

# ***PHYSICS FOR ENGINEERING I***

***(PHYS 1210)***

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- Chapter 2:*** Motion in One Dimension
- Chapter 3:*** Vectors
- Chapter 4:*** Motion in Two Dimensions
- Chapter 5:*** Laws of Motion
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*Chapter 3:*

# Vectors

*Chapter 3:*

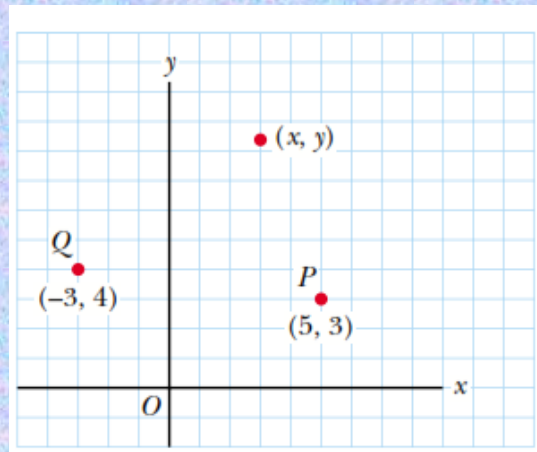
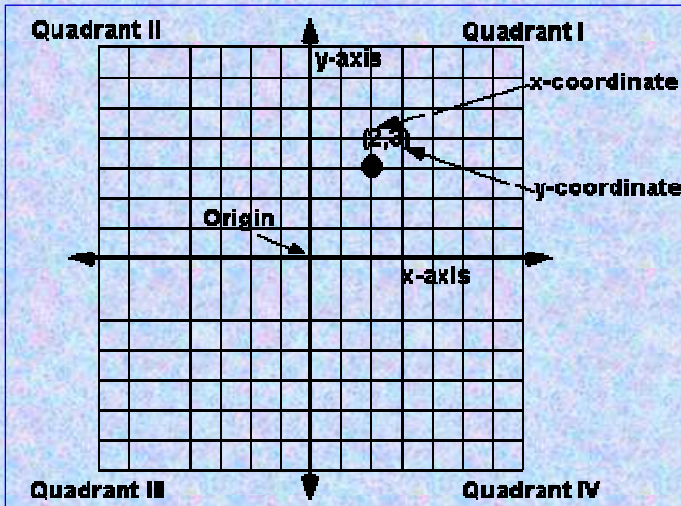
# Vectors

- ✓ **Coordinate systems**
- ✓ **Vectors and scalar quantities**
- ✓ **Properties of vectors**
- ✓ **Components of vectors and unit vectors**

# Coordination System

## 1. Cartesian Coordinate System

- ✓ In this Chapter, we use the **Cartesian Coordinate System**, in which horizontal and vertical axes intersect at a point defined as the origin .
- ✓ Cartesian coordinates are also called *rectangular coordinates*.

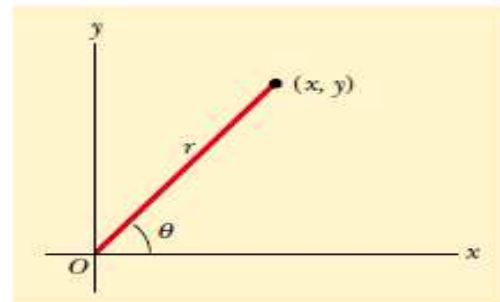


Designation of points in a Cartesian coordinate system. Every point is labelled with coordinates  $(x, y)$ .

## 2. Polar Coordinate System

Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates*  $(r, \theta)$ , as shown in Figure.

In this *Polar Coordinate System*,  $r$  is the distance from the origin to the point having Cartesian coordinates  $(x, y)$ , and  $\theta$  is the angle between a line drawn from the origin to the point and a fixed axis.

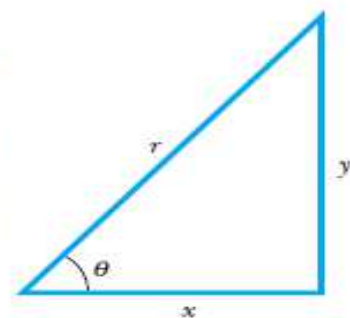


(a)

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



(b)

**Active Figure 3.2** (a) The plane polar coordinates of a point are represented by the distance  $r$  and the angle  $\theta$ , where  $\theta$  is measured counterclockwise from the positive  $x$  axis. (b) The right triangle used to relate  $(x, y)$  to  $(r, \theta)$ .

## The Relation between Cartesian and Polar Coordinate Systems

From the right triangle in the figure, we find that

$$\sin \theta = y/r \quad \text{and,} \quad \cos \theta = x/r$$

Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

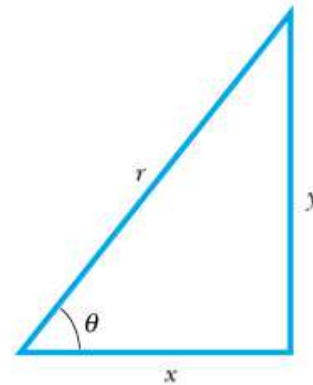
$$\tan \theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



(b)

**Active Figure 3.2** (a) The plane polar coordinates of a point are represented by the distance  $r$  and the angle  $\theta$ , where  $\theta$  is measured counterclockwise from the positive  $x$  axis. (b) The right triangle used to relate  $(x, y)$  to  $(r, \theta)$ .

