PHYS 500: Research Methodology

Excluding some results

Ignoring results-a

- Some times when we repeatedely measure the same quantity a result is quite different than the others. When this occurs the experimentalist has to decide whether this is a consequence of some mistakes during the measurement process and thus it must be ignored or it is an inevitable result and thus it must be taken into account.
- For example, let's consider 6 measurements of the period of a pendulum (in s) and we find:

  3.8 3.5 3.9 3.9 3.4 1.8

- In the above example 1.8 is much different than the rest. We must decide what to do with it.

Ignoring results-b

- We must point out that in the errors and measurements theory it is proved that such a result (if we assume that we have not done errors in the measurement process) **it is possible** all though its likelihood is small.
- When we are convinced for the correctness of our method we must take the final decision, which it cannot be arbitrary because it will considerably influence our result.
- For example, if we do not ignore the sixth result will be:

$$T = \left(3.4 \pm 0.3\right)s$$

- While if we ignore the sixth result we will get:

$$T = \left(3.7 \pm 0.1\right)s$$

The Chauvenet criterion-a

- One way to answer in the above problem is the use of the so called Chauvenet criterion. These criterion includes the following steps:

1. We use **all** the recordings (even the “bad” one).
2. We calculate the average value $\overline{x}$.
3. We find the standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(\overline{x} - x_i\right)^2}{N-1}}$$

4. We find the ratio μ of the absolute value of the difference of the “bad” value from the average value over the standard deviation:

$$\mu = \frac{\left|\overline{x} - x_j\right|}{\sigma}$$

μ	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,00	0,80	1,60	2,39	3,19	3,99	4,78	5,58	6,38	7,17
0,1	7,97	8,76	9,55	10,34	11,13	11,92	12,71	13,50	14,28	15,07
0,2	15,85	16,63	17,41	18,19	18,97	19,74	20,51	21,28	22,05	22,82
0,3	23,58	23,34	25,10	25,86	26,61	27,37	28,12	28,86	29,61	30,35
0,4	31,08	31,82	32,55	33,28	34,01	34,73	35,45	36,16	36,88	37,59
0,5	38,29	38,99	39,69	40,39	41,08	41,77	42,45	43,13	43,81	44,48
0,6	45,15	45,81	46,47	47,13	47,78	48,43	49,07	49,71	50,35	50,98
0,7	51,61	52,23	52,85	53,46	54,07	54,67	55,27	55,87	56,46	57,05
0,8	57,63	58,21	58,78	59,35	59,91	60,47	61,02	61,57	62,11	62,65
0,9	63,19	63,72	64,24	64,76	65,28	65,79	66,29	66,80	67,29	67,78
1,0	68,27	68,57	69,23	69,70	70,17	70,63	71,09	71,54	71,99	72,43
1,1	72,87	73,30	73,73	74,15	74,57	74,99	75,40	75,80	76,20	76,60
1,2	76,99	77,37	77,75	78,13	78,50	78,87	79,23	79,59	79,95	80,29
1,3	80,64	80,98	81,32	81,65	81,98	82,30	82,62	82,93	83,24	83,55
1,4	83,85	84,15	84,44	84,73	85,01	85,29	85,57	85,84	86,11	86,38
1,5	86,64	86,90	87,15	87,40	87,64	87,89	88,12	88,36	88,59	88,82
1,6	89,04	89,26	89,48	89,69	89,90	90,11	90,31	90,51	90,70	90,90
1,7	91,09	91,27	91,46	91,64	91,81	91,99	92,16	92,33	92,49	92,65
1,8	92,81	92,97	93,12	93,28	93,42	93,57	93,71	93,85	93,99	94,12
1,9	94,26	94,39	94,51	94,64	94,76	94,88	95,00	95,12	95,23	95,34
2,0	95,45	95,56	95,66	95,76	95,86	95,96	96,06	96,15	96,25	96,34
2,1	96,43	96,51	96,60	96,68	96,76	96,84	96,92	97,00	97,07	97,15
2,2	97,22	97,29	97,36	97,43	97,49	97,56	97,62	97,68	97,74	97,80
2,3	97,86	97,91	97,97	98,02	98,07	98,12	98,17	98,22	98,27	98,32
2,4	98,36	98,40	98,45	98,49	98,53	98,57	98,61	98,65	98,69	98,72
2,5	98,76	98,79	98,83	98,86	98,89	98,92	98,95	98,98	99,01	99,04
2,6	99,07	99,09	99,12	99,15	99,17	99,20	99,22	99,24	99,26	99,29
2,7	99,31	99,33	99,35	99,37	99,39	99,40	99,42	99,44	99,46	99,47
2,8	99,49	99,50	99,52	99,53	99,55	99,56	99,58	99,59	99,60	99,61
2,9	99,63	99,64	99,65	99,66	99,67	99,68	99,69	99,70	99,71	99,72
3,0	99,73									
3,5	99,95									
4,0	99,994									
4,5	99,9993									
5,0	99,99994									

The values in the table are those of the integral right which cannot be solved analytically.

$$I = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\mu\sigma}^{\mu\sigma} e^{-x^2/2\sigma^2}\, dx$$

The values in the table are given in %

The Chauvenet criterion-b

4. From the previous table we find the probability $P(<\mu)$ to have a recording which is away from the average value less than the “bad” value.

5. We find the probability $P(>=\mu)$ to have a recording that is away from the average more or equal to the “bad” value: $P(>=\mu) =1-P(<\mu)$.

6. We multiply the quantity $P(>=\mu)$ with the number of recordings N and we find the result u. Then:

a) If $u<0.5$ then we reject the “bad” value. We find the new average value of the rest N-1 recordings and its error.

b) If $u>=0.5$ we keep the “bad” value, the average value and its error which we had already found.

The Chauvenet criterion-c

- EXAMPLE: Let's consider 6 measurements of the period of a pendulum (in s) and we find:

  3.8 3.5 3.9 3.9 3.4 1.8

- EXAMPLE: Let's consider 10 measurements of the velocity of an object (in m/s) and we find:

  46 48 44 38 45 47 58 44 45 43