Plane Curve [Parametric Equation]

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1. Parametric Equation
2. Tangent Lines and Arc Length
3. Arc Length and Surface Area

Definition (Swokowsoki,642)

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Example

Sketch the graph of the curve C that has the parametric equations:

1. $x = t^2 + t, \quad y = 2t - 1.$
2. $x = t^2 + t, \quad y = 2t - 1; \quad -1 \leq t \leq 1.$
3. $x = a\cos(t), \quad y = a\sin(t); \quad t \in \mathbb{R}.$

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Note that: It is important to observe that

$$d^2y/dx^2 \neq \frac{d^2y/dt^2}{d^2x/dt^2}$$

Example (Swokowsoki,652)

Let C be the curve with parametrization
$x = t^3 - 3t, y = t^2 - 5t - 1; t \in \mathbb{R}$.

a *Find an equation of the tangent line to C at the point corresponding to $t = 2$.*

b *For what values of t is the tangent line horizontal or vertical.*

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Example (Swokowsoki,653)

Let C be the curve with parametrization $x = e^{-t}, y = e^{2t}; t \in \mathbb{R}$. Find dy/dx and d^2y/dx^2.

Theorem (Swokowsoki,654)

If a smooth curve C *is given parametrically by* $x = f(t), y = g(t); a \leq t \leq b$*, and if* C *does not intersect itself, except possibly for* $t = a$ *and* $t = b$*, then the length* L *of* C *is*

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Example (2)

Determine the surface area of the solid obtained by rotating the following parametric curve about the x-axis and y-axis.

$$x = \sin^2(t), \quad y = \cos^2(t); \quad 0 \leq t \leq \pi/2$$