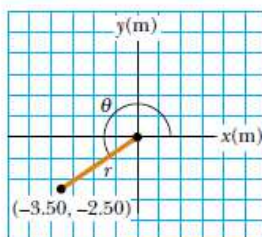
## Example

The Cartesian coordinates of a point in the  $x$ - $y$  plane are  $(x, y) = (-3.50, -2.50)$  m. Find the polar coordinates of this point.


### Example 3.1 Polar Coordinates

The Cartesian coordinates of a point in the  $xy$  plane are  $(x, y) = (-3.50, -2.50)$  m, as shown in Figure 3.3. Find the polar coordinates of this point.

**Solution** For the examples in this and the next two chapters we will illustrate the use of the General Problem-Solving



**Active Figure 3.3** (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

 At the Active Figures link at <http://www.pse6.com>, you can move the point in the  $xy$  plane and see how its Cartesian and polar coordinates change.

Strategy outlined at the end of Chapter 2. In subsequent chapters, we will make fewer explicit references to this strategy, as you will have become familiar with it and should be applying it on your own. The drawing in Figure 3.3 helps us to *conceptualize* the problem. We can *categorize* this as a plug-in problem. From Equation 3.4,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

and from Equation 3.3,

$$\tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216^\circ$$

Note that you must use the signs of  $x$  and  $y$  to find that the point lies in the third quadrant of the coordinate system. That is,  $\theta = 216^\circ$  and not  $35.5^\circ$ .

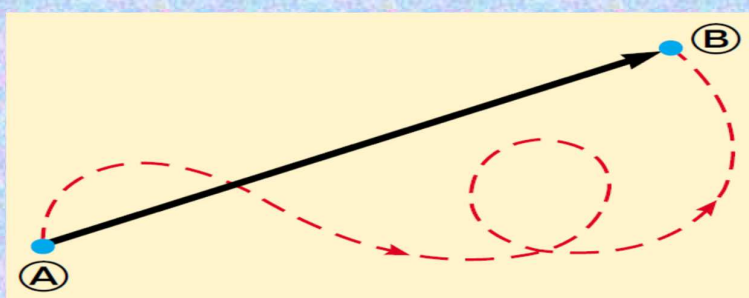
## Vector and Scalar Quantities

**A Scalar Quantity** is completely specified by a single value with an appropriate unit and has no direction.

Examples of scalar quantities are volume, mass, speed, and time intervals.

**A Vector Quantity** is completely specified by a number and appropriate units plus a direction.

Examples of a vector quantity is displacement and velocity.



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In this text, we use a boldface letter, such as  $\mathbf{A}$ , to represent a vector quantity. Another notation is useful when boldface notation is difficult, such as when writing on paper or on a chalkboard—an arrow is written over the symbol for the vector:  $\vec{\mathbf{A}}$ . The magnitude of the vector  $\mathbf{A}$  is written either  $A$  or  $|\mathbf{A}|$ . The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity. The magnitude of a vector is *always* a positive number.

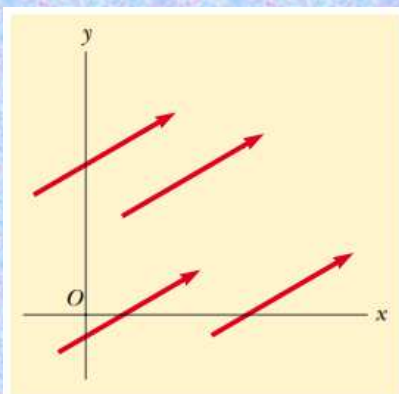
**Quick Quiz 3.1** Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

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# Some Properties of Vectors

## Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is,  $\mathbf{A} = \mathbf{B}$  only if  $A = B$  and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

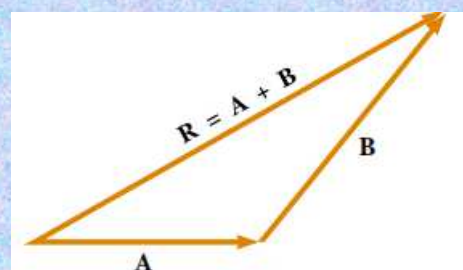
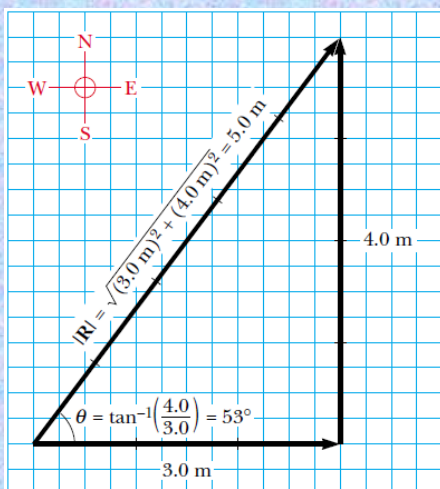


