Random Variables, Distributions and Expectations

Discrete Distributions : Q1 $\frac{\text{العدد}}{\text{العدد الكلي}}$ ذكر في رأس السؤال أن القطعة العملة متزنة أي أن: $P(H) = P(T)$ ولم يمكن تضمينه تعريف المجال:

$\boxed{a}$ $\boxed{b}$ $S = \{H,T\} \times \{H,T\} \times \{H,T\} = \{\overset{3-0=3}{HHH}, \overset{2-1=1}{HHT}, \overset{2-1=1}{HTH}, \overset{2-1=1}{THH}, \overset{1-2=-1}{TTH}, \overset{1-2=-1}{THT}, \overset{1-2=-1}{HTT}, \overset{0-3=-3}{TTT}\}$

X = number of heads − number of tails , $n(S) = 2 \times 2 \times 2 = 8$

$= -3, -1, 1, 3$

$\boxed{c}$

$P(X=-3) = P(\{TTT\}) = \frac{1}{8}$

$P(X=-1) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X=1) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X=3) = P(\{HHH\}) = \frac{1}{8}$

$$F(x) = P(X \le x) = \begin{cases} 0 , & x < -3 \\ 1/8 , & -3 \le x < -1 \\ 4/8 , & -1 \le x < 1 \\ 7/8 , & 1 \le x < 3 \\ 1 , & 3 \le x \end{cases}$$

$\therefore$

X	-3	-1	1	3	Σ
$P(X=x) = f(x)$	$1/8$	$3/8$	$3/8$	$1/8$	1
$P(X \le x) = F(x)$	$1/8$	$4/8$	$7/8$	1	
$x f(x)$	$-3/8$	$-3/8$	$3/8$	$3/8$	$E(X) = \Sigma x f(x) = 0 = \mu$
$x^2 f(x)$	$9/8$	$3/8$	$3/8$	$9/8$	$\Sigma x^2 f(x) = E(X^2) = 3$

$\boxed{d}$

$P(X \le 1) = f(1) + f(-1) + f(-3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

$F(x)$: $F(x) = F(1) = \frac{7}{8}$

$\boxed{e}$

$P(X < 1) = f(-1) + f(-3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

$F(x)$: $P(X < 1) = P(X \le 0) = F(0) = \frac{4}{8} = \frac{1}{2}$

$\boxed{f}$

$E(X) = \mu = 0$

$\boxed{g}$ $Var(X) = \sigma^2 = E(X^2) - [E(X)]^2 = 3 - 0 = 3$

Q2

$\boxed{1}\boxed{2}$ $S = \{M,F\} \times \{M,F\} = \{\overset{0}{MM}, \overset{1}{MF}, \overset{1}{FM}, \overset{2}{FF}\}$, $P(F) = .2$, $P(M) = 1 - .2 = .8$

X = number of females in the committee.

$= 0, 1, 2$

$\boxed{3}$ $P(X=0) = P(\{MM\}) = P(\{M\})P(\{M\}) = (.8)(.8) = .64$

$P(X=1) = P(\{MF, FM\}) = (.8)(.2)(.2) = .32$

$P(X=2) = P(\{FF\}) = (.2)(.2) = .04$

$F(x) = P(X \le x) = \begin{cases} 0 & \longrightarrow x < 0 \\ .64 & \longrightarrow 0 \le x < 1 \\ .96 & \longrightarrow 1 \le x < 2 \\ 1 & \longrightarrow 2 \le x \end{cases}$

$\therefore$

x	0	1	2	Σ
$P(X=x)=f(x)$	.64	.32	.04	1
$P(X \le x) = F(x)$	.64	.96	1	
$x\,f(x)$	0	.32	.08	$E(X) = \Sigma x f(x) = .4$
$x^2 f(x)$	0	.32	.16	$E(X^2) = \Sigma x^2 f(x) = .48$

