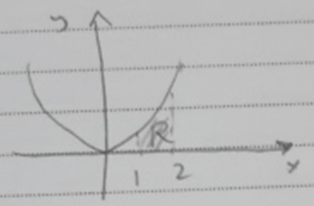
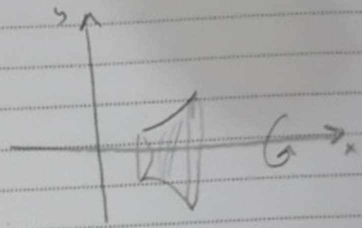
لا يكتب في
هذا الهامش

* The volume of revolution of the region bounded by the curves about the x-axis $y = x^2$, $y=0$, $x=1$, $x=2$





$$V = \pi \int_1^2 \left(x^2 \right)^2 dx$$

$$= \pi \int_1^2 x^4 \, dx = \frac{\pi}{5} \left[x^5 \right]_1^2 = \frac{\pi}{5} \left[32 - 1 \right] = \frac{31\pi}{5}$$

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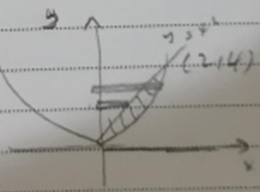
* The volume of revolution of the region bounded by the curves $y = x^2$, $y = 2x$ about the y-axis

Solution:

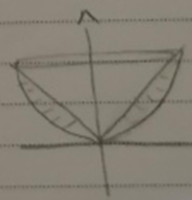


Intersection points:

$x^2 = 2x \Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$x = 0$	$x = 2$
$(0,0)$	$(2,4)$



$y = x^2 \Rightarrow x = \sqrt{y} = f(y)$

$y = 2x \Rightarrow x = \frac{y}{2} = g(y)$

$$V = \pi \int_0^4 \left[(\sqrt{y})^2 - \left(\frac{y}{2}\right)^2 \right] dy = \pi \int_0^4 \left(y - \frac{y^2}{4} \right) dy$$

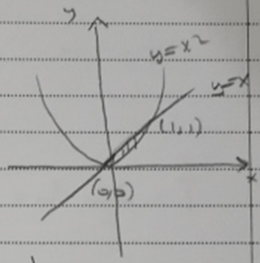
$$= \pi \left[\frac{y^2}{2} - \frac{y^3}{12} \right]_0^4 = \pi \left[\frac{16}{2} - \frac{64}{12} \right]$$

$$= \frac{32\pi}{\ldots}$$

لا يكتب في هذا الهامش

* The area of the region bounded by the curves

$y = x^2$, $y = x$



Solution:

Intersection points:

$y_1 = y_2$

$x^2 = x \Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$x = 0$, $x = 1$

$(0,0)$, $(1,1)$

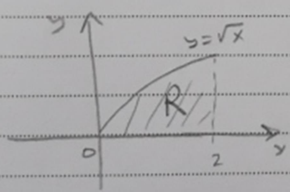
$$A = \int_0^1 (x - x^2)\, dx$$

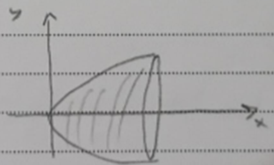
$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

* The volume of revolution of the region bounded by the curves about the x-axis

$y = \sqrt{x}$, $y = 0$, $x = 0$, $x = 2$





$$V = \pi \int_0^2 (\sqrt{x})^2\, dx = \pi \int_0^2 x\, dx$$

$$= \frac{\pi}{2}\left[x^2\right]_0^2 = \frac{4\pi}{2} = 2\pi$$

* $\int x \ln x \, dx$

$u = \ln x \qquad\qquad du = \frac{1}{x}\, dx$

$\Rightarrow$

$dv = x \qquad\qquad v = \int x \, dx = \frac{x^2}{2}$

$I = \frac{x^2}{2} \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + c$

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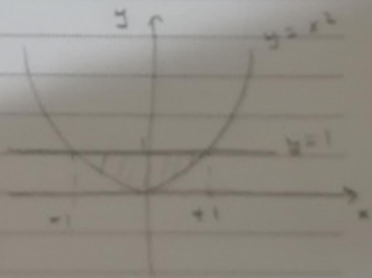
* The area of the region bounded by the curves $y = x^2$, $y = 1$, $x = -1$, $x = 1$.

Solution:

Intersection points



$y_1 = y_2$

$x^2 = 1$

$x = \pm 1$

$A = \int_{-1}^{1} (y_2 - y_1) \, dx$

$= \int_{-1}^{1} (1 - x^2) \, dx = \left[x - \frac{x^3}{3} \right]_{-1}^{1}$

$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$

$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

$\int 5x\, e^x\, dx$

$u = 5x \qquad\qquad du = 5\, dx$

$\Rightarrow$

$dv = e^x\, dx \qquad\qquad v = \int e^x\, dx = e^x$

$I = 5x\, e^x - 5\int e^x\, dx$

$= 5x\, e^x - 5e^x + c$

$\int 3x \sin 2x\, dx$

$du = 3\, dx$

$u = 3x \quad \Rightarrow \quad v = \frac{1}{2}\int 2 \sin 2x\, dx = -\frac{1}{2}\cos 2x$

$dv = \sin 2x\, dx$

$I = -\frac{3x}{2}\cos 2x + \frac{1}{2}\,\frac{1}{2}\int 2\cos 2x\, dx$

$= -\frac{3}{2}x \cos 2x + \frac{1}{4}\sin 2x + c$

① $\int e^x \sqrt{e^x+1}\, dx$

Solution. $\int e^x (e^x+1)^{\frac{1}{2}} dx \qquad u = e^x+1 \Rightarrow du = e^x dx$

$$\int \cancel{e^x}\, u^{\frac{1}{2}} \frac{du}{\cancel{e^x}} \qquad \Rightarrow \frac{du}{e^x} = dx$$

$$\int u^{\frac{1}{2}} du = \frac{u^{3/2}}{3/2} + c = \frac{2}{3}(e^x+1)^{3/2} + c$$

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② $\int \frac{\cos x}{\sqrt{1+\sin x}}\, dx$

Solution: $\int \cos x\,(1+\sin x)^{-\frac{1}{2}} dx \qquad u = 1+\sin x$

$du = \cos x\, dx$

$$\int \cancel{\cos x}\, u^{-\frac{1}{2}} \frac{du}{\cancel{\cos x}} \qquad \frac{du}{\cos x} = dx$$

$$\int u^{-\frac{1}{2}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{1}(1+\sin x)^{\frac{1}{2}} + c$$

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③ $\int \frac{1}{x\sqrt{1+\ln x}}\, dx$

olution: $\int \frac{1}{x}(1+\ln x)^{-\frac{1}{2}} dx \qquad u = 1+\ln x \Rightarrow du = \frac{1}{x} dx$

$x\, du = dx$

$$\int \frac{1}{\cancel{x}}\, u^{-\frac{1}{2}}\, \cancel{x}\, du$$

$$\int u^{-\frac{1}{2}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{1}(1+\ln x)^{\frac{1}{2}} + c$$