**Figure 3.5** These four vectors are equal because they have equal lengths and point in the same direction.

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## Adding Vectors

The rules for adding vectors are conveniently described by graphical methods. To add vector **B** to vector **A**, first draw vector **A** on graph paper, with its magnitude represented by a convenient length scale, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure 3.6. The **resultant vector**  $\mathbf{R} = \mathbf{A} + \mathbf{B}$  is the vector drawn from the tail of **A** to the tip of **B**.

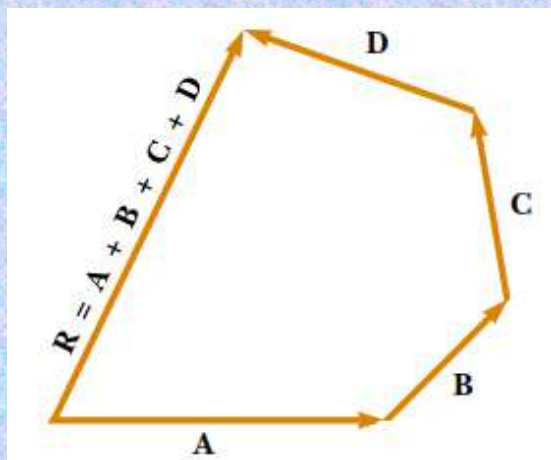


**Active Figure 3.6** When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

**Vector addition. Walking first 3.0 m due east and then 4.0m due north leaves you 5.0 m from your starting point.**

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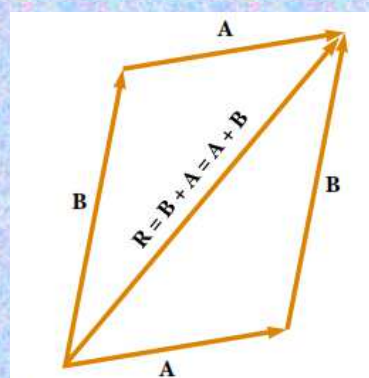
A geometric construction can also be used to add more than two vectors. This is shown in Figure 3.8 for the case of four vectors. The resultant vector  $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$  is the vector that completes the polygon. In other words,  $\mathbf{R}$  is the vector drawn from the tail of the first vector to the tip of the last vector.



**Figure 3.8** Geometric construction for summing four vectors. The resultant vector  $\mathbf{R}$  is by definition the one that completes the polygon.

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, the order is important when vectors are multiplied). This can be seen from the geometric construction in **Figure 3.9** and is known as the commutative law of addition:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

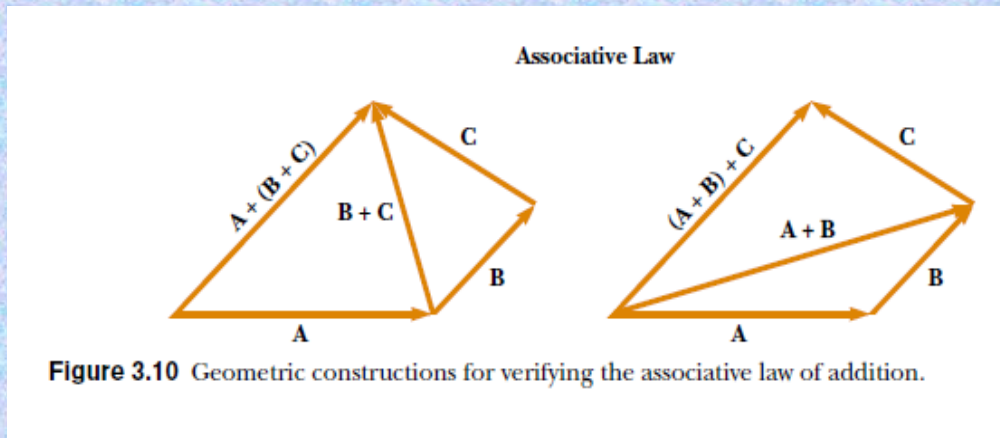


**Figure 3.9** This construction shows that  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ —in other words, that vector addition is commutative.

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 3.10. This is called the **associative law of addition**:

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \quad (3.6)$$

In summary, **a vector quantity has both magnitude and direction and also obeys the laws of vector addition** as described in Figures 3.6 to 3.10. When two or



### Negative of a Vector

The negative of the vector  $\mathbf{A}$  is defined as the vector that when added to  $\mathbf{A}$  gives zero for the vector sum. That is,  $\mathbf{A} + (-\mathbf{A}) = 0$ . The vectors  $\mathbf{A}$  and  $-\mathbf{A}$  have the same magnitude but point in opposite directions.

