PHYS 301
HANDOUT 3A
Dr. Vasileios Lempesis

1. Use the definition of the derivative to calculate the derivative at the corresponding point: a) $f(z)=3z^2+4iz-5+i$, at $z=2$, b) $f(z)=(2z-i)/(z+2i)$, at $z=-i$, c) $f(z)=3z^{-2}$, $z=1+i$.

2. Show that the derivative $d(z^2\overline{z})/dz$, does not exist.

3. Verify that the real and imaginary parts of the following functions satisfy the Cauchy-Riemann conditions: a) $f(z)=z^2+5iz+3-i$, b) $f(z)=ze^{-z}$, c) $f(z)=\sin(2z)$.

4. Verify that the real and imaginary parts of the following functions satisfy the Cauchy-Riemann conditions: a) $f(z)=e^{z^2}$, b) $f(z)=\cos(2z)$, c) $f(z)=\sinh(4z)$.

5. a) Show that the function $u=2x(1-y)$ is harmonic. b) Find a function *v* such that the function $f(z)=u+iv$ is analytic. c) Express $f(z)$ as a function of *z*.

6. Find the orthogonal curve to the curve $x^3y-xy^3=a$.

7. Separating the function $f(z)=z+1/z$ in a real and imaginary part, show that the familes $(r^2+1)\cos\theta=ar$ and $(r^2-1)\sin\theta=\beta r$ represent orthogonal curves.

8. Show that $\nabla^2\left|f(z)\right|^2=4\left|f'(z)\right|^2$. Verify this relation for $f(z)=z^2+iz$.

9. Show that (a) $\frac{\partial}{\partial x}=\frac{\partial}{\partial z}+\frac{\partial}{\partial\overline{z}}$, (b) $\frac{\partial}{\partial y}=i\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial\overline{z}}\right)$.

10. Show that $\nabla=\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}=2\frac{\partial}{\partial\overline{z}}$.

11. Show that $\overline{\nabla}=\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}=2\frac{\partial}{\partial z}$.