}(x) &\iff x = \coth(y)\\
y = \operatorname{sech}^{-1}(x) &\iff x = \operatorname{sech}(y)\\
y = \operatorname{csch}^{-1}(x) &\iff x = \operatorname{csch}(y)
\end{aligned}
$$

$$y = \sinh^{-1}(x) \iff x = \sinh(y)$$
$$y = \cosh^{-1}(x) \iff x = \cosh(y)$$
$$y = \tanh^{-1}(x) \iff x = \tanh(y)$$
$$y = \coth^{-1}(x) \iff x = \coth(y)$$
$$y = \operatorname{sech}^{-1}(x) \iff x = \operatorname{sech}(y)$$
$$y = \operatorname{csch}^{-1}(x) \iff x = \operatorname{csch}(y)$$

Theorem:

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y) \\
y = \cosh^{-1}(x) &\iff x = \cosh(y) \\
y = \tanh^{-1}(x) &\iff x = \tanh(y) \\
y = \coth^{-1}(x) &\iff x = \coth(y) \\
y = \text{sech}^{-1}(x) &\iff x = \text{sech}(y) \\
y = \text{csch}^{-1}(x) &\iff x = \text{csch}(y)
\end{aligned}
$$

Theorem:

- $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$.

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y) \\
y = \cosh^{-1}(x) &\iff x = \cosh(y) \\
y = \tanh^{-1}(x) &\iff x = \tanh(y) \\
y = \coth^{-1}(x) &\iff x = \coth(y) \\
y = \operatorname{sech}^{-1}(x) &\iff x = \operatorname{sech}(y) \\
y = \operatorname{csch}^{-1}(x) &\iff x = \operatorname{csch}(y)
\end{aligned}
$$

Theorem:

- $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$.
- $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1}), x \geq 1$.

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y)\\
y = \cosh^{-1}(x) &\iff x = \cosh(y)\\
y = \tanh^{-1}(x) &\iff x = \tanh(y)\\
y = \coth^{-1}(x) &\iff x = \coth(y)\\
y = \operatorname{sech}^{-1}(x) &\iff x = \operatorname{sech}(y)\\
y = \operatorname{csch}^{-1}(x) &\iff x = \operatorname{csch}(y)
\end{aligned}
$$

Theorem:

- $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$.
- $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1}), x \geq 1$.
- $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), |x| < 1$.

$$
\begin{aligned}
y = \sinh^{-1}(x) &\iff x = \sinh(y) \\
y = \cosh^{-1}(x) &\iff x = \cosh(y) \\
y = \tanh^{-1}(x) &\iff x = \tanh(y) \\
y = \coth^{-1}(x) &\iff x = \coth(y) \\
y = \operatorname{sech}^{-1}(x) &\iff x = \operatorname{sech}(y) \\
y = \operatorname{csch}^{-1}(x) &\iff x = \operatorname{csch}(y)
\end{aligned}
$$

Theorem:

- $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$.
- $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1}), x \geq 1$.
- $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), |x| < 1$.
- $\operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), 0 < x \leq 1$.

Derivatives	Integrals
$\frac{d}{dx}\left(\sinh^{-1}(u)\right) = \frac{u'}{\sqrt{u^2+1}}$	$\int \frac{1}{\sqrt{u^2+a^2}}du = \sinh^{-1}(\frac{u}{a}) + c$
$\frac{d}{dx}\left(\cosh^{-1}(u)\right) = \frac{u'}{\sqrt{u^2-1}}$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}(\frac{u}{a}) + c$
$\frac{d}{dx}\left(\tanh^{-1}(u)\right) = \frac{u'}{1-u^2}$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + c$
$\frac{d}{dx}\left(\coth^{-1}(u)\right) = \frac{-u'}{1-u^2}$	$\int \frac{-1}{a^2-u^2}du = -\frac{1}{a}\tanh^{-1}(\frac{u}{a}) + c$
$\frac{d}{dx}\left(\operatorname{sech}^{-1}(u)\right) = \frac{-u'}{u\sqrt{1-u^2}}$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = \frac{-1}{a}\operatorname{sech}^{-1}(\frac{\lvert u\rvert}{a}) + c$
$\frac{d}{dx}\left(\operatorname{csch}^{-1}(u)\right) = \frac{-u'}{\lvert u\rvert\sqrt{u^2+1}}$	$\int \frac{-1}{\lvert u\rvert\sqrt{u^2+a^2}}du = -\frac{1}{a}\operatorname{csch}^{-1}(\frac{u}{a}) + c$

