PHYS 500: Research Methodology

Theory of Errors – What is an error in a physical measurement?

Measurement in Physics

- An essential part and ultimate aim of an experiment is the **measurement**.
- By measuring a physical quantity we actually mean its **comparison** with a similar physical quantity which is taken as a **prototype** or **unit**.
- For example we can use the length of our foot if we wish to measure the length of an object.

Errors-a

- No measurement is exact; there is always some **uncertainty** due to limited instrument accuracy and difficulty reading results.
- The photograph to the right illustrates this – it would be difficult to measure the width of this object to better than a **millimeter**.

Errors-b

- The term **error** means: the inevitable lack of accuracy in the measurement of any physical quantity in all the experiments as well as the imperfections of our instruments or methods.
- Example: Three different ways to measure the height of a door.
- A) An experienced carpenter just using his eyes says that the height is around 210 cm plus or minus 5 cm. If we write down his estimation in a "scientific way" the height is $\left(210 \pm 5\right) cm$.
- B) The carpenter then uses his meter and finds that the height of the door is 211.3 cm. Taking into account factors like contraction/compression due to heating or not perfect matching of his meter to the door we can write that the height of the door is $\left(211.30 \pm 0.05\right) cm$
- C) The owner of the house is a physicists and uses an interferometer of high accuracy and takes the following recording for the height of the door $\left(211.3001580 \pm 0.0000005\right) cm$

Errors-c

- **Question**: What is then the **“correct”** measurement for the height of the door?
- ***Answer***: All of them!!!! Their difference is the size of the accuracy.
- **Question**: What is then the height of the door?
- ***Answer***: It depends on the reason for which we need to know the height of the door. Case A may cause trouble to the carpenter. Case B seems OK for him. Case C contains information which is useless for a carpenter.

Errors-d

- As we shall see later some types of errors can be avoided. But we cannot do an experiment with no errors at all.
- **Question**: Are the errors in an experiment fundamentally unavoidable?
- ***Answer:*** Although we can build experimental devices of high accuracy there is a fundamental reason. This is the limit imposed by Heisenberg's uncertainty principle $\Delta x \cdot \Delta p \geq \hbar / 2\pi$.

Errors-e

- **Question:** Why we should know the errors in a measurement?
- ***Answer:*** Many times (especially when a recording is very close to a value given in the literature) we think that it is not necessary to quote the error of the measurement. Unfortunately if we do not know the errors we can not be sure about the results of our measurements.
- **Example:** Let's assume that we perform an experiment to investigate whether the resistance of a coil depends on temperature. We do two measurements and we get the following recordings:

$$200.025 \text{ Ohm at } 10^0 \text{ C}$$

$$200.034 \text{ Ohm at } 20^0 \text{ C}$$

Errors-f

- **Question:** Is there any difference in these two recordings?
- ***Answer***: If we do not know the error we cannot answer.
- If the error is 0.01 Ohm we cannot answer and we have to increase the accuracy of our experiment. If, on the contrary, the error is 0.001 Ohm we can say that these measured values are indeed different.
- **Example:** Which of the values given below are equal? a_1 and a_2 or b_1 and b_2?

$$a_1 = 3.62,\ \ a_2 = 3.38$$
$$b_1 = 2.820,\ \ b_2 = 2.880$$

- The obvious answer is that b_1 and b_2 are almost equal while a_1 and a_2 are not. But what about if we take into account the errors?

Errors-g

- If the values are given with their errors as follows,

$$a_1 = 3.62 \pm 0.29, \quad a_2 = 3.38 \pm 0.26$$
$$b_1 = 2.820 \pm 0.006, \quad b_2 = 2.880 \pm 0.008$$

then it is obvious that our previous conclusion is completely reversed!

Accuracy and Precision

- It is very important to distinguish between the terms **accuracy** and **precision**.
- The **accuracy** of an experiment is a measure of how close the result of the experiment is to the true value.
- The **precision** is a measure of how well the result has been determined, without reference to its agreement with the true value. The precision is also a measure of the reproducibility of the result in a given experiment.