$\boxed{4}$ P(at least one female in the committee) $\longrightarrow$ $F(x)$: $1 - P(X < 1) = 1 - P(X \le 0) = 1 - .64 = .36$

$= P(X \ge 1) = P(X=1) + P(X=2) = f(1) + f(2) = .32 + .04 = .36$

or $1 - P(X < 1) = 1 - \{P(X=0)\} = 1 - .64 = .36$

$\boxed{5}$ P(at most one female in the committee) $\longrightarrow$ $F(x)$: $F(1) = .96$

$= P(X \le 1) = P(X=0) + P(X=1) = .64 + .32 = .96$

$\boxed{6}$ $\mu = E(X) = \Sigma x f(x) = .4$

$\boxed{7}$ $Var(X) = \sigma^2 = E(X^2) - [E(X)]^2 = .48 - (.4)^2 = .48 - (.16) = .32$

Q3

$\boxed{i}\boxed{ii}$ 40 of 5, 60 of 10, total: 100

2 card with replacement (indep.), $S = \{5,10\} \times \{5,10\} = \{\overset{10}{(5,5)}, \overset{15}{(5,10)}, \overset{15}{(10,5)}, \overset{20}{(10,10)}\}$

$P(5) = \frac{40}{100} = .4$, $P(10) = \frac{60}{100} = .6$

X = the total sum of the two cards.

$= 10, 15, 20$

$\boxed{iii}$ $P(X=10) = P(\{(5,5)\}) = (.4)(.4) = .16$

$P(X=15) = P(\{(5,10),(10,5)\}) = P(\{(5,10)\}) + P(\{(10,5)\})$

$= (.4)(.6) + (.4)(.6) = (2)(.4)(.6) = .48$

$P(X=20) = P(\{(10,10)\}) = (.6)(.6) = .36$

$\therefore$

x	10	15	20	Σ
$P(X=x)=f(x)$	.16	.48	.36	1
$x\,f(x)$	1.6	7.2	7.2	$E(X) = 16$
$x^2 f(x)$	16	108	144	$E(X^2) = 268$

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$\boxed{iv}$ $p(X=0)=0$

$\boxed{v}$ $p(X>10) = p(X=15) + p(X=20) = .48 + .36 = .84$

$\boxed{vi}$ $\mu = E(X) = 16$

$\boxed{vii}$ $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = 268 - (16)^2 = 12$

Q4

$\boxed{1}$ $\mu = E(X) = (-3)(.1) + (6)(.5) + (4)(.4) = 6.3$

$\boxed{2}$ $E(X^2) = (-3)^2(.1) + (6)^2(.5) + (4)^2(.4) = 51.3$

$\boxed{3}$ $Var(X) = \sigma_X^2 = E(X^2) - [E(X)]^2 = (51.3) - (6.3)^2 = 11.61$

$\boxed{4}$ $\mu_{2X+1} = E(2X+1) = E(2X) + E(1) = 2E(X) + 1 = 2(6.3) + 1 = 13.6$

$\boxed{5}$

$\sigma^2_{2X+1} = Var(2X+1) = Var(2X) + Var(1) = 4\,Var(X) + 0 = 4(11.61) = 46.44$

Q5

$\boxed{A}$ $f(x) = \frac{x+1}{10} \Rightarrow f(0) = \frac{1}{10} < 1$

$f(1) = \frac{2}{10} < 1$

$f(2) = \frac{3}{10} < 1$

$f(3) = \frac{4}{10} < 1$

$f(4) = \frac{5}{10} < 1$

and $\Sigma f(x) = \frac{1+2+3+4+5}{10} = \frac{15}{10} \neq 1$

$\therefore$ it is not probability

$\boxed{B}$ $f(x) = \frac{x-1}{10} \Rightarrow f(0) = \frac{-1}{5} < 0$

$\therefore$ it is not probability

$\boxed{C}$ $f(x) = \frac{1}{5} \Rightarrow f(0) = f(1) = f(2) = f(3) = f(4) = \frac{1}{5} < 1$

and $\Sigma f(x) = \frac{1+1+1+1+1}{5} = \frac{5}{5} = 1$

$\therefore$ it is probability

6 $\boxed{1}$ $E(X) = \mu = \Sigma x f(x) = (0)(\frac{1}{3}) + 1(\frac{1}{3}) + 2(\frac{1}{3}) = \frac{1+2}{3} = \frac{3}{3} = 1$

