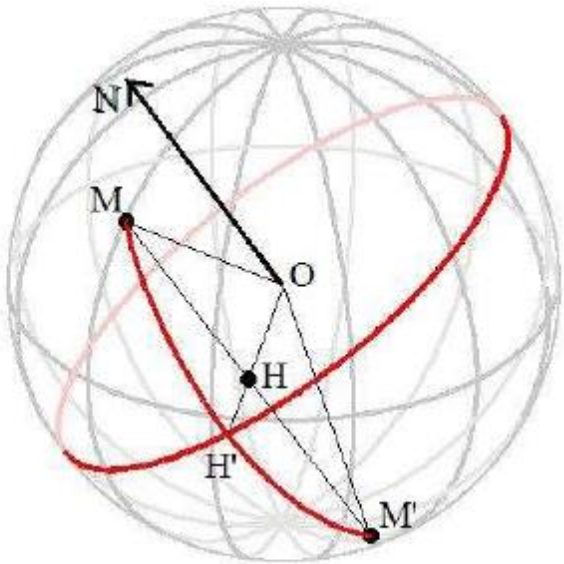
## Reflection on $S^2$ about great circle




$$OM' = OM - 2\langle OM | N \rangle N$$

## Rotation about axis passing through the center O with an angle $\varphi$ on the sphere $S^2$

We define a **rotation** $Rot$ by a unit vector $N(a,b,c)$ of its axis and an angle $\varphi$. So the matrix of the rotation is $A_\varphi = (1-\cos\varphi)\begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} + \cos\varphi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin\varphi \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$

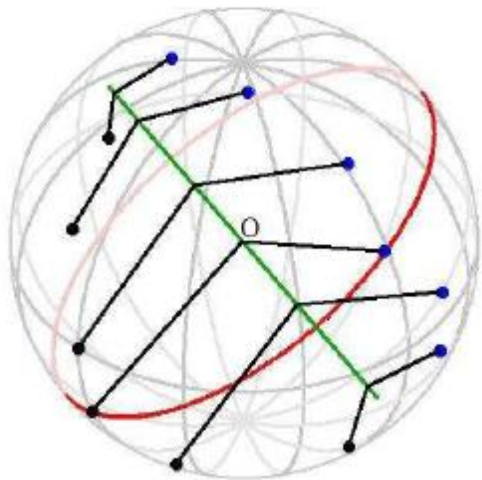
$$Rot(M(x,y,z)) = M'(x',y',z') \Leftrightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = A_\varphi \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Prove:

$\overrightarrow{OM} = \overrightarrow{OM}_{\parallel} + \overrightarrow{OM}_{\perp} = \left(\langle \overrightarrow{OM} | N \rangle N\right) + \left(\overrightarrow{OM} - \langle \overrightarrow{OM} | N \rangle N\right)$ then

$\overrightarrow{OM}' = \overrightarrow{OM}_{\parallel} + \cos\varphi \ \overrightarrow{OM}_{\perp} + \sin\varphi \ \left(N \times \overrightarrow{OM}_{\perp}\right)$. So

$\overrightarrow{OM}' = (1-\cos\varphi).\langle \overrightarrow{OM} | N \rangle N + \cos\varphi.\overrightarrow{OM} + \sin\varphi.\left(N \times \overrightarrow{OM}\right)$.




Rotation about green axis with an angle 100°

**Evolution of equilateral spherical triangle**