Random Errors

- Always in an experiment there is a difference between the **recorded** value and the “perfect” or “ correct” or “theoretical” value.
- Errors are classified as **random** or **systematic**.
- A) Random errors are evident when repeated measurements of the same quantity in the same situation give different readings. Sources of random errors include

  a) The readability of the instrument

  b) The abilities of the observer

  c) Changes in the surroundings
- **If the random errors result from instrumental uncertainties we may reduce them by using more precise and reliable instruments.**
- **If they come from from statistical fluctuations because of a limited number of measurements then repeating readings reduce random errors.**
- **Random errors can be positive or negative. They are always present in an experiment**.

Systematic Errors

- Sources of systematic errors include:

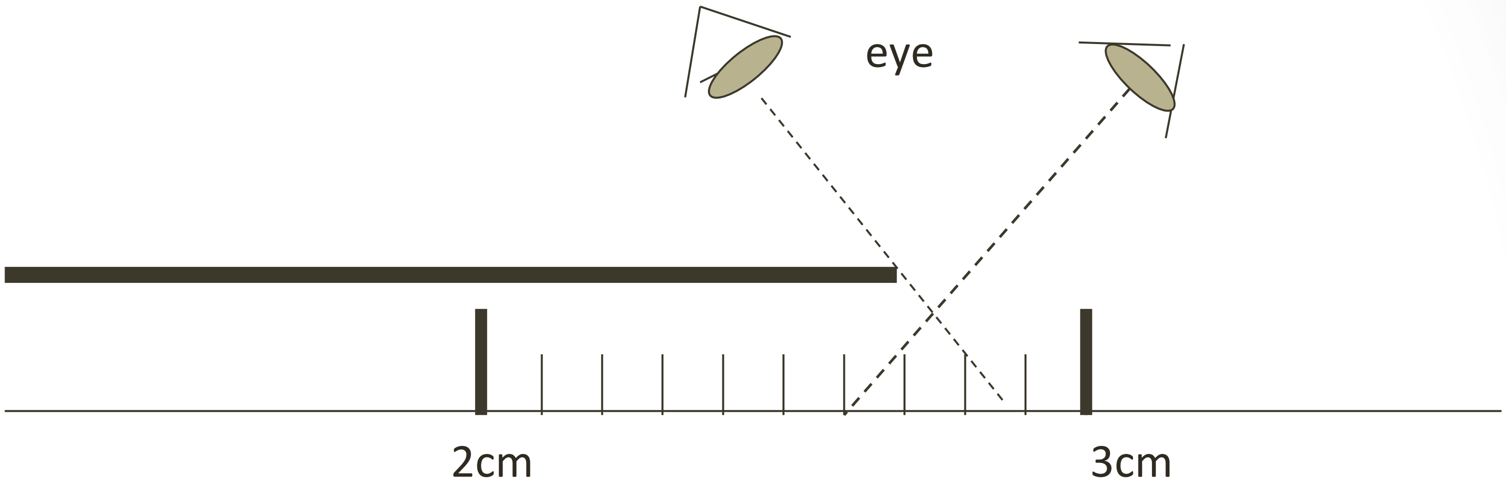
  a) An instrument with **zero** error

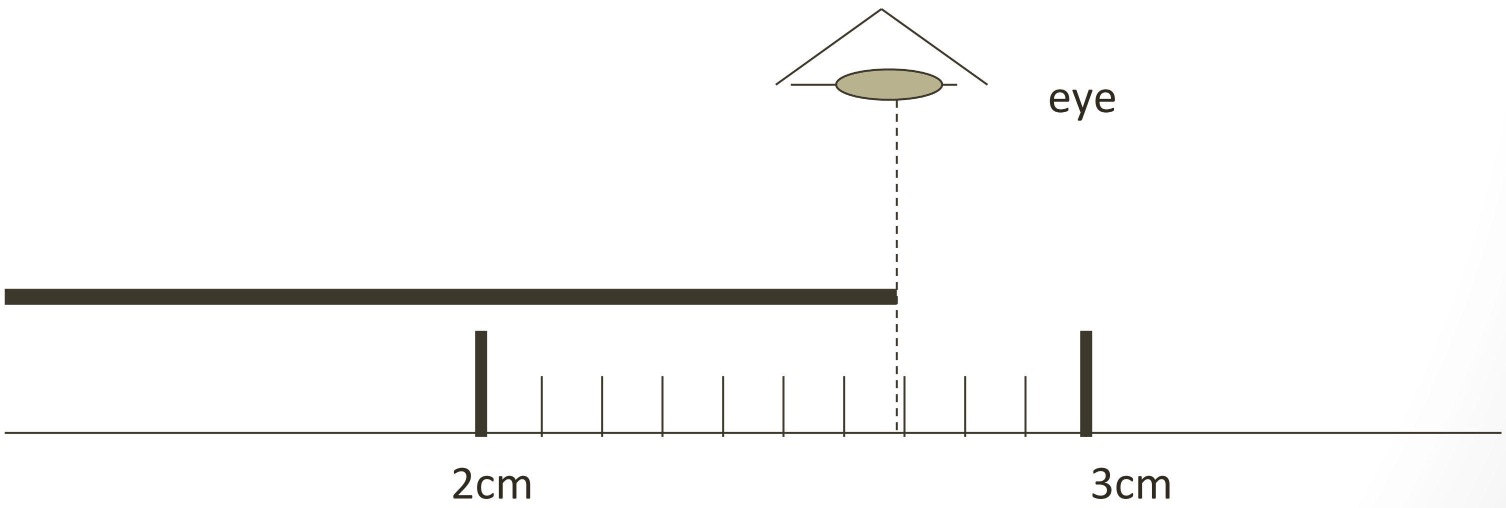
  b) An instrument being wrongly **calibrated**

