PHYS 301
HANDOUT 8

1. Expand in Laurent series around the point $z_0 = 0$ the function $f(z) = e^{1/z}$.

   (Ans: $\sum_{n=0}^{\infty} \frac{(1/z)^n}{n!}$)

2. Expand in Laurent series around the point $z_0 = i$ the function $f(z) = 1/(z-i)^2$.

3. Expand in Laurent series the function $f(z) = \frac{-1}{(z-1)(z-2)}$.

4. Find the Laurent series of the function $f(z) = 1/(1+z)$ for $|z| > 1$.

5. Find the Laurent series for the function $f(z) = \frac{1}{(z+1)(z+3)}$ for a) $1 < |z| < 3$, b) $|z| > 3$, c) $0 < |z+1| < 2$, d) $|z| < 1$.

6. Find the first two, non-zero, terms of the Laurent series of the function $f(z) = \tan z$ around $z = \pi/2$.

7. Show that $1/z^2 = \sum_{n=0}^{\infty} (-1)^n (n+1)(z-1)^n$ (for $|z-1| < 1$).

8. Laurent series holds in a circular sector $r_1 < |z - z_0| < r_2$ for a given point z_0. The power series coefficients a_n are unique. If z_0 changes then the coefficients a_n change and the series takes a different form. Study the Laurent series expansion of the function $f(z) = 1/[z(z-1)]$ around $z_0 = 0$, $z_0 = 1$ and $z_0 = i$ respectively.