PHYS 301
HANDOUT 5
Dr. Vasileios Lempesis

1. Show that the Average Value Theorem is not valid in the complex numbers analysis.

2. Calculate the integral $\int_0^1 (1+it)^2\, dt$.

3. Calculate the integral $\int_0^{\pi/4} e^{it}\, dt$.

4. Calculate the integral $\int_1^2 \left(\dfrac{1}{t} - i\right)^2 dt$.

5. Calculate the integral $\int_0^{\pi/6} e^{i2t}\, dt$.

6. Calcluate the integrals $\int_1^{\pi} e^x \cos x\, dx$, $\int_1^{\pi} e^x \sin x\, dx$ by using the following relation:

$$\int_0^{\pi} e^{(1+i)x}\, dx = \int_0^{\pi} e^x \cos x\, dx + i\int_0^{\pi} e^x \sin x\, dx$$

7. Let the complex function $w(t) = u(t) + iv(t)$ which is defined in the interval $[-a,\, a]$.
   (a) If $w(t)$ is even (i.e. $w(-t) = w(t)$ in this interval) then show that

$$\int_{-a}^{a} w(t)dt = 2\int_0^a w(t)dt.$$

   (b) If $w(t)$ is odd (i.e. $w(-t) = -w(t)$ in this interval) then show that

$$\int_{-a}^{a} w(t)dt = 0$$