  c) The observer being less than perfect in the same way every measurement

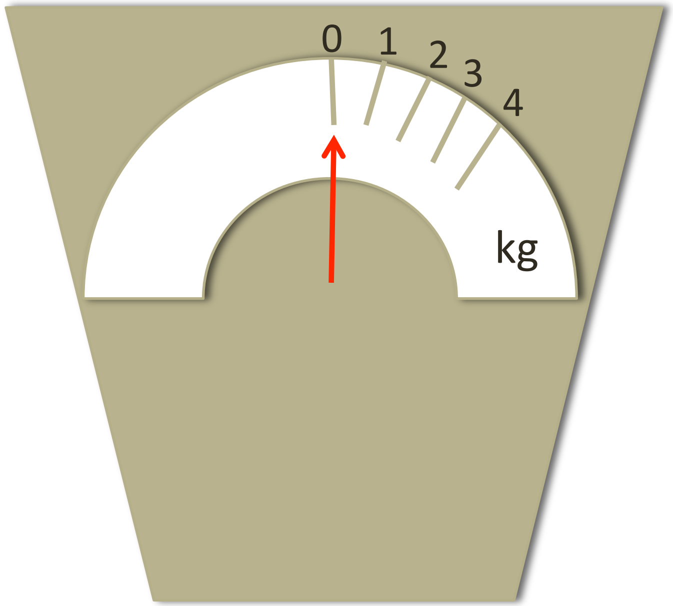
Repeating readings do not reduce systematic errors!

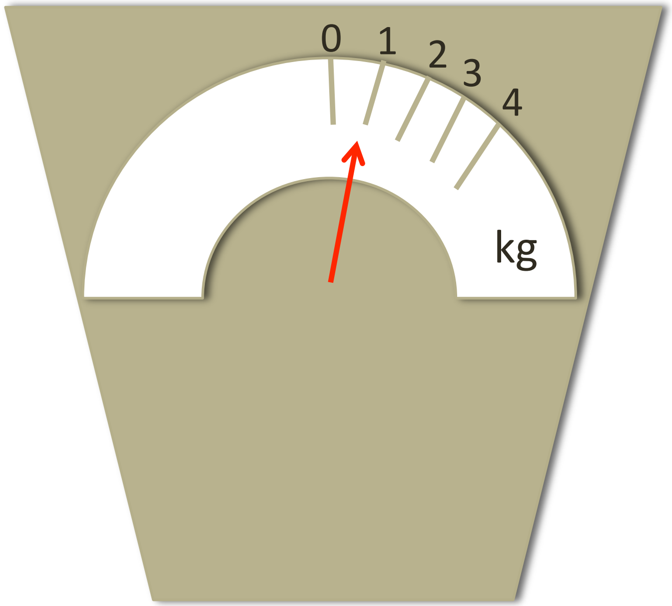
The parallax error: An example of random error