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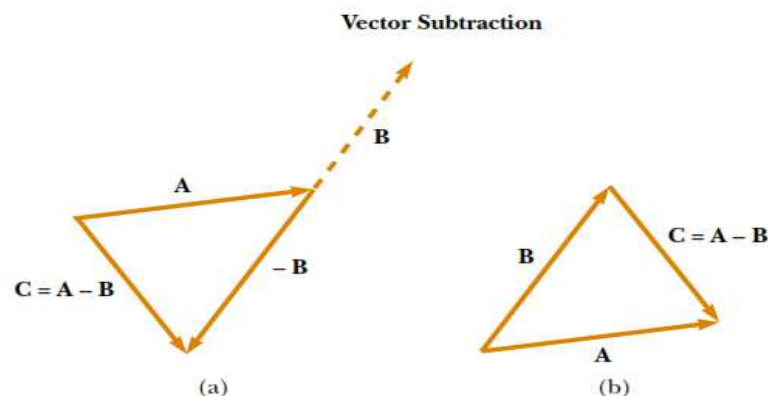
### Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation  $\mathbf{A} - \mathbf{B}$  as vector  $-\mathbf{B}$  added to vector  $\mathbf{A}$ :

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \quad (3.7)$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.11a.

Another way of looking at vector subtraction is to note that the difference  $\mathbf{A} - \mathbf{B}$  between two vectors  $\mathbf{A}$  and  $\mathbf{B}$  is what you have to add to the second vector to obtain the first. In this case, the vector  $\mathbf{A} - \mathbf{B}$  points from the tip of the second vector to the tip of the first, as Figure 3.11b shows.



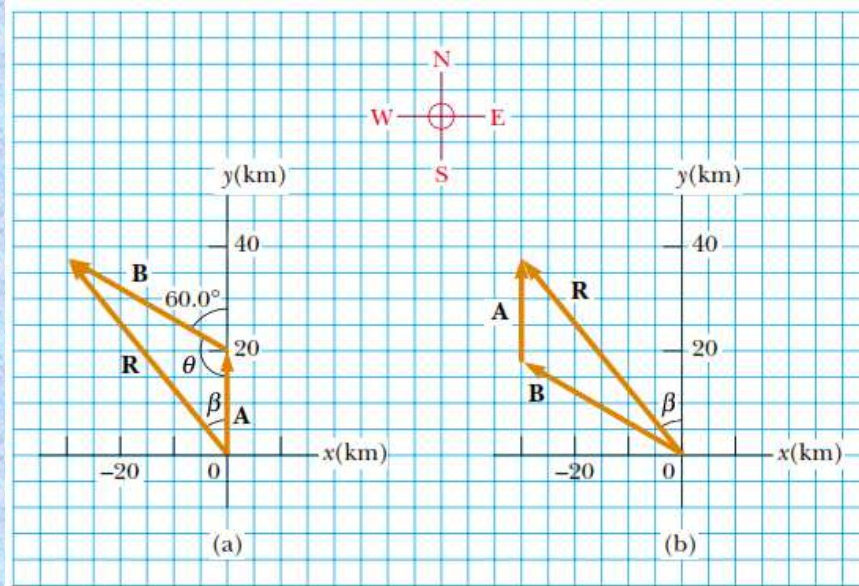
**Figure 3.11** (a) This construction shows how to subtract vector  $\mathbf{B}$  from vector  $\mathbf{A}$ . The vector  $-\mathbf{B}$  is equal in magnitude to vector  $\mathbf{B}$  and points in the opposite direction. To subtract  $\mathbf{B}$  from  $\mathbf{A}$ , apply the rule of vector addition to the combination of  $\mathbf{A}$  and  $-\mathbf{B}$ : Draw  $\mathbf{A}$  along some convenient axis, place the tail of  $-\mathbf{B}$  at the tip of  $\mathbf{A}$ , and  $\mathbf{C}$  is the difference  $\mathbf{A} - \mathbf{B}$ . (b) A second way of looking at vector subtraction. The difference vector  $\mathbf{C} = \mathbf{A} - \mathbf{B}$  is the vector that we must add to  $\mathbf{B}$  to obtain  $\mathbf{A}$ .

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### Example 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction  $60.0^\circ$  west of north, as shown in Figure 3.12a. Find the magnitude and direction of the car's resultant displacement.



**Figure 3.12** (Example 3.2) (a) Graphical method for finding the resultant displacement vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$ . (b) Adding the vectors in reverse order ( $\mathbf{B} + \mathbf{A}$ ) gives the same result for  $\mathbf{R}$ .