$$\int uv' = uv - \int vu'$$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$
- $$\cos(2x) = \cos^2(x) - \sin^2(x)$$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$
- $$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 2\cos^2(x) - 1 \end{aligned}$$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$
- $$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 2\cos^2(x) - 1 \\ &= 1 - 2\sin^2(x). \end{aligned}$$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$
- $$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 2\cos^2(x) - 1 \\ &= 1 - 2\sin^2(x). \end{aligned}$$
- $\sec^2(x) = 1 + \tan^2(x).$

In this section, we will discuss integrals of functions that are product of powers of the trigonometric functions, so they are of the form:

$$\int \sin^n(x)dx$$
$$\int \cos^n(x)dx$$
$$\int \sin^n(x)\cos^m(x)dx$$
$$\int \tan^n(x).\sec^m(x)dx.$$

We need some formulas that would help us to find these integrals:

- $\cos^2(x) + \sin^2(x) = 1.$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}.$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}.$
- $$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x)\\ &= 2\cos^2(x) - 1\\ &= 1 - 2\sin^2(x).\end{aligned}$$
- $\sec^2(x) = 1 + \tan^2(x).$

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$,

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int \left(sin^2(x)\right)^n \sin(x)dx, \quad \int \left(\cos^2(x)\right)^n \cos(x)dx.$$

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int (sin^2(x))^n \sin(x)dx, \quad \int (\cos^2(x))^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1 - \cos^2(x)$

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int \left(sin^2(x)\right)^n \sin(x)dx, \quad \int \left(\cos^2(x)\right)^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1-\cos^2(x)$ (for $\cos^2(x)$, we replace by $1-\cos^2(x)$). so the integrals become

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int \left(sin^2(x)\right)^n \sin(x)dx, \quad \int \left(\cos^2(x)\right)^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1-\cos^2(x)$ (for $\cos^2(x)$, we replace by $1-\cos^2(x)$). so the integrals become

$$I_1 = \int \left(1-\cos^2(x)\right)^n \sin(x)dx, \quad I_2 = \int \left(1-\sin^2(x)\right)^n \cos(x)dx.$$

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int \left(sin^2(x)\right)^n \sin(x)dx, \quad \int \left(\cos^2(x)\right)^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1-\cos^2(x)$ (for $\cos^2(x)$, we replace by $1-\cos^2(x)$). so the integrals become

$$I_1 = \int \left(1-\cos^2(x)\right)^n \sin(x)dx, \quad I_2 = \int \left(1-\sin^2(x)\right)^n \cos(x)dx.$$

Finally, we substitute by $u = cos(x)$ for I_1 (so $du = -\sin(x)dx$)

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int \left(sin^2(x)\right)^n \sin(x)dx, \quad \int \left(\cos^2(x)\right)^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1-\cos^2(x)$ (for $\cos^2(x)$, we replace by $1-\cos^2(x)$). so the integrals become

$$I_1 = \int \left(1-\cos^2(x)\right)^n \sin(x)dx, \quad I_2 = \int \left(1-\sin^2(x)\right)^n \cos(x)dx.$$

Finally, we substitute by $u = cos(x)$ for I_1 (so $du = -\sin(x)dx$) and $u = \sin(x)$ for I_2 (so $du = \cos(x)dx$.