An example of systematic error



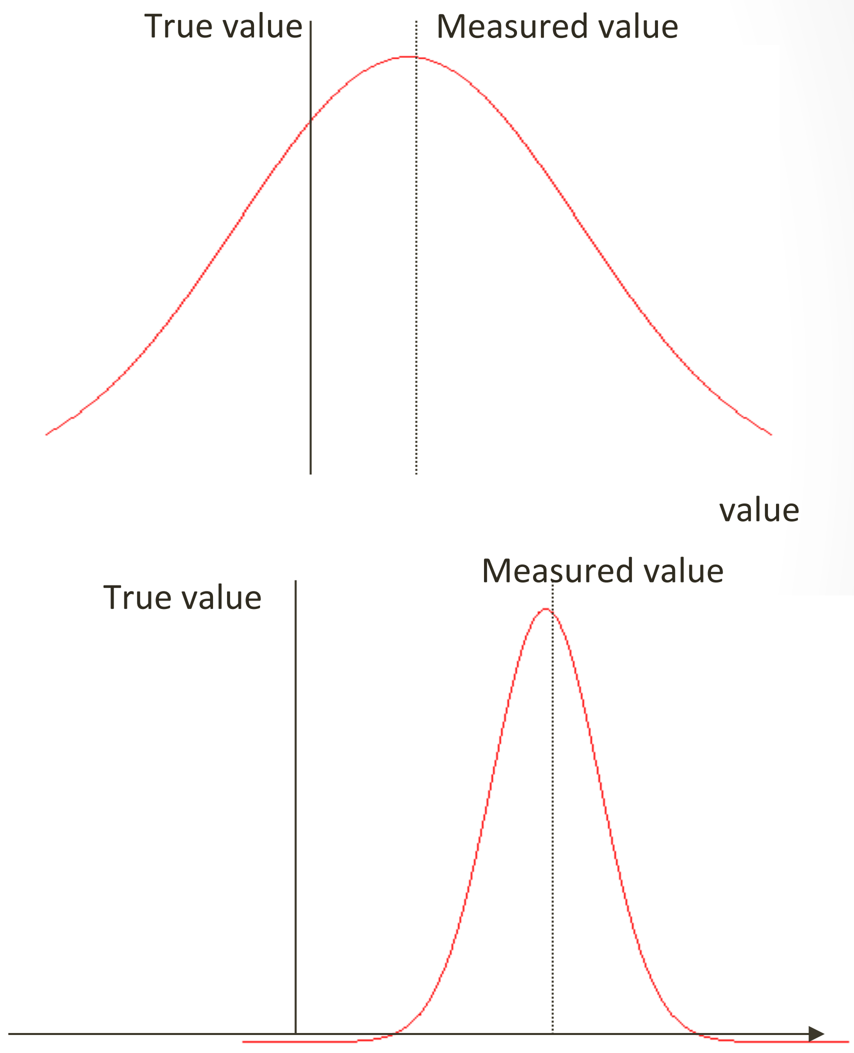


Weighting scale with correct calibration

Weighting scale with wrong calibration. The reading will have a systematic error of 1 kg.

Difference between Accuracy & Precision

- An **accurate** experiment is one that has a small systematic error
- A **precise** experiment has is one that has a small random error
- The first graph shows an accurate experiment of low precision, while the second shows a less accurate but more precise experiment