$\boxed{2}$ $E(X^2) = \Sigma x^2 f(x) = (0)^2(\frac{1}{3}) + (1)^2(\frac{1}{3}) + (2)^2(\frac{1}{3}) = \frac{1+4}{3} = \frac{5}{3} = 1.67$

$\therefore Var(X) = E(X^2) - [E(X)]^2 = 1.67 - 1 = 0.67$

Q6 $f(x) = kx, \ x = 1, 2, 3$

$\boxed{1}$ we know that $\Sigma f(x) = 1 \Rightarrow k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

$\therefore f(x) = \frac{x}{6}, \ x = 1, 2, 3$

$\boxed{2}$ $F(1) = P(X \le 1) = P(X=1) = f(1) = \frac{1}{6}$

$F(2) = P(X \le 2) = P(X=2) + P(X=1) = \frac{3}{6}$

$F(3) = P(X \le 3) = P(X=3) + P(X=2) + P(X=1) = 1$

x	1	2	3
$f(x)$	1/6	2/6	3/6
$F(x)$	1/6	3/6	1

3

$$\therefore F(x) = P(X \le x) = \begin{cases} 0 & x < 1 \\ 1/6 & 1 \le x < 2 \\ 3/6 = 1/2 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

$\boxed{3}$ $P(.5 < X \le 2.5) = P(X \le 2.5) - P(X \le .5) = F(2.5) - F(.5)$

$= \frac{1}{2} - 0 = \frac{1}{2}$

or by $f(x)$: $P(.5 < X \le 2.5) = P(X=1) + P(X=2)$

$= \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

Q7

$\boxed{1}$ we have $F(x) = \begin{cases} 0 & , x < 0 \\ .25 & , 0 \le x < 1 \\ .6 & , 1 \le x < 2 \\ 1 & , 2 \le x \end{cases}$

$f(0) = .25 - 0 = .25$

$f(1) = .6 - .25 = .35$

$f(2) = 1 - .6 = .4$

$$\therefore f(x) = \begin{cases} .25 & , x = 0 \\ .35 & , x = 1 \\ .4 & , x = 2 \\ 0 & , \text{other ways} \end{cases}$$

$\boxed{2}$

$P(1 \le X < 2) = \begin{cases} P(X=1) = .35 = f(1) & , \text{by used } f(x) \\ F(2^-1) - F(1^-1) = F(1) - F(0) = .6 - .25 = .35 & , \text{by used } F(x) \end{cases}$

$\boxed{3}$ $P(X > 2) = 1 - P(X \le 2) = \begin{cases} 1 - (f(0) + f(1) + f(2)) = 0 & , \text{by used } f(x) \\ 1 - F(2) = 1 - 1 = 0 & , \text{by used } F(x) \end{cases}$

أو على طول من جدول $f(x)$

$= 0$

Q8 we know that $\sum f(x) = 1$

$\Rightarrow (.4) + c + (.3) + (.1) = 1 \Rightarrow .8 + c = 1 \Rightarrow c = 1 - .8 = .2$

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$$Var(X) = E(X^2) - [E(X)]^2 = .6$$

$$\sum_{i=1}^{3} x_i^2 f(x_i) = 4.6 \qquad \sum_{i=1}^{3} x_i f(x_i) = 2$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 1.6$$

$$\sum_{i=1}^{5} y_i^2 f(y_i) = 5.6 \qquad \sum_{i=1}^{5} y_i f(y_i) = 2$$

So, $Var(X) = .6 < Var(Y) = 1.6$

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Continuous distributions:

