PHYS 505 – Fall 2019
Homework 2-Solutions
Prof. Vasileios Lempesis
Hand in: Tuesday 8th of October 2019

1. A body slides down an inclined plane and we did the following ten measurements for the time of its motion.

Ten values of the time for a body to slide down an inclined plane										
Time (s)	0.64	0.64	0.59	0.58	0.70	0.61	0.68	0.55	0.57	0.63

Find the average value of the time, its error and quote the final result with the correct number of significant digits

Solution

The average value is 0.619 and the error is 0.0152. Thus we the rounded error is 0.015 and the average value is $(0.619 \pm 0.015)s$.

2. In survey for the annual income of the passengers travelling in a flight we found the following data

Number of passengers	Annual Income in SAR
5	800,000
10	600,000
15	400,000
15	350,000
40	250,000
55	150,000
15	100,000

Find the mean (average value), the mode, the median and the standard deviation of the above income distribution.

Solution

There are total N= 155 passengers. The probabilities of the given incomes are:

Vasileios Lembessis 9/10/2019 17:50
Comment [1]: This is a faster way when you have a large number of data.

Probability (N_i /N)	Annual Income in SAR
0.032	800,000
0.064	600,000
0.097	400,000
0.097	350,000
0.258	250,000
0.355	150,000
0.097	100,000

Thus the average value is

(0.032×800,000)+ (0.064×600,000)+ (0.097×400,000)+ (0.097×350,000)+ (0.258×250,000)+ (0.355×150,000)+ (0.097×100,000) =

25600+38400+38800+33950+64500+53250+9700=264,200 SAR

The mode is the income with the highest probability, so it is 150,000 SAR.

To find the median we have: The median is (N+1)/2 = (155+1)/2=78 the 78th recording has income 250,000 SAR so this is the median.

To find the standard deviation we have:

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(N_i - \overline{N})^2 = \frac{1}{155-1}\sum_{i=1}^{155}(N_i - \overline{N})^2 =$$

$$\frac{1}{154}\Big\{5\times\left(800{,}000-264{,}200\right)^2 + 10\times\left(600{,}000-264{,}200\right)^2 + 15\times\left(400{,}000-264{,}200\right)^2 +$$
$$15\times\left(350{,}000-264{,}200\right)^2 + 40\times\left(250{,}000-264{,}200\right)^2 + 55\times\left(150{,}000-264{,}200\right)^2$$
$$+15\times\left(100{,}000-264{,}200\right)^2\Big\}$$

$$=2.65\times10^{10}$$

$$s = \sqrt{2.65\times10^{10}} = 1.62\times10^5$$

3. A continuous probability distribution is given by:

$$f(x) = \begin{cases} \dfrac{2}{\sqrt{\pi}}\exp(-x^2) & 0 \le x \le \infty \\ 0 & else \end{cases}$$

Find the mean (average value), the mode, the median and the standard deviation of the above probability distribution.

$$\langle x\rangle = \int_0^\infty x f(x)\,dx = \int_0^\infty \frac{2}{\sqrt{\pi}} x\exp(-x^2)\,dx = \frac{1}{\sqrt{\pi}}$$
$$\langle x^2\rangle = \int_0^\infty x^2 f(x)\,dx = \int_0^\infty \frac{2}{\sqrt{\pi}} x^2\exp(-x^2)\,dx = \frac{1}{2}$$

The mode is the maximum value of the distribution

$$f^{'}(x)=\frac{2}{\sqrt{\pi}}\left[\exp(-x^2)\right]^{'}=-\frac{4x\exp(-x^2)}{\sqrt{\pi}}$$

This is equal to 0 at x=0. Thus the mode is $f(0)=\frac{2}{\sqrt{\pi}}$

The median is the value of x for which

$$\int_0^x f(x)=0.5\Rightarrow erf(x)=0.5$$

$x\approx 0.52$

The standard deviation is:

$$\sigma^2=\int_0^\infty\left(x-\langle x\rangle\right)^2 f(x)\,dx=\frac{2}{\sqrt{\pi}}\int_0^\infty\left(x-\frac{1}{\sqrt{\pi}}\right)^2\exp(-x^2)\,dx=\frac{2}{\sqrt{\pi}}\frac{1}{4}\frac{(\pi-2)}{\sqrt{\pi}}=\frac{(\pi-2)}{2\pi}=0.181$$

$$\sigma=\sqrt{0.181}=0.425$$

For integrations you may use also online tools like
https://www.integral-calculator.com/
https://www.wolframalpha.com/calculators/integral-calculator/

For the error function you may use

https://keisan.casio.com/exec/system/1180573449

Vasileios Lembessis 9/10/2019 11:04
Comment [2]: Some of you did not calculate the mode. The mode is the value of the distribution function at the point where it has a maximum (here at x=0)

Vasileios Lembessis 9/10/2019 11:05
Comment [3]: Some of you took that the integral of e^{-x^2} is e^{-x^2}. This is not true.