Difference between Accuracy & Precision

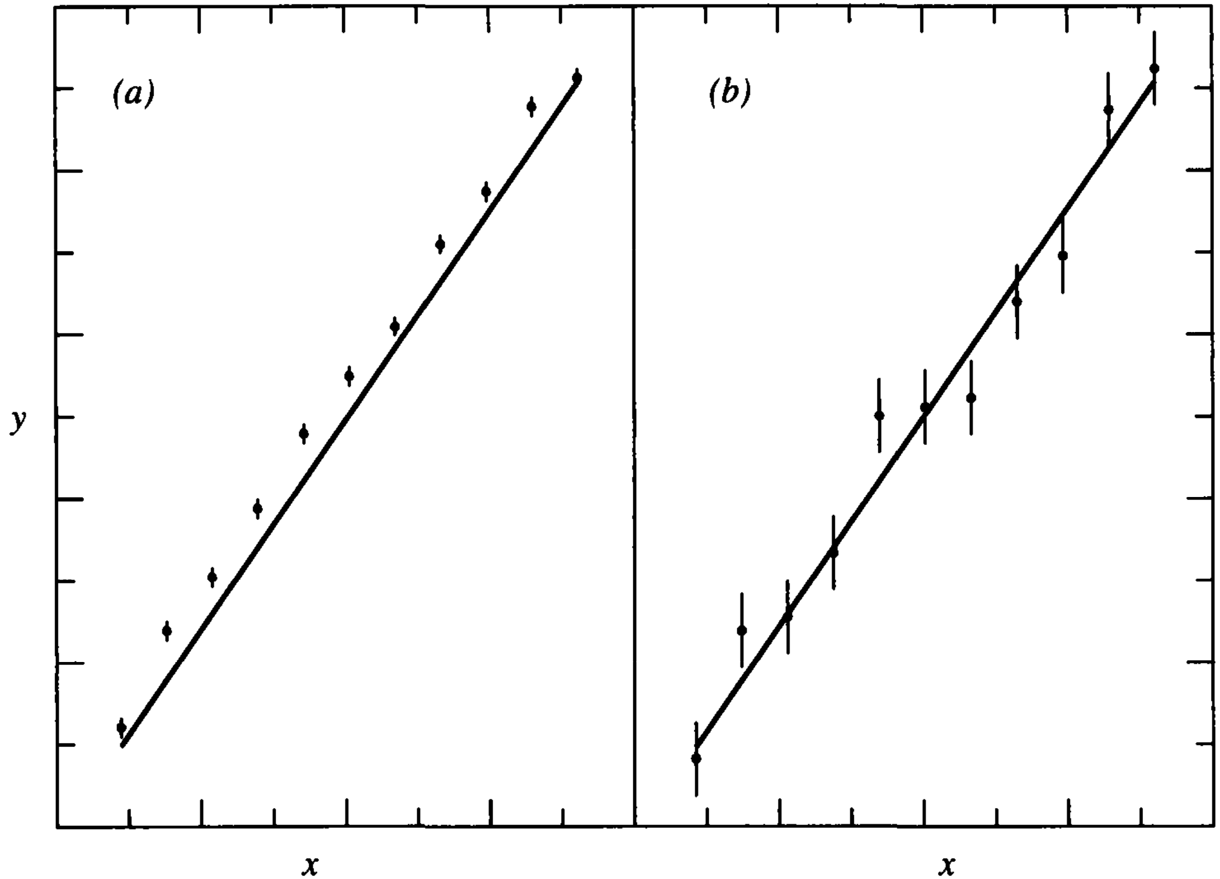
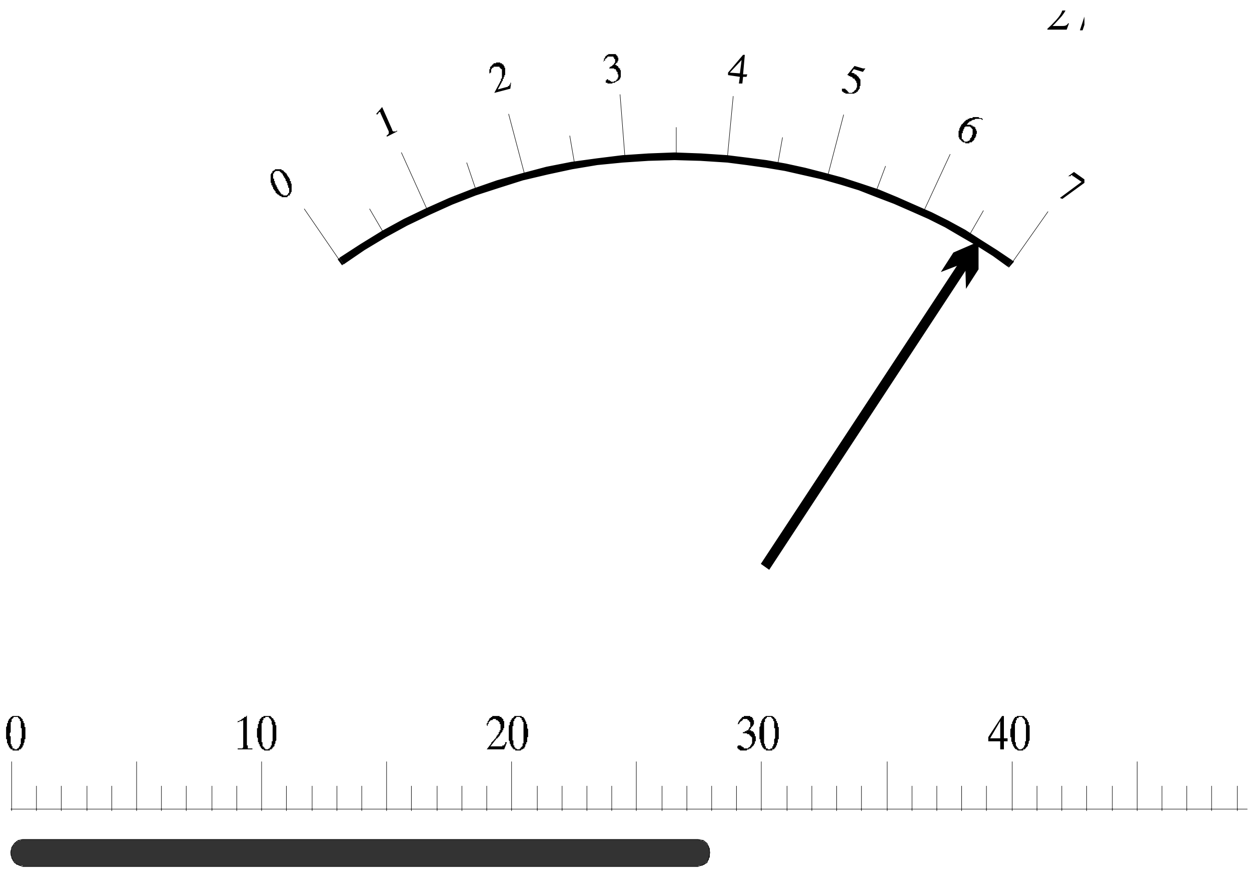