Q1 $P(X=16)=0$ (because X is Continuous random)

$$f(x)=\begin{cases}\frac{3}{2}\sqrt{x}, & 0<x<1\\ 0, & \text{o.w}\end{cases}$$

Q2 $\boxed{1}$ we know $\int_{-\infty}^{\infty} f(x)\,dx=1$

$$\Rightarrow \int_0^1 k\sqrt{x}\,dx=1 \Rightarrow k\left[\frac{x^{3/2}}{3/2}\right]_0^1=1 \Rightarrow \frac{2}{3}k=1 \Rightarrow k=\frac{3}{2}=1.5$$

$$F(x)=\begin{cases}0, & x<0\\ \int_0^x f(t)\,dt=\int_0^x \frac{3}{2}\sqrt{t}\,dt=x^{\frac{3}{2}}, & 0\le x<1\\ 1, & 1\le x\end{cases}$$

$\boxed{2}$ $P(.3<X\le .6)\overset{f(x)}{=}\int_{.3}^{.6} 1.5\sqrt{x}\,dx=1.5\left[\frac{x^{3/2}}{3/2}\right]_{.3}^{.6}=.3004$

$\overset{F(x)}{=} F(.6)-F(.3)=.6^{3/2}-.3^{3/2}$

$\boxed{3}$ $E(X)=\int_0^1 x\,f(x)\,dx=\int_0^1 x(1.5\sqrt{x})\,dx=1.5\int_0^1 x^{3/2}\,dx=1.5\left[\frac{x^{5/2}}{5/2}\right]_0^1=.6$

Q3 $\boxed{1}$ $P(0<X<2)=F(2)-F(0)=\frac{2}{2+1}-\frac{0}{0+1}=\frac{2}{3}=.667$

$f(x)$ [illegible]

$\boxed{2}$ $P(X\le k)=.5 \Rightarrow F(k)=.5 \Rightarrow \frac{k}{k+1}=.5 \Rightarrow k=.5k+.5 \Rightarrow .5k=.5 \Rightarrow k=1$

if $f(x)=F'(x)=\frac{(x+1)-x}{(x+1)^2}=\frac{1}{(1+x)^2}$, $0\le x$. Then,

$$P(0<X<2)=\int_0^2 \frac{1}{(1+x)^2}\,dx=\int_1^3 \frac{1}{u^2}\,du$$

$f(x)$ [illegible]

$u=1+x$
$du=dx$
$x=0 \Rightarrow u=1$
$x=2 \Rightarrow u=3$

$$=\int_1^3 u^{-2}\,du=\frac{1}{-1}\,u^{-2+1}\Big|_1^3$$

$$=-\frac{1}{u}\Big|_1^3=-\left[\frac{1}{3}-\frac{1}{1}\right]=-\left[\frac{1-3}{3}\right]=-\left[\frac{-2}{3}\right]=\frac{2}{3}$$

$P(X\le k)=.5 \Leftrightarrow \int_0^k f(x)\,dx=.5 \Leftrightarrow \int_0^k \frac{1}{(1+x^2)}\,dx=.5 \Leftrightarrow \frac{k}{k+1}=.5 \Leftrightarrow k=1$

أبرز القوانين

① $\int_a^b e^{cx}\,dx = \frac{1}{c} e^{cx}\Big|_a^b \quad , c\in\mathbb{R}$

② $\int_a^b x^n e^{kx}\,dx = \left[\frac{x^n}{k} - \frac{\frac{d}{dx}x^n}{k^2} + \frac{\frac{d^2}{dx^2}x^n}{k^3} - \cdots\right.$

$\left. \cdots + (-1)^n \frac{\frac{d^n}{dx^n}x^n}{k^{n+1}}\right] e^{kx}\Big|_a^b$

$\int_a^b x^n e^{-kx}\,dx = -\left[\frac{x^n}{k} + \frac{\frac{d}{dx}x^n}{k^2} + \frac{\frac{d^2}{dx^2}x^n}{k^3} + \cdots + \frac{\frac{d^n}{dx^n}x^n}{k^{n+1}}\right] e^{-kx}\Big|_a^b$

whae $n\in\mathbb{N}$, $k\in\mathbb{R}^+$ and $a,b\in\mathbb{R}$ which can a take $-\infty$ and b can also take ∞.