Now evaluate $\int \sin^{2n+1}(x)dx$ or $\int \cos^{2n+1}(x)dx$, we write them as follows

$$\int (sin^2(x))^n \sin(x)dx, \quad \int (\cos^2(x))^n \cos(x)dx.$$

The we replace $\sin^2(x)$ by $1-\cos^2(x)$ (for $\cos^2(x)$, we replace by $1-\cos^2(x)$). so the integrals become

$$I_1 = \int (1-\cos^2(x))^n \sin(x)dx, \quad I_2 = \int (1-\sin^2(x))^n \cos(x)dx.$$

Finally, we substitute by $u = cos(x)$ for I_1 (so $du = -\sin(x)dx$) and $u = \sin(x)$ for I_2 (so $du = \cos(x)dx$.

Examples:

$\int \sin^3(x)dx$. [2] $\int \sin^3(x)dx$. [3] $\int \sin^5(x)dx$.

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$,

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

The we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

The we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ (for $\cos^2(x)$, we replace by $\frac{1+\cos(2x)}{2}$). so the integrals become

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

The we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ (for $\cos^2(x)$, we replace by $\frac{1+\cos(2x)}{2}$). so the integrals become

$$\int \left(\frac{1-\cos(2x)}{2}\right)^n dx, \quad \int \left(\frac{1-\cos(2x)}{2}\right)^n dx.$$

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

The we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ (for $\cos^2(x)$, we replace by $\frac{1+\cos(2x)}{2}$). so the integrals become

$$\int \left(\frac{1-\cos(2x)}{2}\right)^n dx, \quad \int \left(\frac{1-\cos(2x)}{2}\right)^n dx.$$

Finally, we expand $\left(\frac{1-\cos(2x)}{2}\right)^n$ (and $\left(\frac{1+\cos(2x)}{2}\right)^n$).

Now to evaluate $\int \sin^{2n}(x)dx$ or $\int \cos^{2n}(x)dx$, we write them as follows

$$\int \left(\sin^2(x)\right)^n dx \quad \int \left(\cos^2(x)\right)^n dx$$

The we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ (for $\cos^2(x)$, we replace by $\frac{1+\cos(2x)}{2}$). so the integrals become

$$\int \left(\frac{1-\cos(2x)}{2}\right)^n dx, \quad \int \left(\frac{1-\cos(2x)}{2}\right)^n dx.$$

Finally, we expand $\left(\frac{1-\cos(2x)}{2}\right)^n$ (and $\left(\frac{1+\cos(2x)}{2}\right)^n$).

Examples:

[1] $\int \sin^4(x)dx$. [2] $\int \sin^6(x)dx$. [3] $\int \cos^6(x)dx$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$,

To evaluate $\int \sin^k(x) \cos^t(x) dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m+1}(x)\cos^{2n}(x)dx$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m+1}(x)\cos^{2n}(x)dx = \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$,

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$,

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n + 1$.

(II) If $k = 2m + 1$ and $t = 2n + 1$, we write the integral as follow:

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m + 1$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m+1}(x)\cos^{2n}(x)dx = \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx$$
$$= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n + 1$.

(II) If $k = 2m + 1$ and $t = 2n + 1$, we write the integral as follow:

$$\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx = \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx &= \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n+1}(x)\sin(x)dx.\end{aligned}$$

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx &= \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n+1}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$,

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx &= \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n+1}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx &= \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n+1}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$. We also, can use the substitution $u = \sin(x)$, How??.

To evaluate $\int \sin^k(x)\cos^t(x)dx$, we consider three cases:

(I) If $k = 2m+1$ and $t = 2n$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n}(x)dx &= \int (\sin^2(x))^m \cos^{2n}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$.
Similarly, if $k = 2m$ and $t = 2n+1$.

(II) If $k = 2m+1$ and $t = 2n+1$, we write the integral as follow:

$$\begin{aligned}\int \sin^{2m+1}(x)\cos^{2n+1}(x)dx &= \int (\sin^2(x))^m \cos^{2n+1}(x)\sin(x)dx \\ &= \int (1-\cos^2(x))^m \cos^{2n+1}(x)\sin(x)dx.\end{aligned}$$

Then we use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$. We also, can use the substitution $u = \sin(x)$, How??.