Illustration of the difference between precision and accuracy. (*a*) Precise but inaccurate data. (*b*) Accurate but imprecise data. True values are represented by the straight lines.

Errors when reading an instrument-a



Errors when reading an instrument-b

- An important class of errors is the error we do when we read an instrument. Let's assume we wish to read the voltage in (a) or the length of the rod (b). There are no strict rules for this. We can consider it as equal to half or one division of the instrument. So we write as our answers:

$$l = 28.0 \pm 0.5 \ cm$$

$$V = 6.7 \pm 0.1 \ \ V$$

- Note 1: Every instrument has an error given by its manufacturer. We must compare this to the reading error in order to check which one we are going to consider. **If it is of the same order as the reading error we must ignore the reading error.**
- Note 2: We must not confuse the reading error with the parallax error. The parallax error may be eliminated by placing a mirror behind the pointer. If we look at the pointer in such a way that its image on the mirror is totally covered by the pointer then we have eliminated the parallax error.

Errors when reading an instrument-c

- If the instrument used is a digital one then we have two case:

a. If the recorded value is not constant because the last digit "changes" continuously then our recording is the average value of what we see and the "width" of the last digit changes is taken as the error.

b. If the recorded value is constant then as error we take the 0.5 of the last digit. The same we do if we use an electronic calculator.

Errors when reading an instrument-c

$$V = (234.5 \pm 0.5)\ V$$

Significant Figures-a

- The precision of an experimental result is implied by the number of digits recorded in the result, although generally the uncertainty should be quoted specifically as well. The number of **significant figures** in a result is defined as follows:
- The leftmost non-zero digit is significant and is in fact the most significant digit in the number.
- If the number has no decimal point, the right most non-zero digit is significant.
- If the number does have a decimal point, the least significant digit is the rightmost digit (which may be zero).
- The number of significant digits of a number is the number of digits from the most to the least significant digit.

Significant Figures-b

- For example all the following numbers each have four significant digits: 1234, 123,400, 123.4, 1001, 1000., 10.10, 0.0001010, 100.0.
- If there is no decimal point, there are ambiguities when the rightmost digit is 0. Thus, the number 1010 is considered to have only three significant digits even though the last digit might be physically significant. To avoid ambiguity, is better to supply decimal points or to write such numbers in **scientific notation**, that is, as an argument in decimal notation multiplied by the appropriate power of 10. Thus, our example of 1010 would be written as 1010. or 1.010×10^{3} if all four digits are significant.