③ $\int_0^\infty x^{n-1} e^{-kx}\,dx = \frac{\Gamma n}{k^n}$, n and $k\in\mathbb{R}^+$

where $\Gamma n = (n-1)!$ if $n\in\mathbb{N}$

④ $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2}\,dx = 1$

⑤ $\Gamma\frac{1}{2} = \sqrt{\pi}$ and $\Gamma\frac{3}{2} = \frac{1}{2}\sqrt{\pi}$

ⓐ $f_1(x) = c(2x - x^3)$, $0 < x < \frac{5}{2}$

we have that $0 < 2x < 5$ and $-\frac{5^3}{2^3} < -x^3 < 0$

so, $-\frac{5^3}{2^3} < 2x - x^3 < 5$

we can see that it is impossible to multiply by c to make

$f_1(x) \geq 0$ on $0 < x < \frac{5}{2}$

ⓒ $f_3(x) = c(2x^2 - 4x)$, $0 < x < 3$

we have that $0 < 2x^2 < 18$ and $-12 < -4x < 0$

so, $-12 < 2x^2 - 4x < 8$

we can see that it is impossible to multiply by c to make

$f_3(x) \geq 0$ on $0 < x < 3$

Q5

(a) $\int_{-1}^{1} f(x)\,dx = 1 \Leftrightarrow \int_{-1}^{1} c(1-x^2)\,dx = 1$

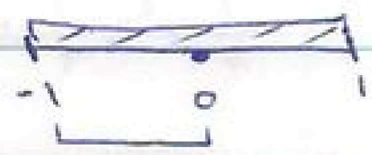
$\Leftrightarrow c\left[x - \frac{x^3}{3}\right]\Big|_{-1}^{1} = c\left[(1-\frac{1}{3}) - (-1+\frac{1}{3})\right] = 1 \Leftrightarrow c\left[\frac{2}{3}+\frac{2}{3}\right] = \frac{4}{3}c = 1$

$\Leftrightarrow c = \frac{3}{4}$

$$\therefore f(x) = \begin{cases} \frac{3}{4}(1-x^2) & , -1<x<1 \\ 0 & , \text{o.w} \end{cases}$$

(b)

(i) $P(X<0) = \frac{3}{4}\int_{-1}^{0}(1-x^2)\,dx$

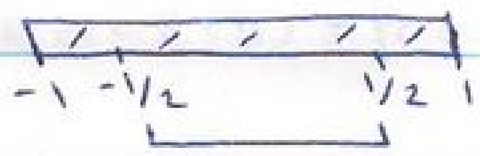


$= \frac{3}{4}\left[x - \frac{x^3}{3}\right]\Big|_{-1}^{0} = \frac{3}{4}\left[0 - (-1+\frac{1}{3})\right] = \frac{1}{2}$

(ii) $P(X \geq \frac{1}{2}) = \frac{3}{4}\int_{1/2}^{1}(1-x^2)\,dx$

$= \frac{3}{4}\left[x - \frac{x^3}{3}\right]\Big|_{1/2}^{1} = \frac{3}{4}\left[(1-\frac{1}{3}) - (\frac{1}{2}-\frac{1}{24})\right] = \frac{5}{32}$

(iii) $P(-\frac{1}{2} \leq x < \frac{1}{2}) = \frac{3}{4}\left[x - \frac{x^3}{3}\right]\Big|_{-1/2}^{1/2}$



