PHYS 301
HANDOUT 7

1. **1.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = e^z$.

   (Ans: $\sum_{n=0}^{\infty} \frac{z^n}{n!}$)

2. **2.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = z^2 e^{3z}$.

   (Ans: $\sum_{n=0}^{\infty} \frac{3^n}{n!} z^{n+2}$)

3. **3.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = \sin z$.

   (Ans: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$)

4. **4.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = \sinh z$.

   (Ans: $\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}$)

5. **5.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = \cos z$.

   (Ans: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n}$)

6. **6.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = \cosh z$.

   (Ans: $\sum_{n=0}^{\infty} \frac{1}{(2n)!} z^{2n}$)

7. **7.** Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = 1/z$.

   (Ans: $\sum_{n=0}^{\infty} z^n, \ (|z| < 1)$)

8. **8.** Could we have a Maclaurin series for the function $f(z) = (1+2z^2)/(z^3+z^5)$? Discuss this case.

9. **9.** Expand in Taylor series around the point $z_0 = 0$ the function $\ln(1+z)$.

(Ans: $\sum_{n=0}^{\infty}(-)^{n-1}\frac{z^n}{n}$)

10. If a function $f(z)$ can be expanded in a Taylor series around the point $z_0 = 0$ and the coefficients a_n are all real, show that $f^*(z) = f(z^*)$.

11. Expand in Taylor series around the point $z_0 = 0$ the function $f(z) = (1+z)e^z$.

(Ans: $\sum_{n=0}^{\infty}\frac{n+1}{n!}z^n$)

12. Expand in Taylor series around the point $z_0 = 2$ the function $f(z) = 1/z^2$.

(Ans: $\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$)

13. Expand in Taylor series around the point $z_0 = 2$ the function $u(x,y) = e^x \cos y$. Express your result in polar coordinates.

14. Expand the function $f(z) = 1/(1+z^2)$ around the point $z_0 = 0$. (Hint: use the formula $1/(1-q) = \sum_{n=0}^{\infty} q^n$.

15. Prove that an odd function has an odd Taylor series representation.

16. Prove that an even function has an even Taylor series representation.

17. Derive the Taylor series for the function $f(z) = (1+i)z^2 - 2z + 4i$ around the point $z_0 = 1 - i$.

18. Prove de l' Hospital rule applying the Taylor expansion.