Significant Figures-c

- When quoting an experimental result, the number of significant figures should be approximately one more than that dictated by the experimental precision. The reason for including the extra digit is to avoid errors that might be caused by rounding errors in later calculations.
- **Question:** When we find a result after processing our data how many decimal points shall we keep? For example let's say we have found the average value of the length of a cylinder to be x=7.333333 *mm.* How many decimal digits we must keel?
- **Answer:** We must know the error of our measurements. In this case we round the error up to one non-zero digit. For example if the error in the above measurement is δx=0.06273273. Then we round it to δx=0.06 (unless the first non-zero digit is 1 or 2 where we keep two digits in the error). And from our measurement we shall keep the decimal of the same order by doing rounding. Thus we write

$$x = 7.33 \pm 0.06$$

Significant Figures-d

- **When multiplying or dividing, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures.**
- Example: 25.57 m x 2.45 m = 62.6 m^2
  - The 2.45 m limits your result to 3 significant figures
- **When adding or subtracting, the number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.**
- Example: 135 cm + 3.25 cm = 138 cm
  - The 135 cm limits your answer to the units decimal value

Roundoff-a

- When insignificant digits are dropped from a number, the last digit retained should be rounded off for the best accuracy. To round off a number to fewer significant digits than were specified originally, we truncate the number as desired and treat the next digits as follows:

1. If the next digit is larger than 5 then we increase the last digit we wish to keep by 1 and all the rest are put equal to zero.
2. If the next digit is smaller than 5 then we keep the last digit we wish to keep as it is and all the rest are put equal to zero.
3. If the next digit is equal to 5 we check if there is after this any non-zero digit at any position. If there is then we increase the last digit we wish to keep by 1 and all the rest are put equal to zero. If there is no then we do anything we like.
4. If the next digit is equal to 5 and there are no digits after it then we could increase the last digit we wish to keep by 1 or keep it the same. If we have a lot of cases like this in half of them we increase the last digit and in the other half we keep it the same.

Roundoff-b

- In a calculations involving several arithmetic steps, it is very good advice not to round numbers until all the calculations have been completed; otherwise the rounding process itself can have a large effect on the numbers that emerge from the calculations

Significant Figures & Scientific Notation-a

- It is not always clear how many figures in a number are significant. By changing the unit in which a number is expressed, it can appear that the number of significant figures changes. For example, suppose in an experiment a time interval was recorded as 346s. We could choose to write the time in other units such as milliseconds or microseconds. These would be written as 346000ms and 346000000 μs, respectively. In both cases the number of significant figures remains as three.
- However, if someone asked you for your value for the time interval to be expressed in ms, how would they know that, in your value of 346 000 ms, only the first three figures were significant? It is possible that you used a timing device capable of a resolution of 1 ms and that the time interval came out, to the nearest millisecond, as 346 000 ms; that is, all six figures are significant.

Significant Figures & Scientific Notation-b

- The way to get around this difficulty is to present numbers in scientific notation . In scientific notation the first non-zero figure that appears is followed by a decimal point, so that 346 becomes 3.46. To bring the number back to its original value we must multiply 3.46 by 100 or 10^2 . We can now write the time interval as 3.46×10^2 s. In terms of milliseconds and microseconds this becomes 3.46×10^5 ms and 3.46×10^5 μs, respectively
- The number of significant figures is equal to the number of figures that appear to the left of the multiplication sign. In situations where a number lies between 1 and 10, for example 7.15, we could write this as 7.15×10^0. Though this is technically correct, it is far more usual for the number to be expressed simply as 7.15. Table 2.7 contains a variety of numbers and their representation in scientific notation (we assume here that all the figures given are significant).

Examples of numbers expressed in scientific notation	
Number	Scientific Notation
12.65	1.265×10^{1}
0.00023	2.3×10^{-4}
342.5	3.245×10^{2}
34001	3.4001×10^{4}

On-Line Quizes

Below you can find nice on-line quizes on significant figures.

- http://www.dallassd.com/our%20schools/high%20school/chemsite/chem/hotpot/sf.htm
- http://ths.sps.lane.edu/chemweb/unit1/problems/significantfigures/
- http://slc.umd.umich.edu/slconline/SIGF/lastpage.html
- http://www.sciencegeek.net/APchemistry/APtaters/sigfigs.htm
- http://www.chemistrywithmsdana.org/wp-content/uploads/2012/07/SigFig.html