PHYS 500-FALL 2019
Homework 4
(24 marks)
Hand in: Saturday 30th November at 23:59

1. In an experiment to measure the volume of a cube of side *a* we got the following recordings:

Recordings	Length of side *a* in cm (±0.3 cm)
1	8.2
2	5.3
3	7.9
4	8.1
5	8.2
6	7.8

A) The second measurement looks very odd compared with the others. Apply Chauvenet's Criterion to check if we should keep it or reject it.

Solution:

We find the average value

$$a_{avg} = \frac{\sum_{i=1}^{6} a_i}{6} = 7.583$$

We find the standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{6}\left(a_i - a_{avg}\right)^2}{6-1}} = 1.13$$

We find the ratio μ

$$\mu = \frac{\left|a_{avg} - 5.3\right|}{\sigma} = \frac{\left|7.583 - 5.3\right|}{1.33} = 2.02$$

From the table we have that $P\left(< \mu\right) = 0.9566$ so

$P\left(\geq \mu\right) = 1 - 0.9566 = 0.0434$.

Then we multiply with the number of recordings 6

$u = 6P\left(\geq \mu\right) = 0.2604$

This is smaller than 0.5 so the "bad" recording 5.3 cm is rejected.

After completing the first step we follow two different methods to find the volume of the cube:

B) (i) Find the average side length and its error.

Solution:

After the rejection of the second recording we have the following 5 recordings:

Recordings	Length of side a in cm (±0.3 cm)
1	8.2
2	7.9
3	8.1
4	8.2
5	7.8

$$a_{avg} = \frac{\sum_{i=1}^{5} a_i}{5} = 8.04 \text{ cm}$$

The error in the average value is

$$\delta a = \sqrt{\frac{\sum_{i=1}^{5}\left(a_i - a_{avg}\right)^2}{5(5-1)}} = 0.0812 \text{ cm}$$

Thus the average value is:

$a_{avg} = \left(8.04 \pm 0.08\right)$ cm.

(ii) Find the volume of the cube and its error using as a length for the side a the value you found in the previous question B(i).

Solution

The volume of the cube is given by:

$$V1 = a_{avg}^3 = 519.7184640 \text{ cm}^3$$

The error for the volume is given by:

$$\delta V1 = \sqrt{\left(\frac{\partial V1}{\partial a_{avg}}\delta a_{avg}\right)^2} = \frac{\partial V1}{\partial a_{avg}}\delta a_{avg} = 3a_{avg}^2\delta a_{avg} = 15.513 \text{ cm}^3$$

Thus the volume calculated with this first method is:

$$V1 = \left(520 \pm 16\right) \text{ cm}^3.$$

(iii) Find the relative error for the volume.

Solution

$$\frac{\delta V1}{V1} = \frac{16}{520} = 0.03 \text{ or } 3\%$$

C) (i) For each recorded value for the side length given in the table find the volume of the cube. Find the error in each recording and round properly your results.

Solution:

With this different method it is helpful to construct the following table:

Recordings	Length of side a in cm (±0.3 cm)	Volume in cm^3	Error in volume in cm^3	Volume rounded in cm^3
1	8.2	551.368	60.516	551±61
2	7.9	493.039	56.169	493±56
3	8.1	531.441	59.049	531±59
4	8.2	551.368	60.516	551±61
5	7.8	474.552	54.756	475±55

Vasileios Lembessis 1/12/2019 13:39

Comment [1]: Most of you calculated wrongly the error for the volume. You considered the cube as a rectangular parallepiped with volume V=abc and then at the end you put a=b=c. This gave you wrong result. The volume of the cube is $V=a^3$. And $dV/da=3a^2$

Vasileios Lembessis 1/12/2019 13:43

Comment [2]: Almos all of you did mistake in the calculation of volume errors. You put in your calculation that δa=0.08 cm!!!! Where did you see this??? This is the error of the average value of the cube side. Here we follow another method of calculating the average volume by finding the volume for each measurement separately. So you need to use the side error you have for each measurement and which in the table I give you that the error is δa=0.3 cm.

(ii) Find the average value of the volumes you found in the previous question C(i) and its error.

Solutions:

We find the average value of the volume

$$V_{avg} = \frac{\sum_{i=1}^{5} V_i}{5} = 520.273 \text{ cm}^3$$

Then we find the error of this average volume

$$\delta V_{avg} = \sqrt{\frac{\sum_{i=1}^{5} \left(V_i - V_{avg}\right)^2}{5(5-1)}} = 15.487 \text{ cm}^3$$

Thus the volume of the cube is

$V2 = \left(520 \pm 15\right) \text{ cm}^3$.

(iii) Find the relative error for the volume average value you found in the previous question C(ii)

Solution:

$$\frac{\delta V2}{V2} = \frac{15}{520} = 0.03 \text{ or } 3\%$$

D) In steps B) and C) we show two different methods for measuring the same thing. What is the value of the volume of the cube and its final error?

Solution:

$$w1 = \frac{1}{\delta V1^2} = \frac{1}{16^2} = 0.0039, \quad w2 = \frac{1}{\delta V2^2} = \frac{1}{15^2} = 0.0044$$

$$V = \frac{w1 \cdot V1 + w2 \cdot V2}{w1 + w2} = 520.000 \text{ cm}^3$$

$$\delta V = \sqrt{\frac{1}{w1 + w2}} = 10.943 \text{ cm}^3$$

So the final estimated value for the volume is

$$V2 = (520 \pm 11)\ \text{cm}^3$$

Vasileios Lembessis 1/12/2019 13:45

Comment [3]: You need to do this step. Also a student wrote that the first method of calculating the volume is more correct than the second one. This is wrong. They both have the same relative error! Also I did not ask this question.