PHYS 505 MSc

HANDOUT 9 – *Identical Particles*

1. Assume the case of a four-particle state $\psi(\xi_1, \xi_2, \xi_3, \xi_4) = (3\xi_4/\xi_2\xi_3)e^{-i\xi_1}$. Find the result of the successive application of the permutation operators $\hat{P}_{12}\hat{P}_{14}\psi(\xi_1, \xi_2, \xi_3, \xi_4)$ and $\hat{P}_{14}\hat{P}_{12}\psi(\xi_1, \xi_2, \xi_3, \xi_4)$. What is the conclusion?

2. Specify the symmetry of the following functions:

(i) $\psi(x_1, x_2) = 4(x_1 - x_2)^2 + \frac{10}{x_1^2 + x_2^2}$, (ii) $\Phi(x_1, x_2) = \frac{1}{x_2+3} e^{-|x_1|}$

3. Find the energy levels and wave functions of a system of four distinguishable spinless particles placed in an infinite potential well of size *a.* Use this result to infer the energy and the wave function of the ground state and the first excited state.

4. Prove that the permutation operator is a constant of motion.

5. Discuss the symmetry of a wave function of a system of *N* particles if we take into consideration the spin degree of freedom.

6. Find the wave functions of two systems of identical, non-interacting particles: the first consists of two bosons, and the second of two spin 1/2 fermions.

7. Three imaginary "spinless" fermions are confined to a one-dimensional well of infinite depth and of length *L.* We assume that there is no interaction between the fermions. (i) What is the ground state of the system? (ii) Find the state of the system.

8. Repeat problem 7 for three electrons. Ignore the Coulomb interactions between the electrons.

9. Two non-identical particles, each of mass m, are confined in one dimension to an impenetrable box of length *L*. What are the wave functions and energies of the three lowest-energy states of the system (i.e., in which at most one particle is excited out of its ground state)? If an interaction potential of the form $V_{12} = \lambda\delta(x_1 - x_2)$ is added, calculate to first order in λ the energies of these three lowest states and their wave functions to zeroth order in X.

10. Two particles of mass *m* are placed in a rectangular box of sides *a* > *b* > *c* in the lowest energy state of the system compatible with the conditions below. The particles interact with each other according to the potential $V = A\delta(x_1 - x_2)$. Use first order perturbation theory to calculate the energy of the system under the following conditions:

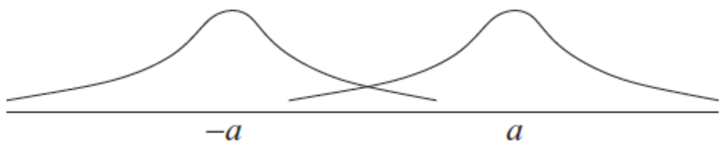
(a) Particles not identical.
(b) Identical particles of spin zero,
(c) Identical particles of spin one-half **with spins parallel**.

11. Consider a pair of **free identical** particles of mass m. For simplicity, suppose that they are moving in one dimension and neglect their spin variables. Each particle is described in terms of a **real** wave function, well localized around points $+a$ and $-a$ respectively; see Fig. 38. For definiteness, take $\psi_{\pm}(x) = (\beta/\pi)^{1/4} exp\left[-\frac{\beta}{2}(x \mp a)^2\right]$. A well-localized state corresponds to $\beta \gg 1/a^2$.

(i) Write down the wave function of the system and calculate the expectation value of the energy.

(ii) Show that if the two particles are fermions then there is an effective repulsion between them.

(iii) Compare with the case of two identical bosons.



12. The interaction potential of two electrons is given by the relation:

$$V(x_1, x_2) = g\mathbf{s}_1 \cdot \mathbf{s}_2 (x_1 - x_2)^2$$

Find the energy spectrum and the degeneracy of the bound states.

13. The state of an electron is described by the wavefunction:

$$\psi(\mathbf{r},\text{spin}) = \varphi(\mathbf{r}) \cdot X = A\left[\sqrt{x^2+y^2+z^2} - 2\sqrt{3}x + \sqrt{3}z\right] e^{-a\sqrt{x^2+y^2+z^2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

a) What will be the outcome of a measurement of the magnitude of the angular momentum and of its projection along the *z*-axis.

b) What will be the outcome of a measurement of the spin projection along the *z*-axis and what of the spin projection along the *y*-axis.