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The second way to solve the problem is to analyze it algebraically. The magnitude of  $\mathbf{R}$  can be obtained from the law of cosines as applied to the triangle (see Appendix B.4). With  $\theta = 180^\circ - 60^\circ = 120^\circ$  and  $R^2 = A^2 + B^2 - 2AB \cos \theta$ , we find that

$$\begin{aligned} R &= \sqrt{A^2 + B^2 - 2AB \cos \theta} \\ &= \sqrt{(20.0 \text{ km})^2 + (35.0 \text{ km})^2 - 2(20.0 \text{ km})(35.0 \text{ km}) \cos 120^\circ} \\ &= 48.2 \text{ km} \end{aligned}$$

$$\frac{\sin \beta}{B} = \frac{\sin \theta}{R}$$

$$\sin \beta = \frac{B}{R} \sin \theta = \frac{35.0 \text{ km}}{48.2 \text{ km}} \sin 120^\circ = 0.629$$

$$\beta = 39.0^\circ$$

The resultant displacement of the car is 48.2 km in a direction  $39.0^\circ$  west of north.

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### 3.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

From Figure 3.13 and the definition of sine and cosine, we see that  $\cos \theta = A_x/A$  and that  $\sin \theta = A_y/A$ . Hence, the components of **A** are

$$A_x = A \cos \theta \tag{3.8}$$

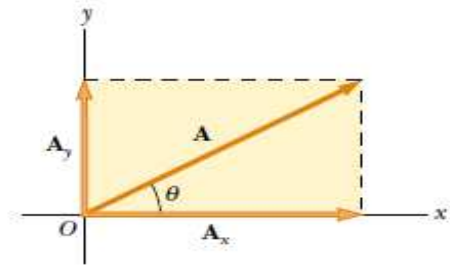
$$A_y = A \sin \theta \tag{3.9}$$

These components form two sides of a right triangle with a hypotenuse of length *A*. Thus, it follows that the magnitude and direction of **A** are related to its components through the expressions

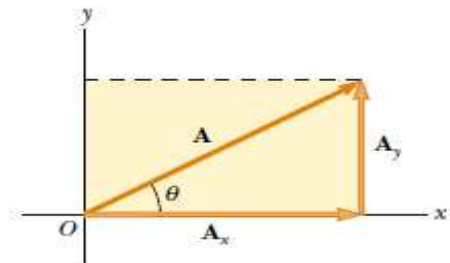
$$A = \sqrt{A_x^2 + A_y^2} \tag{3.10}$$

$$\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) \tag{3.11}$$

Note that **the signs of the components  $A_x$  and  $A_y$  depend on the angle  $\theta$** . For example, if  $\theta = 120^\circ$ , then  $A_x$  is negative and  $A_y$  is positive. If  $\theta = 225^\circ$ , then both  $A_x$  and  $A_y$  are negative. Figure 3.14 summarizes the signs of the components when **A** lies in the various quadrants.



(a)



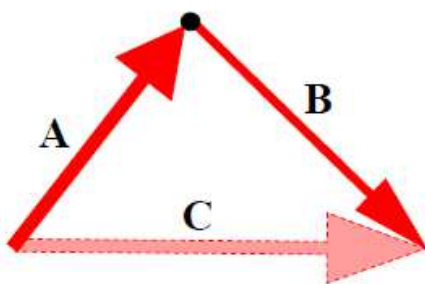
(b)

**Figure 3.13** (a) A vector **A** lying in the *xy* plane can be represented by its component vectors  $A_x$  and  $A_y$ . (b) The *y* component vector  $A_y$  can be moved to the right so that it adds to  $A_x$ . The vector sum of the component vectors is **A**. These three vectors form a right triangle.

## Vectors: Rules of addition

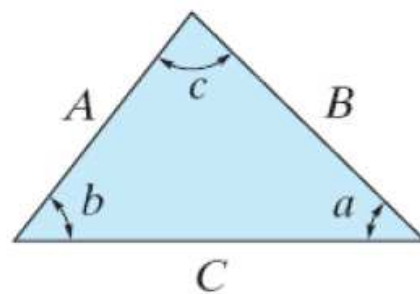
### • Trigonometric Rule

- Law of Sines
- Law of Cosine



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

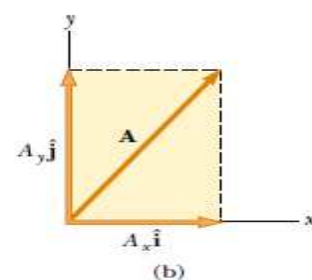
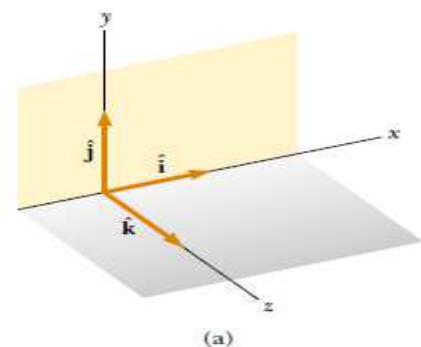
**Figure 3.14 summarizes the signs of the components when  $\mathbf{A}$  lies in the various quadrants.**

	$y$	
$A_x$ negative		$A_x$ positive
$A_y$ positive		$A_y$ positive
		$x$
$A_x$ negative		$A_x$ positive
$A_y$ negative		$A_y$ negative

**Figure 3.14** The signs of the components of a vector  $\mathbf{A}$  depend on the quadrant in which the vector is located.