$= \frac{3}{4}\left[(\frac{1}{2}-\frac{1}{24}) - (\frac{-1}{2}+\frac{1}{24})\right] = \frac{11}{16}$

(d)

$$F(x) = P(X \leq x) = \begin{cases} 0 & , x<-1 \\ \int_{-1}^{x}\frac{3}{4}(1-t^2)\,dt = \frac{3}{4}(t - \frac{1}{3}t^3)\Big|_{-1}^{x} = \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2} & , -1 \leq x < 1 \\ 1 & , +1 \leq x \end{cases}$$

(e) $P(X<0) = F(0) = \frac{3}{4}(0) - \frac{1}{4}(0)^3 + \frac{1}{2} = \frac{1}{2}$

$P(X \geq \frac{-1}{2}) = 1 - P(X < -\frac{1}{2}) = 1 - F(-\frac{1}{2}) = 1 - \left[\frac{3}{4}(\frac{-1}{2}) - \frac{1}{4}(\frac{-1}{2})^3 + \frac{1}{2}\right] = 1 - \frac{5}{32} = \frac{27}{32}$

$P(-1/2 < X \leq 1/2) = F(\frac{1}{2}) - F(-\frac{1}{2}) = \left[\frac{3}{4}(\frac{1}{2}) - \frac{1}{4}(\frac{1}{2})^3 + \frac{1}{2}\right] - \frac{5}{32} = \frac{27}{32} - \frac{5}{32} = \frac{22}{32} = \frac{11}{16}$

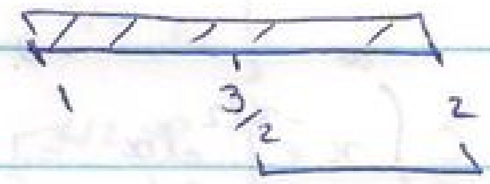
$P(X>1) = 1 - P(X<1) = 1 - F(1) = 1 - 1 = 0$

Q 6

ⓐ $1 = \int_1^2 cx^2\,dx = c\left(\frac{1}{3}x^3\Big|_1^2\right) = c\left(\frac{8}{3} - \frac{1}{3}\right) = \frac{7}{3}c \Rightarrow c = \frac{3}{7}$

$$\therefore f(x) = \begin{cases} \frac{3}{7}x^2 , & 1 < x < 2 \\ 0 , & \text{o.w} \end{cases}$$

ⓑ $P(X > \frac{3}{2}) = \int_{3/2}^{2} \frac{3}{7}x^2\,dx = \frac{1}{7}x^3\Big|_{3/2}^{2} = \frac{8}{7} - \frac{27}{56} = \frac{37}{56}$

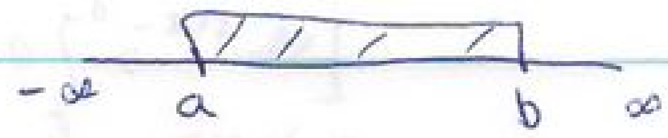


ⓒ $$F(x) = P(X \le x) = \begin{cases} 0 , & x < 1 \\ \int_1^x \frac{3}{7}t^2\,dt = \frac{1}{7}t^3\Big|_1^x = \frac{1}{7}x^3 - \frac{1}{7} , & 1 \le x < 2 \\ 1 , & 2 \le x \end{cases}$$

ⓓ $P(X > \frac{3}{2}) = 1 - P(X < \frac{3}{2}) = 1 - F(\frac{3}{2}) = 1 - (\frac{1}{7}\cdot\frac{27}{8} - \frac{1}{7}) = \frac{37}{56}$

Q 7

$$P(a \le X \le b) = \int_a^b f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx = F(b) - F(a)$$



where $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$ and we know that

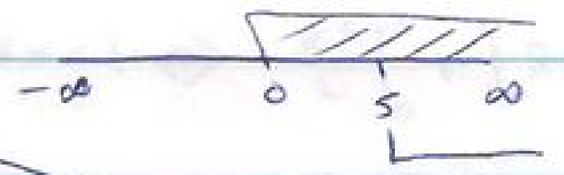
$$\int_a^b f(x)\,dx = P(a < X < b) = P(a \le X < b) = P(a < X \le b) = P(a \le X \le b).$$

