

Poisson Distribution

*Poisson distribution $X \sim \text{Poisson}(\lambda)$ then its pmf is given by

$$f(x) = f(x; \lambda) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!} & ; x=0,1,2,\dots \\ 0 & \text{otherwise} \end{cases}$$

Parameter of the Distribution: $\lambda > 0$ (The average)

Mean and Variance

If X is a discrete random variable has Poisson distribution with parameter λ then,

$$E(X) = V(x) = \lambda.$$

Example 3.11 "from slides"

Suppose that the number of typing errors per page has a Poisson distribution with average 6 typing errors. What is the probability that

- I. the number of typing errors in a page will be 7.
- II. the number of typing errors in a page will be at least 2.
- III. in 2 pages there will be 10 typing errors.
- IV. in a half page there will be no typing errors.

outstanding
job!
very clear and
easy to
understand!

Solution

Let X represents the no. of typing errors per page. Therefore, $\lambda_x = 6 \Rightarrow X \sim \text{Poisson}(6)$.

I. $P(X = 7) = 0.1377$.

II. $P(X \geq 2) = f(2) + f(3) + \dots = \underbrace{1 - P(X < 2)} = 1 - f(0) - f(1) = 0.9826$.

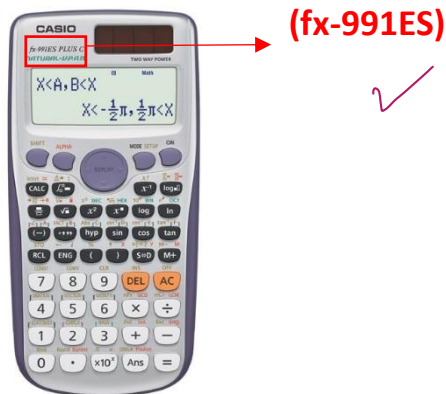
III. Let Y represents the no. of typing errors in 2 pages. Therefore, $\lambda_y = \lambda_x t = 6 \cdot 2 = 12 \Rightarrow Y \sim \text{Poisson}(12)$.

$$P(Y = 10) = 0.1048.$$

IV. Let Z represents the no. of typing errors in a half pages. Therefore, $\lambda_z = \lambda_x t = 6 \cdot 1/2 = 3 \Rightarrow Z \sim \text{Poisson}(3)$.










$$P(Z = 0) = 0.0498.$$

#Now a way to solve the Poisson distribution question with the calculator type (fx-991ES)




















#steps:

- **Solve** $P(X = 7) = 0.1377$.

first click  then , , , write (-6),  click right ,write (6) then click  write (7) then  click double right and write (7) then  then  finally click "=" to have a solution 0.1377 .



- **Solve** $P(X \geq 2) = f(2) + f(3) + \dots = 1 - P(X < 2) = 0.9826$.

First write (1-) "Subtract " , now click  then  so now write the function for Poisson distribution between "(" so click  then , , , write (-6),  click right then click X "Multiply" ,write (6) then click ,  then  then  click double right and click again ,  then  then  so now we done from function go to summation Σ by click  double right and write (0) then  click right and write (1) , so now click "=" to have a solution 0.9826 .



***So complete in this way with any question just change λ value and check if it needs to write x or use summation , about the mean and variance don't needs for calculator because both of them have same λ value.**

Excellent!

"poisson Approximation to binomial distribution"

Example:

A factory produces a particular electrical component and on average 1 in 50 is faulty. in a batch of 300 components taken at random. what is the probability of having at least eight faulty components?

$$\begin{aligned}P(X \geq 8) &= 1 - P(X \leq 7) \\&= 1 - [P(X=0) + P(X=1) + \dots + P(X=7)] \\&= 1 - \left[{}^{300}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{300} + {}^{300}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^{299} \right. \\&\quad \left. + \dots + {}^{300}C_7 \left(\frac{1}{50}\right)^7 \left(\frac{49}{50}\right)^{293} \right] \\&= 1 - 0.74538 = 0.25461\end{aligned}$$

*
mean $\rightarrow \mu = E(X) = \lambda = np$

Variance $\rightarrow \sigma^2 = E(X)$

$$P(x) = \frac{e^{-(np)} (np)^x}{x!}$$

نویسہ لایہ: $P_x = \frac{e^{-np} (np)^x}{x!}$

Step:

First click $\frac{\square}{\square}$ then shift In, write the sample and probability of success then click (and write sample and probability of success then \downarrow then click shift then ENG and write X_i .