### Unit Vectors

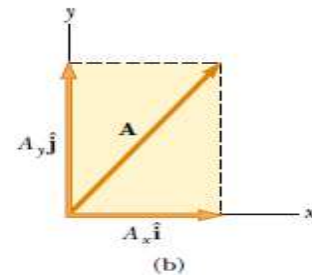
Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$ , and  $\hat{\mathbf{k}}$  to represent unit vectors pointing in the positive  $x$ ,  $y$ , and  $z$  directions, respectively. (The “hats” on the symbols are a standard notation for unit vectors.) The unit vectors  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$ , and  $\hat{\mathbf{k}}$  form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure 3.16a. The magnitude of each unit vector equals 1; that is,  $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$ .



**Active Figure 3.16** (a) The unit vectors  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$ , and  $\hat{\mathbf{k}}$  are directed along the  $x$ ,  $y$ , and  $z$  axes, respectively. (b) Vector  $\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$  lying in the  $xy$  plane has components  $A_x$  and  $A_y$ .

Consider a vector  $\mathbf{A}$  lying in the  $xy$  plane, as shown in Figure 3.16b. The product of the component  $A_x$  and the unit vector  $\hat{\mathbf{i}}$  is the vector  $A_x\hat{\mathbf{i}}$ , which lies on the  $x$  axis and has magnitude  $|A_x|$ . (The vector  $A_x\hat{\mathbf{i}}$  is an alternative representation of vector  $\mathbf{A}_x$ .) Likewise,  $A_y\hat{\mathbf{j}}$  is a vector of magnitude  $|A_y|$  lying on the  $y$  axis. (Again, vector  $A_y\hat{\mathbf{j}}$  is an alternative representation of vector  $\mathbf{A}_y$ .) Thus, the unit-vector notation for the vector  $\mathbf{A}$  is

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \quad (3.12)$$



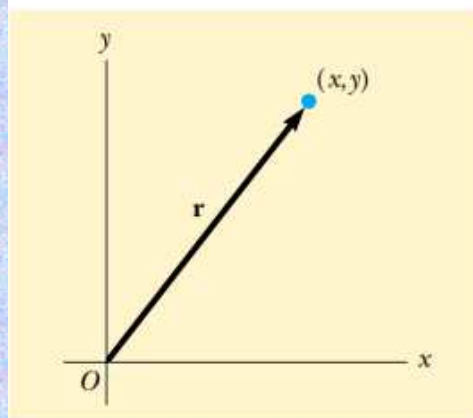
**Active Figure 3.16** (a) The unit vectors  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$ , and  $\hat{\mathbf{k}}$  are directed along the  $x$ ,  $y$ , and  $z$  axes, respectively. (b) Vector  $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$  lying in the  $xy$  plane has components  $A_x$  and  $A_y$ .

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For example, consider a point lying in the  $xy$  plane and having Cartesian coordinates  $(x, y)$ , as in Figure 3.17. The point can be specified by the **position vector**  $\mathbf{r}$ , which in unit-vector form is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \quad (3.13)$$

This notation tells us that the components of  $\mathbf{r}$  are the lengths  $x$  and  $y$ .



**Figure 3.17** The point whose Cartesian coordinates are  $(x, y)$  can be represented by the position vector  $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ .

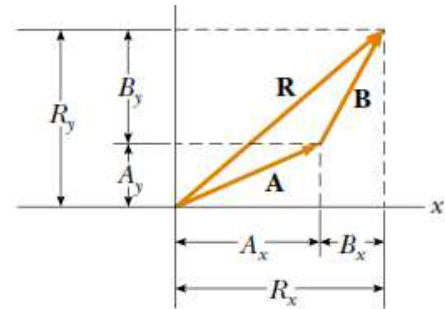
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We obtain the magnitude of  $\mathbf{R}$  and the angle it makes with the  $x$  axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \quad (3.16)$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \quad (3.17)$$

We can check this addition by components with a geometric construction, as shown in Figure 3.18. Remember that you must note the signs of the components when using either the algebraic or the graphical method.



**Figure 3.18** This geometric construction for the sum of two vectors shows the relationship between the components of the resultant  $\mathbf{R}$  and the components of the individual vectors.

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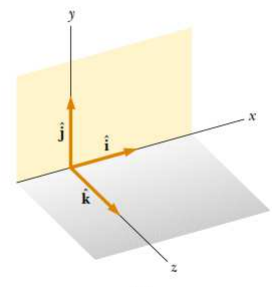
At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If  $\mathbf{A}$  and  $\mathbf{B}$  both have  $x$ ,  $y$ , and  $z$  components, we express them in the form

$$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}} \quad (3.18)$$

$$\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}} \quad (3.19)$$

The sum of  $\mathbf{A}$  and  $\mathbf{B}$  is

$$\mathbf{R} = (A_x + B_x) \hat{\mathbf{i}} + (A_y + B_y) \hat{\mathbf{j}} + (A_z + B_z) \hat{\mathbf{k}} \quad (3.20)$$



Note that Equation 3.20 differs from Equation 3.14: in Equation 3.20, the resultant vector also has a  $z$  component  $R_z = A_z + B_z$ . If a vector  $\mathbf{R}$  has  $x$ ,  $y$ , and  $z$  components, the magnitude of the vector is  $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$ . The angle  $\theta_x$  that  $\mathbf{R}$  makes with the  $x$  axis is found from the expression  $\cos \theta_x = R_x/R$ , with similar expressions for the angles with respect to the  $y$  and  $z$  axes.

