

PHYS 301
HANDOUT 3
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1. Find the derivative of the complex function $f(z) = z^2$.
2. Check if the function $f(z) = z + (z - z^*)/2$ is differentiable.
3. Check if the function $f(z) = |z|^2$ is differentiable.
4. The complex function $f(z) = z^2$ is differentiable. Verify the Cauchy-Riemann conditions.
5. Discuss the Cauchy-Riemann conditions for the complex function $f(z) = |z|^2$.
6. For the complex function $f(z) = e^x(\cos y + i \sin y)$, show that $f'(z) = f(z)$.
7. Find the derivative of the complex function $f(z) = \frac{1}{z} = \frac{1}{r} e^{-i\theta}$.
8. The function $f(z) = x + iy$ is analytic. What happens with the function $f(z) = z^*$? (Ver. 29)
9. The function $f(z) = u(x, y) + iv(x, y)$ is analytic. What happens with the function $f^*(z^*)$?
10. If there is some common region in which $w_1 = u(x, y) + iv(x, y)$ and $w_2 = w_1^* = u(x, y) - iv(x, y)$ are both analytic, prove that $u(x, y)$ and $v(x, y)$ are constants. (Arf. 364)
11. The function $f(z)$ is analytic. Show that the derivative of $f(z)$ with respect to z^* vanishes.
Hint. Use the chain rule and take $x = (z + z^*)/2$, $y = (z - z^*)/2i$.
Note: This result emphasizes that our analytic function $f(z)$ is not just a complex function of two real variables x and y . It is a function of the complex variable $z = x + iy$. (Arf. 365)
12. Show that the function $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. (b) What is the function v , such that $f(z) = u + iv$ is to be analytic? (Sch. 73).
13. You are given the complex function $f(x, y) = x^2 + ixy$. Find the function $f(z)$.

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