(III) If $k = 2m$ and $t = 2n$,

(III) If $k = 2m$ and $t = 2n$, we write the integral as follow:

(III) If $k = 2m$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m}(x)\cos^{2n}(x)dx$$

(III) If $k = 2m$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m}(x)\cos^{2n}(x)dx = \int (\sin^2(x))^m(\cos^2(x))^n \sin(x)dx.$$

(III) If $k = 2m$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m}(x)\cos^{2n}(x)dx = \int (\sin^2(x))^m(\cos^2(x))^n \sin(x)dx.$$

Then we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ and we replace $\cos^2(x)$ by $\frac{1+\cos(2x)}{2}$,

(III) If $k = 2m$ and $t = 2n$, we write the integral as follow:

$$\int \sin^{2m}(x)\cos^{2n}(x)dx = \int (\sin^2(x))^m(\cos^2(x))^n \sin(x)dx.$$

Then we replace $\sin^2(x)$ by $\frac{1-\cos(2x)}{2}$ and we replace $\cos^2(x)$ by $\frac{1+\cos(2x)}{2}$, then we simplify the brackets.

To evaluate the integrals $\int \tan^n(x)dx$ and $\int \sec^n(x)dx$, To evaluate the integrals $\int \tan^n(x)dx$ and $\int \sec^n(x)dx$, we use the relation:

$$\sec^2(x) = \tan^2(x) + 1.$$

To evaluate the integrals $\int \tan^n(x)dx$ and $\int \sec^n(x)dx$, we use the relation:

$$\sec^2(x) = \tan^2(x) + 1.$$

Example: evaluate the following:

(1) $\int \tan(x)dx$. (2) $\int \tan^2(x)dx$.
(3) $\int \tan^3(x)dx$. (4) $\int \tan^4(x)dx$.
(5) $\int \sec(x)dx$. (6) $\int \sec^2(x)dx$.
(7) $\int \sec^3(x)dx$. (8) $\sec^4(x)dx$.

Now to evaluate integration of the form:

$$\int \tan^n(x) \sec^m(x) dx.$$

We use the following substitution:

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd,

Now to evaluate integration of the form:

$$\int \tan^n(x) \sec^m(x) dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x) \tan(x) dx$.

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.

(b) If m is even,

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.

(b) If m is even, we put $u = \tan(x)$, so $du = \sec^2(x)dx$.

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.

(b) If m is even, we put $u = \tan(x)$, so $du = \sec^2(x)dx$.

(c) If n is odd and m is even we use the substitution in (a) or (b).

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.
(b) If m is even, we put $u = \tan(x)$, so $du = \sec^2(x)dx$.
(c) If n is odd and m is even we use the substitution in (a) or (b).
(d) If $n = 2k$ is even and $m = 2t + 1$ is odd,

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.

(b) If m is even, we put $u = \tan(x)$, so $du = \sec^2(x)dx$.

(c) If n is odd and m is even we use the substitution in (a) or (b).

(d) If $n = 2k$ is even and $m = 2t + 1$ is odd, then we write the integral as follow:

$$\int \tan^{2k}(x)\sec^{2t+1}(x)dx = \int (\tan^2(x))^k \sec^{2t+1}(x)dx,$$

Now to evaluate integration of the form:

$$\int \tan^n(x)\sec^m(x)dx.$$

We use the following substitution:

(a) If n is odd, we put $u = \sec(x)$, so $du = \sec(x)\tan(x)dx$.

(b) If m is even, we put $u = \tan(x)$, so $du = \sec^2(x)dx$.

(c) If n is odd and m is even we use the substitution in (a) or (b).

(d) If $n = 2k$ is even and $m = 2t + 1$ is odd, then we write the integral as follow:

$$\int \tan^{2k}(x)\sec^{2t+1}(x)dx = \int (\tan^2(x))^k \sec^{2t+1}(x)dx,$$

then we replace $\tan^2(x)$ by $\sec^2(x) - 1$.