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### Example 3.3 The Sum of Two Vectors

Find the sum of two vectors  $\mathbf{A}$  and  $\mathbf{B}$  lying in the  $xy$  plane and given by

$$\mathbf{A} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \text{ m} \quad \text{and} \quad \mathbf{B} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}}) \text{ m}$$

**Solution** You may wish to draw the vectors to *conceptualize* the situation. We *categorize* this as a simple plug-in problem. Comparing this expression for  $\mathbf{A}$  with the general expression  $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ , we see that  $A_x = 2.0$  m and  $A_y = 2.0$  m. Likewise,  $B_x = 2.0$  m and  $B_y = -4.0$  m. We obtain the resultant vector  $\mathbf{R}$ , using Equation 3.14:

$$\begin{aligned}\mathbf{R} &= \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\hat{\mathbf{i}} \text{ m} + (2.0 - 4.0)\hat{\mathbf{j}} \text{ m} \\ &= (4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}) \text{ m}\end{aligned}$$

or

$$R_x = 4.0 \text{ m} \quad R_y = -2.0 \text{ m}$$

The magnitude of  $\mathbf{R}$  is found using Equation 3.16:

$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0 \text{ m})^2 + (-2.0 \text{ m})^2} = \sqrt{20} \text{ m} \\ &= 4.5 \text{ m}\end{aligned}$$

We can find the direction of  $\mathbf{R}$  from Equation 3.17:

$$\tan \theta = \frac{R_y}{R_x} = \frac{-2.0 \text{ m}}{4.0 \text{ m}} = -0.50$$

Your calculator likely gives the answer  $-27^\circ$  for  $\theta = \tan^{-1}(-0.50)$ . This answer is correct if we interpret it to mean  $27^\circ$  clockwise from the  $x$  axis. Our standard form has been to quote the angles measured counterclockwise from the  $+x$  axis, and that angle for this vector is  $\theta = 333^\circ$ .

### Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements:  $\mathbf{d}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$  cm,  $\mathbf{d}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$  cm and  $\mathbf{d}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$  cm. Find the components of the resultant displacement and its magnitude.

**Solution** Three-dimensional displacements are more difficult to imagine than those in two dimensions, because the latter can be drawn on paper. For this problem, let us *conceptualize* that you start with your pencil at the origin of a piece of graph paper on which you have drawn  $x$  and  $y$  axes. Move your pencil 15 cm to the right along the  $x$  axis, then 30 cm upward along the  $y$  axis, and then 12 cm *vertically away* from the graph paper. This provides the displacement described by  $\mathbf{d}_1$ . From this point, move your pencil 23 cm to the right parallel to the  $x$  axis, 14 cm parallel to the graph paper in the  $-y$  direction, and then 5.0 cm vertically downward toward the graph paper. You are now at the displacement from the origin described by  $\mathbf{d}_1 + \mathbf{d}_2$ . From this point, move your pencil 13 cm to the left in the  $-x$  direction, and (finally!) 15 cm parallel to the graph paper along the  $y$  axis.

Your final position is at a displacement  $\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$  from the origin.

Despite the difficulty in conceptualizing in three dimensions, we can *categorize* this problem as a plug-in problem due to the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way:

$$\begin{aligned}\mathbf{R} &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 \\ &= (15 + 23 - 13)\hat{\mathbf{i}} \text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}} \text{ cm} \\ &\quad + (12 - 5.0 + 0)\hat{\mathbf{k}} \text{ cm} \\ &= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}}) \text{ cm}\end{aligned}$$

The resultant displacement has components  $R_x = 25$  cm,  $R_y = 31$  cm, and  $R_z = 7.0$  cm. Its magnitude is

$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2 + R_z^2} \\ &= \sqrt{(25 \text{ cm})^2 + (31 \text{ cm})^2 + (7.0 \text{ cm})^2} = 40 \text{ cm}\end{aligned}$$

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

**(A)** Determine the components of the hiker's displacement for each day.

**Solution** We *conceptualize* the problem by drawing a sketch as in Figure 3.19. If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.19. Drawing the resultant **R**, we can now *categorize* this as a problem we've solved before—an addition of two vectors. This should give you a hint of the power of categorization—many new problems are very similar to problems that we have already solved if we are careful to conceptualize them.