Q 8

$$P(X \ge c) = \begin{cases} 1 - P(X \le c) = 1 - F(c) \\ \int_c^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} f(x)\,dx - \int_{-\infty}^{c} f(x)\,dx = 1 - P(X < c) = 1 - F(c) \end{cases}$$

Q9

(a) $P(X \geq 5) = C\int_5^{\infty} x e^{-x/2}\, dx$



بالقانون or بالتجزئ

$= -C\left[\frac{x}{1/2} + \frac{1}{(1/2)^2}\right] e^{-x/2}\Big|_5^{\infty}$

$= -C[2x+4]\, e^{-x/2}\Big|_5^{\infty}$

$= -C\left[2xe^{-x/2} + 4e^{-x/2}\right]\Big|_5^{\infty}$

$= 0 - (-C)\left[10e^{-5/2} + 4e^{-5/2}\right]$

$= C\, 14e^{-5/2} = 1.149C$

$u = x \Rightarrow du = dx$ و $dv = e^{-x/2}dx \Rightarrow v = -2e^{-x/2}$

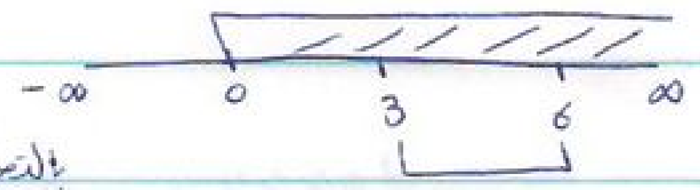
$= C\left[-2xe^{-x/2}\Big|_5^{\infty} + 2\int_5^{\infty} e^{-x/2}\, dx\right]$

$= C\left[-2\left[0 - 5e^{-5/2}\right] - 4e^{-x/2}\Big|_5^{\infty}\right]$

$= C\left[10e^{-5/2} - 4(0 - e^{-5/2}) = 10e^{-5/2} + 4e^{-5/2}\right]$

$= C\left[14e^{-5/2}\right] = 1.149C$

(b) $P(3 < X < 6) = C\int_3^{6} x e^{-x/2}\, dx$



بالقانون or بالتجزئ

$= -C\left[\frac{x}{1/2} + \frac{1}{(1/2)^2}\right] e^{-x/2}\Big|_3^{6}$

$= -C[2x+4]\, e^{-x/2}\Big|_3^{6}$

$= C\left[10e^{-3/2} - 16e^{-3}\right]$

$= 1.435C$

$u = x \Rightarrow du = dx$ و $dv = e^{-x/2}dx \Rightarrow v = -2e^{-x/2}$

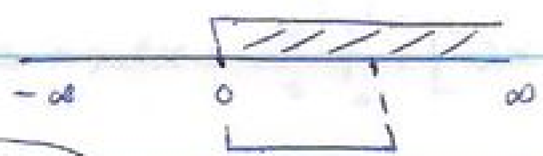
$= C\left[-2xe^{-x/2}\Big|_3^{6} + 2\int_3^{6} e^{-x/2}\, dx\right]$

$= C\left[-12e^{-3} + 6e^{-3/2} - 4e^{-x/2}\Big|_3^{6}\right]$

$= C\left[-12e^{-3} + 6e^{-3/2} - 4e^{-3} + 4e^{-3/2}\right]$

$= C\left[10e^{-3/2} - 16e^{-3}\right] = 1.435C$

(c) $P(X < 1) = C\int_0^{1} x e^{-x/2}\, dx$



بالقانون or

$= -C\left[(2x+4)\, e^{-x/2}\Big|_0^1\right]$

$= -C\left[6e^{-1/2} - 4\right]$

$= .360816C$

ويمكن حله كذلك بالتجزئ
كما في الفقرات السابقة

Q10

(a) $P(Y \leq 5) = F(5) = 1 - \frac{9}{25} = \frac{16}{25} = .64$

(b) $P(Y > 8) = 1 - P(Y \leq 8) = 1 - F(8) = 1 - \left(1 - \frac{9}{64}\right) = \frac{9}{64} = .1406$

