PHYS 505
HANDOUT 1 – On time independent perturbation theory.

1. Consider a particle of mass m in a one-dimensional infinite potential well of width a:

$$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}.$$

The particle is subject to perturbation of the form

$$W(x) = a\omega_0 \delta\left(x - a/2\right).$$

Calculate the changes in the energy levels of the particle in the first order of ω_0. (Sch)

2. Consider a particle of mass m and charge e in the central potential:

$$V(r) = \begin{cases} -e^2/r, & 0 < r < R \\ -e^2 \exp\left[-\lambda\left(r - R\right)\right]/r, & R < r < \infty \end{cases}$$

The potential differs from the Coulomb potential only in the region $r > R$, where the Coulomb force is *screened.* The difference becomes negligible if the parameter $\lambda \to 0$. Consider this difference as a perturbation and calculate the first-order correction to the energy of the ground state.

3. A particle of mass m moves in one dimension subject to a harmonic oscillator potential $\frac{1}{2}m\omega^2 x^2$. The particle oscillation is perturbed by an additional weak an-harmonic force, described by the potential $\Delta V = \lambda \sin \kappa x$. Find the corrected ground state. You may need the identity $e^{\mathbf{A}+\mathbf{B}} = e^{\mathbf{A}} e^{\mathbf{B}} e^{-[\mathbf{A},\,\mathbf{B}]/2}$

4. A particle of mass m moves in one dimension subject to an an-harmonic potential that is close to but not exactly a harmonic oscillator potential, namely

$$V(x) = \frac{m\omega^2 x^2}{2}\left(\frac{x}{a}\right)^{2\lambda}$$

where a is a parameter with the dimensions of length and $\lambda << 1$ is a dimensionless exponent. We can write this potential as

$$V(x) = \frac{m\omega^2 x^2}{2} + \Delta V$$

with

$$\Delta V\left(x\right) = \frac{m\omega^2 x^2}{2}\left[\left(\frac{x}{a}\right)^{2\lambda} - 1\right].$$

Treating ΔV as a small perturbation, calculate the first-order correction to the ground state energy.

5. A particle of mass m and charge q moves in one dimensional infinite square potential well:

$$V(x) = \begin{cases} 0, & |x| < L \\ \infty, & |x| > L \end{cases}.$$

We consider a weak uniform electric field of strength E that acts on the particle given by:

$$E = E_0 x / L, \qquad -L < x < L.$$

Calculate the first non-trivial correction to the particle's ground state energy.

6. Apply the first order perturbation theory to calculate the first correction at the energy eigenvalues of a simple harmonic oscillator which is caused by the presence of the term $V\left(x\right) = gx^4$.

7. Consider a charged particle with charge q, which has a mass m and is attached to a spring of spring constant k. We apply a uniform electric field E. Calculate its energy eigenvalues.

8. A particle of mass m moves inside the potential $U(x) = m\omega^2 x^2 / 2 + \lambda^2 \delta\left(x\right)$. Apply the perturbation theory of first order to find the energy spectrum. (Lag. 110)

9. The Hamiltonian of a quantum system is given by:

$$H_0 = E\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

We add to this the perturbative term:

$$V = \varepsilon\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

where ($\varepsilon << E$). Find: a) the first order correction to the energy, b) find the exact values for the energies of the total Hamiltonian $H = H_0 + V$, c) show that to the first order of the small term ε the above answers agree. (Menis 404).

10. Find the energy spectrum to the first order of the parameter ε of the Hamiltonian (Menis 401):

$$H = \begin{pmatrix} E & E+\varepsilon & 0 \\ E+\varepsilon & E & \varepsilon \\ 0 & \varepsilon & E \end{pmatrix}.$$

11. The potential of a simple harmonic oscillator gets a perturbations given by:

$$V = \begin{cases} gx & -a \le x \le a \\ 0 & |x| > a \end{cases}.$$

Find the first order correction in the energy spectrum (Menis 416).

12. Write the Hamiltonian of a simple harmonic (one-dimensional) at the approximation where the relativistic corrections are non-zero. What are the corrections in the ground state energy? (*Hint: in the theory of SHO we have the so called creation and annihilation operators given by* $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$, $a|n\rangle = \sqrt{n}|n-1\rangle$. *The position and momentum operator are given by:* $x = (\hbar/2m\omega)^{1/2}(a + a^+)$, $p = i(\hbar m\omega/2)^{1/2}(a^+ - a)$).

13. Discuss the method of variations in the case of a one dimensional SHO.

14. Discuss the method of variations in the case of a one dimensional potential given by $V(x) = gx^4$. Compare it with the real value of $0.668(g^{1/3}\hbar^{4/3}/m^{2/3})$ and explain the difference of your results.

15. Discuss the issue of the normalization of the perturbed wavefunction $\psi_n = \psi_n^o + \lambda\psi_n^1 + \lambda^2\psi_n^2 + ...$

16. A system for a given energy *E* has two degenerate states $\psi_1^{(0)}, \psi_2^{(0)}$. We apply a perturbation *V* on the system represented by a matrix:

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

Find the energy eigenstates and eigenvalues.