We will *analyze* this problem by using our new knowledge of vector components. Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive x axis. From Equations 3.8 and 3.9, its components are

$$A_x = A \cos(-45.0^\circ) = (25.0 \text{ km})(0.707) = 17.7 \text{ km}$$

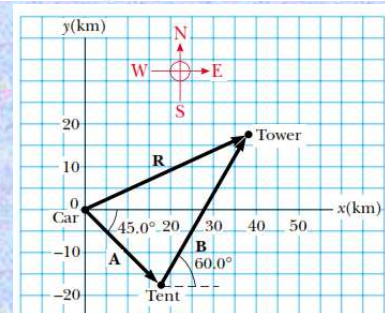
$$A_y = A \sin(-45.0^\circ) = (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$$

The negative value of  $A_y$  indicates that the hiker walks in the negative y direction on the first day. The signs of  $A_x$  and  $A_y$  also are evident from Figure 3.19.

The second displacement **B** has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B \cos 60.0^\circ = (40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B \sin 60.0^\circ = (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$



**Figure 3.19** (Example 3.5) The total displacement of the hiker is the vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$ . 29

**(B)** Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms of unit vectors.

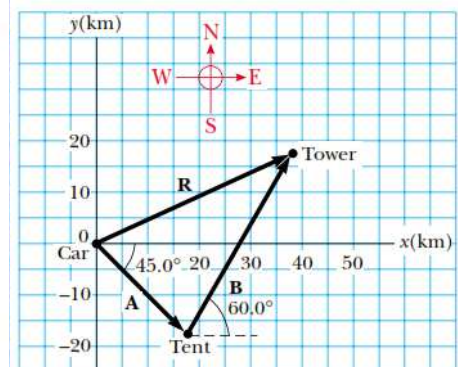
**Solution** The resultant displacement for the trip  $\mathbf{R} = \mathbf{A} + \mathbf{B}$  has components given by Equation 3.15:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = 37.7 \text{ km}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 16.9 \text{ km}$$

In unit-vector form, we can write the total displacement as

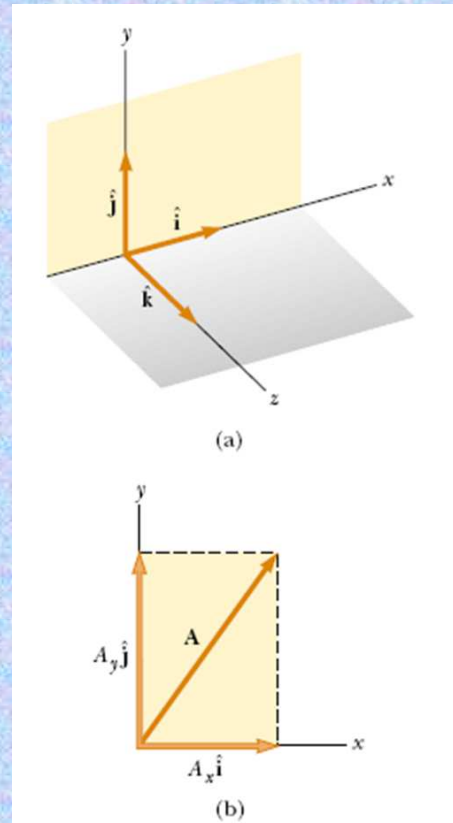
$$\mathbf{R} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}}) \text{ km}$$



**Figure 3.19** (Example 3.5) The total displacement of the hiker is the vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$ .

# Unit vector in 3-D

✓ Unit vectors in x, y, z coordinate are i, j, k



# Dot Product

Let  $\mathbf{a}$  and  $\mathbf{b}$  be two vectors defined as

$\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$  and  $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$  then the

dot product is a scalar value defined as

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{Or} \quad = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta$$

**Example :** Calculate the dot product of  $\mathbf{C} = (-4, -9)$  and  $\mathbf{D} = (-1, 2)$ .

**Solution:**

$$\mathbf{C} \cdot \mathbf{D} = -4(-1) - 9(2) = 4 - 18 = -14.$$



# Cross Product

Let  $\mathbf{a}$  and  $\mathbf{b}$  be two vectors defined as

$\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$  and  $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$  then the cross

product is a vector value defined as

$$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - a_3 b_2) \mathbf{i} + (a_1 b_3 - a_3 b_1) \mathbf{j} + (a_1 b_2 - a_2 b_1) \mathbf{k}$$

$$\begin{array}{ccc} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array}$$

**Example :** Calculate the cross product between  $\mathbf{a}=(3,-3,1)$  and  $\mathbf{b}=(4,9,2)$ .

**Solution:** The cross product is

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 1 \\ 4 & 9 & 2 \end{vmatrix} \\ &= \mathbf{i}(-3 \cdot 2 - 1 \cdot 9) - \mathbf{j}(3 \cdot 2 - 1 \cdot 4) + \mathbf{k}(3 \cdot 9 + 3 \cdot 4) \\ &= -15\mathbf{i} - 2\mathbf{j} + 39\mathbf{k} \end{aligned}$$