(c) $f(y) \underset{\text{اشتقاق}}{\overset{\text{تكامل}}{\rightleftarrows}} F(y)$

$f(y) = \frac{d}{dy}F(y) = (F(y))' = \frac{18y}{y^4} = \frac{18}{y^3}$

$\therefore f(y) = \begin{cases} \frac{18}{y^3}, & y > 3 \\ 0, & \text{o.w} \end{cases}$

(11)

2 #1

ⓐ $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Leftrightarrow \int_0^1 x\,dx + \int_1^c (2-x)\,dx = 1 \Leftrightarrow \frac{x^2}{2}\Big|_0^1 + \left[2x - \frac{x^2}{2}\right]\Big|_1^c = 1$

$\Leftrightarrow \frac{1}{2} + \left[(2c - \frac{c^2}{2}) - (2 - \frac{1}{2})\right] = 1 \Leftrightarrow \frac{1}{2} + \left[2c - \frac{c^2}{2} - \frac{3}{2}\right] = 1$

$\Leftrightarrow -\frac{1}{2}c^2 + 2c - 1 = 1 \Leftrightarrow -\frac{1}{2}c^2 + 2c - 2 = 0$

or

بإكمال المربع

$a = -\frac{1}{2},\ b = 2,\ c = -2$

$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4-4}}{2(-\frac{1}{2})} = \frac{-2}{-1} = 2$

$-\frac{1}{2}\left[c^2 - 4c + 4\right] = 0$

$\Leftrightarrow c^2 - 4c + 4 = 0 \Leftrightarrow (c-2)^2 = 0$

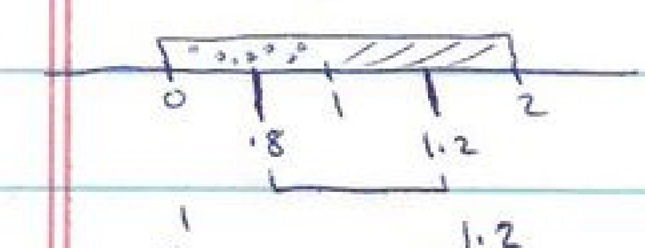
$\Leftrightarrow c - 2 = 0 \Leftrightarrow c = 2$

$\therefore f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , \text{o.w} \end{cases}$

ⓑ $F(x) = P(X \le x) = \begin{cases} 0 & , x < 0 \\ \int_0^x x\,dx = \frac{1}{2}x^2\Big|_0^x = \frac{1}{2}x^2 & , 0 \le x < 1 \\ \int_0^1 x\,dx + \int_1^x (2-x)\,dx = \frac{x^2}{2}\Big|_0^1 + \left[2x - \frac{x^2}{2}\right]\Big|_1^x = 2x - \frac{x^2}{2} - 1 & , 1 \le x < 2 \\ 1 & , 2 \le x \end{cases}$

ⓒ $P(.8 < X < .6c) = P(.8 < X < 1.2)$, where $.6c = .6(2) = 1.2$

عن طريقة دالة f(x)



$= \int_{.8}^{1} x\,dx + \int_{1}^{1.2} (2-x)\,dx$

عن طريق F(x)

$= P(X < 1.2) - P(X < .8)$

$= F(1.2) - F(.8)$

$= \left[2(1.2) - \frac{(1.2)^2}{2} - 1\right] - \left[\frac{1}{2}(.8)^2\right]$

$= .68 - .32 = .36$